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Perception – a cognitive component of the self-organization of mathematics learning process

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Abstract. The article considers one of the main mental processes - perception by students of educational content in mathematics, having an abstract character, through which its initial assimilation is achieved, as well as perception of the process of solving a certain type of problems. This is done from the point of view of the reflexive approach, aimed at realizing the ideal purpose of education – self-actualization of the subjects – both trainees and trainers. For this purpose, different views of perception are analyzed and illustrative examples from the teaching practice are presented. These examples aim a more in-depth and conscious knowledge to be achieved by the trainees. The paper is intended for current teachers and students who are preparing for mathematics teachers.
Keywords: *perception, education in mathematics, knowledge, problems.*

Introduction. Self-organization, as one of the main components of the reflective approach, related specifically to the teaching of mathematics and aimed mainly at the formation of praxiological reflection[1] in learners (pupils, university students, PhD students), and even teachers, as well as in order to achieve the ideal goal of education - achieving self-actualization of the subjects, includes the mental processes of perception, comprehension, generalization, understanding and memorization. The expected final result is the achievement of a new positive structure of the „trainer - trainee" system. Here, self-actualization is understood as the aspiration of the subject (pupil, student, etc.) to the full use of his possibilities in order to reach „alone (without outside help) the peaks of his spiritual and creative potential. At the same time, the conscious choice of goals in favor of growth – towards self-actualization, is carried out in problematic situations" (Vasilev, Dimova & Kolarova-Kancheva, 2005).According to psychologists, the activity of each person is determined by the depth of his psyche. The psyche is a property of thought to reflect the objective world in the human mind. From an educational and scientific point of view, it is interesting to trace the process of building a person's psyche. For this purpose, in several articles we have considered and analyzed the mental processes of perception, comprehension, generalization, understanding, memorization, which are essential in the realization of reflexivity in the learning process in order to achieve optimal learning of the studied content.

The purpose of the article is to consider the process of **perception** of learning content in teaching mathematics at school.

Materials and methods. Lerner's statement that „Learning, as an interaction between the trainer and the trainees, is conditioned both by its purpose to ensure the assimilation by the young generation of the social experience gained by society, embodied in the content of education, and by the opportunities of the trainees at the time of training" (Lerner, 1981), is still relevant today.

Perception occupies an important place in the cognitive activity of person, in particular the learning subject. This is especially true for the education in mathematics, because mathematical concepts and statements have a high degree of abstraction and there are complex logical connections between them. Krutecki also says that „to deny the importance of active, organized, systematic perception, of observation in the learning process would be a mistake. In mastering knowledge, students observe specific objects and phenomena, their images, acquire ideas" (Krutetski, 1976).

In the twentieth century, significant changes took place in psychology, perception is no longer seen as a combination of atomic sensory sensations, but began to be understood as a structural and integral phenomenon. For example, the psychologist J. Gibson interprets perception as „an active process of appropriating information from the world, which includes a real study of perceived information"[2].

Generally said, perception is a mental process that provides information about the observed objects and phenomena that people seek to know. For example, the psychologist T. Trifonov believes that „perception is a cognitive mental process that reflects the properties of objects from the outside world in their immediate impact; perception reflects objects and phenomena in general" (Trifonov, 2018). In other words, expressed through the perception in the human mind are reflected as spatial characteristics of the objects – shape, size, relief; as well as qualitative characteristics of the objects – color, density, taste, smell, as well as intensity characteristics – the force with which the objects affect its sensory organs. As a result of this cognitive mental process, a reflected mental image of the respective object arises in the human brain. For the teaching of mathematics, and especially of geometry, the spatial characteristics of the studied objects are of main importance.

Physiological basis of perception are analyzers. Perceptions perform mainly an orienting and regulating

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[1] V. Vassilev defines praxiological reflection as „reflections through which the subject selects the necessary and most appropriate knowledge to carry out a practical activity, mental procedures through which prepares, regulates and controls the transformation of this knowledge into a means of solving professional and life practical tasks ...; the regulation, control and comprehension of the effectiveness of the use of pragmatized knowledge and actions ... and all this is constantly correlated with the peculiarities of the thinking and acting subject" (Vasilev, 2006, p. 181).

[2] Perception in psychology. https://bg.psichiatria.org/vzpriyatie/#i

function in the practical activity and behavior of the person. Through perceptions, various data, which come from all our senses are synthesized and organized. Perception is one of the necessary conditions for the implementation of the learning process. Of course, human perception is not only a sensory image, but also an awareness of the perceived object, because the past experience that the learner has acquired and possesses also participates in the perception.

Initially, in the early childhood, perception is not purposeful. With advancing age, during the school period, the basic functions of the psyche (cognitive, regulatory, creative and reflexive) gradually increase their capabilities, as a result of which mental processes, incl. and perception, are strengthened and carried out more consciously and purposefully. As an active mental process, perception exits then and insofar as the certain stimulus presents and acts, which affects the respective receptor.

There are different ways of perceiving information. Especially for the teaching of mathematics the most important are the visual, auditory and logical way, and less – the tactile – in people with special educational needs.

The visual perception of space is based on the perception of the size and shape of the object, due to the synthesis of visual, muscular, tactile sensations. The integrity, the complexity of perception gives more information, in contrast to the sensation, which reflects only one individual characteristic of the object. Perception is not just a summation of human sensations and it doesn't react immediately to them. The subject perceives a generalized structure, which is essentially isolated from sensations and is formed at a certain time. Human perception has a very close connection with his thinking. „Although the process of perception arises under the influence of the direct impact of the object on the human senses, perception always has a semantic meaning“[3]. Therefore, meaningful perception plays an important role.

The ways in which people perceive information largely determine the quality and durability of its assimilation by the subject. Of course, usually, there are no people who receive all new knowledge entirely and only with the help of one sensory organ or even a group of sensory organs, because they always act as a complex means of perceiving information. Therefore, knowing which senses have a dominant role in a student, significantly increases the ability to provide him in the optimal way the necessary information from the teacher or other participants in the learning process, and the student can more effectively organize and implement the process of their self-learning. For example, for students with a predominance of visual perception, it is appropriate to present the information in a more visual form, using, for example, figures, models, drawings, diagrams, schemes, formulas and etc.

With a few examples in the article, we will illustrate different ways to help students perceive new learning content that is abstract in nature, or to provide a more accessible perception of the problem-solving process.

Example 1. For easier perception by students in 7th grade of the formula $(a+b)^2 = a^2 + 2ab + b^2$, it is appropriate to illustrate it in the way shown in fig. 1.

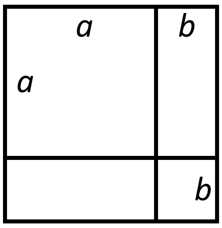

Fig. 1.

Here, the „outer“ square, which contains the two rectangles and the two squares having lengths of their sides a and b, respectively, obviously has a side with length $(a+b)$. Then the left side of the formula above expresses the face of the „outer“ square, the first and last addend of its right side – the faces of the two „inner“ squares, respectively, and the middle addend represents the sum of the faces of the two rectangles. Thus, since the geometric statement is fulfilled: „the face of the „outer“ square of the considered composite figure is equal to the sum of the faces of its constituent two squares and two rectangles“, it can be said, that this illustrative model is essentially one geometric proof of the considered formula, which was used in antiquity.

Thus, through this illustration, not only a better perception of the formula for the square of the sum of two addends is achieved, but also the inverse formula: $a^2 + 2ab + b^2 = (a+b)^2$ for representing a quadratic trinomial of addends through an exact square, and its more complete and in-depth understanding and assimilation is also achieved, and connections between algebra and geometry are also made.

Example 2. In order to make it easier for students to understand the theorem about cofunctions and especially its variations, it is appropriate to use a **visualization** of formulas through a trigonometric circle, for which an appropriate model (or a specialized software) can be used in school to represent the angles considered, to mark the corresponding directed line segments, the algebraic measures of which represent the different trigonometric functions of the angles involved in certain formulas, and consider the corresponding pairs of equal triangles. In this way an accessible proof of the studied formulas is realized.

Thus, through the activities and tools described above, as well as with the help of modern IT, good opportunities are provided to enable students to perceive more easily and remember more permanently the formulas expressing the content of the so-called cofunctions theorem: „If the sum of two angles α and β is equal to 90°, then each function of one angle is equal to its corresponding cofunction of the other angle”. Then, writing the condition of the theorem in the form: $\beta = 90^{\circ} - \alpha$, its symbolic record is: $\sin(90^{\circ} - \alpha) = \cos\alpha$, $\cos(90^{\circ} - \alpha) = \sin\alpha$, $\operatorname{tg}(90^{\circ} - \alpha) = \operatorname{cotg}\alpha$, $\operatorname{cotg}(90^{\circ} - \alpha) = \operatorname{tg}\alpha$.

In a similar way, by the same activities, changing the condition, the mentioned different variations of the theorem on the cofunctions are formulated and established. We will indicate the following here:

a) If the subtraction of two angles α and β (where α is an acute positive angle) is equal to 90°, i.e. $\beta - \alpha = 90^{\circ}$, which can be written in the form $\beta = 90^{\circ} + \alpha$, the following equations are satisfied:

$\sin(90^{\circ} + \alpha) = \cos\alpha$, $\cos(90^{\circ} + \alpha) = -\sin\alpha$, $\operatorname{tg}(90^{\circ} + \alpha) = -\operatorname{cotg}\alpha$ и $\operatorname{cotg}(90^{\circ} + \alpha) = -\operatorname{tg}\alpha$. Here

[3] Perception in psychology. https://bg.psichiatria.org/vzpriyatie/#i

we mean that if β is an obtuse angle (Why?), i.e. its second arm is in the second quadrant, then only $\sin\beta > 0$.

b) If the sum of two angles α and β is equal to 270^o, i.e. $\beta = 270^o - \alpha$, then each function of one angle is expressed by its corresponding cofunction of the other angle, taking into account the fact that if α is an acute positive angle, i.e. β is a convex angle with a second arm in quadrant III, then the signs of $\sin\beta$ and $\cos\beta$ are negative, and $\operatorname{tg}\beta$ and $\operatorname{cotg}\beta$ are positive. Symbolically written, these statements have the form: $\sin(270^o - \alpha) = -\cos\alpha$, $\cos(270^o - \alpha) = -\sin\alpha$, $\operatorname{tg}(270^o - \alpha) = \operatorname{cotg}\alpha$ и $\operatorname{cotg}(270^o - \alpha) = \operatorname{tg}\alpha$.

c) If the subtraction of two angles α and β (where α is an acute positive angle) is equal to 270^o, i.e. $\beta - \alpha = 270^o$, which can be written in the form $\beta = 270^o + \alpha$, the following equations are satisfied: $\sin(270^o + \alpha) = -\cos\alpha$, $\cos(270^o + \alpha) = \sin\alpha$, $\operatorname{tg}(270^o + \alpha) = -\operatorname{cotg}\alpha$ и $\operatorname{cotg}(270^o + \alpha) = -\operatorname{tg}\alpha$.

Due to the periodicity of the trigonometric functions, the variations above are sufficient.

Of course, a radian measure can be used instead of a degree measure. Then the theorem on the cofunctions is written as follows:

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$, $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$, $\operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{cotg}\alpha$, $\operatorname{cotg}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{tg}\alpha$.

The formulas of options a), b) and c) are recorded similarly.

a) $\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$, $\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$, $\operatorname{tg}\left(\frac{\pi}{2} + \alpha\right) = -\operatorname{cotg}\alpha$ и $\operatorname{cotg}\left(\frac{\pi}{2} + \alpha\right) = -\operatorname{tg}\alpha$.

b) $\sin\left(\frac{3\pi}{2} - \alpha\right) = \cos\alpha$, $\cos\left(\frac{3\pi}{2} - \alpha\right) = \sin\alpha$, $\operatorname{tg}\left(\frac{3\pi}{2} - \alpha\right) = \operatorname{cotg}\alpha$ и $\operatorname{cotg}\left(\frac{3\pi}{2} - \alpha\right) = \operatorname{tg}\alpha$.

c) $\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos\alpha$, $\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin\alpha$, $\operatorname{tg}\left(\frac{3\pi}{2} + \alpha\right) = -\operatorname{cotg}\alpha$ и $\operatorname{cotg}\left(\frac{3\pi}{2} + \alpha\right) = -\operatorname{tg}\alpha$.

Example 3. Solve the system of inequalities

$$\begin{cases} y > x^2 \\ x + y - 2 \le 0. \end{cases}$$

Solution. In order to provide a more accessible perception of the process of solving this system, consisting of two inequalities with two unknowns, we will use the graphical method. For this purpose, it is appropriate to present the given system in the form:

$$\begin{cases} y > x^2 \\ y \le 2 - x. \end{cases}$$

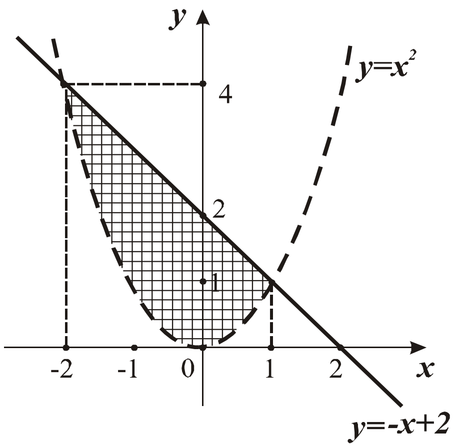

Fig. 2.

The reason for this presentation and the choice of this method is the fact that the functions $y = x^2$ and $y = 2 - x$ are well known to students. Their graphs (parabola and line) are constructed in fig. 2.

To find the coordinates of their intersection points we use the fact that at the point of intersection of the graphs of two (or more) functions they have a common, one and the same, functional value (which is achieved at one and the same value of the argument), which leads to create an equation for the argument x (in this case the equation is quadratic: $x^2 = 2 - x$). Its roots are -2 and 1. These are the abscissas of the intersection points of the graphs of the two functions. Thus, the intersection points of the parabola and the line have coordinates $(-2; 4)$ and $(1; 1)$.

Since the first inequality in the system is strict, taking into account its direction, it follows that its solutions represent the set M_1 of all ordered pairs of real numbers, the images of which in the coordinate plane are the points „above" the parabola $y = x^2$. And since the second inequality is not strict (mixed), then taking into account its direction, it is concluded that its solutions represent the set M_2 of the ordered pairs of real numbers, the images of which are the points of the line $y = 2 - x$ and those that are „under" it. Then the solutions of the given system of inequalities will be those ordered pairs of real numbers which belong simultaneously to both indicated sets M_1 and M_2, i.e. of their section $M_1 \cap M_2 = M$. The geometric images of the solutions of the given system are represented by the points from the shaded part of fig. 2, and analytically the solutions are recorded as follows:

$$\begin{cases} -2 < x < 1 \\ x^2 < y \le 2 - x. \end{cases} \quad \textbf{(1)}$$

When realizing the fourth stage of the methodology for solving problems, which provides a more complete perception of the process of looking for and finding a solution, the question of finding other ideas for solving is usually commented. In this problem we can point out that the same result is reached if for the inequalities in the system the transitive property of the one-way inequalities is used, as a result of which the quadratic inequality is obtained $x^2 < 2 - x$, i.e. $x^2 + x - 2 < 0$, whence it follows that че $-2 < x < 1$.

The process of perceiving the discovery of the solutions of the problem and their layout can be continued by taking a „look back" (Polya, 1972) on the solution above. This „look back" actually analyzes the solution process, that is why some authors call it „Analysis of the way the problem was solved" (Slavov, 1978) or „Additional work on the problem after its solution" (Portev & Nikolov, 1987). This analysis in the specific example allows us to draw some conclusions:

1) Knowledge of *graphs of functions* (in this case linear and quadratic) is a means of solving systems of inequalities with two unknowns.

2) Each of the double inequalities in the last system (1), which analytically represents the solutions of the given system, in turn, defines a geometrically defined subset in the coordinate plane – the first defines the strip bounded by the lines which pass through the intersections of the two graphs and are parallel to the ordinate axis *Oy*; and the second inequality determines the set of points that belong to the specified section $M = M_1 \cap M_2$ of the two subsets of points described in the decision above: one is M_1 and consists of those points that are „above" the parabola $y = x^2$, and the other is M_2 – of the points that are on the line $y = 2 - x$

itself and those that are „below“ it. It should be noted that the section M of these two subsets of points in the coordinate plane is at the same time located in the mentioned strip, which can also be considered as a set of permissible values for the second inequality of the last system (1).

Conclusion. In conclusion we note that, in order to achieve not only a better initial perception by students of the studied mathematical content, which has an abstract character, but also to form more lasting knowledge and skills for its application in solving problems, it's appropriate in similar ways, through concrete examples and illustrations, the accompanying learning activity in mathematics to be illustrated.

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