Science and Education a New Dimension. Natural and Technical Sciences, IX(33), Issue: 262, 2021 Dec. www.seanewdim.com
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The $\mathbf{F}^{\omega}$-covering Subgroups of Finite Groups

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Paper received 01.12.21; Accepted for publication 16.12.21.

https://doi.org/10.31174/SEND-NT2021-262IX33-05

Abstract. Only finite groups and classes of finite groups are considered. Let ω be a non-empty set of primes and $\mathbf{F}$ be a non-empty class of finite groups. A subgroup H of a group G is called an F^{ω}-covering subgroup if H $\in \mathbf{F}$ and the property that H $\leq$ U $\leq$ G, V is a normal ω-subgroup of U with U/V $\in \mathbf{F}$ implies that U = HV. A class of groups is called a formation if it is closed under homomorph images and under subdirect products. In the article we have obtained properties of $\mathbf{F}^{\omega}$-covering subgroups of a finite group G where $\mathbf{F}$ is an ω-local formation of finite groups.

Keywords: *finite group, $\mathbf{F}^{\omega}$-covering subgroup, $\mathbf{F}^{\omega}$-projector, class of groups, ω-local formation.*

Introduction. We deal only with finite groups. A set of groups is called a class of groups if with its group G this class contains every group which is isomorphic to the group G. For the class of groups $\mathbf{F}$ Gaschutz defined an $\mathbf{F}$-covering subgroup of the solvable group G [5] and an $\mathbf{F}$-projector of G [6]. These concepts were the natural extension of the concepts of Hall and Carter subgroups, namely, in a solvable group G the set of all $\mathbf{E}_{\pi}$-covering ($\mathbf{N}$-covering) subgroups coincide with the set of all its π-Hall (nilpotent) subgroups where $\mathbf{E}_{\pi}$ ($\mathbf{N}$) is the class of all π-groups (the class of all nilpotent groups). We note that in the universe $\mathbf{S}$ of all solvable groups the concepts of an $\mathbf{F}$-covering subgroup and of an $\mathbf{F}$-projector coincide.

Among classes of finite groups the central place belongs to formations which was introduced by Gaschutz in [5]. A class of groups is called a formation if it is closed under homomorph images and under subdirect products. Gaschutz using function methods formed local formations and proved the existence and conjugacy of $\mathbf{F}$-covering subgroups in a solvable group G for the class $\mathbf{F}$ which is a local formation [5]. Another important properties of $\mathbf{F}$-covering subgroups and $\mathbf{F}$-projectors of groups for the local formation $\mathbf{F}$ were obtained by Carter, Hawkes, Doerk, Huppert, L.A. Shemetkov, A.F. Shmigirev, Shmid, V.A. Vedernikov, S.F. Kamornikov and others (see [2, 7-9, 12, 14, 16, 18], for instance). Schunck in [13] proved that every local (saturated) formation is a primitively closed homomorph. Properties of $\mathbf{F}$-covering subgroups and $\mathbf{F}$-projectors of groups for the primitively closed homomorph $\mathbf{F}$ are studied in [3, 4, 13] and others.

The natural generalization of the concept of a local formation is the concept of an ω-local formation introduced by L.A. Shemetkov in [15] where ω is a non-empty set of primes. In the article [20] $\mathbf{F}^{\omega}$-covering subgroups and $\mathbf{F}^{\omega}$-projectors of groups were introduced for a non-empty class of groups $\mathbf{F}$ and their crucial properties were obtained (existence, conjugacy, embedding and others) for the class $\mathbf{F}$ which is an ω-local formation or an ω-primitively closed homomorph. This work continues the investigations in this direction. For an ω-local formation $\mathbf{F}$ we have established the conditions under which an $\mathbf{F}$-subgroup of a group G is contained only in one its $\mathbf{F}^{\omega}$-covering subgroup (Theorem 1). As corollaries these theorems imply the result of Carter, Hawkes on $\mathbf{F}$-covering subgroups and $\mathbf{F}$-projectors. Theorem proofs use classical methods of the theory of groups, as well as methods of the theory of classes of finite groups.

Preliminary Information. Used definitions and notations for groups are standard (see [2, 10, 11], for instance). Let us give only some notations and definitions. A note A := B means that the equality A=B is true by the definition.

Denote characterization of the class $\mathbf{F}$ by $\chi(\mathbf{F})$, i.e. $\chi(\mathbf{F})$ is a set of all primes p such that there exists a non-identity p-group in $\mathbf{F}$ (see [11], for instance); $\pi(\mathbf{F}) = \bigcup_{G\in\mathbf{F}} \pi(G)$. A class $\mathbf{F}$ is called *closed under homomorph images*, or briefly, *homomorph* if $G \in \mathbf{F}$ and $N \triangleleft G$ imply that $G/N \in \mathbf{F}$. A homomorph $\mathbf{F}$ is called *a formation*, if $\mathbf{F}$ is closed under subdirect products, i.e. $G/A \in \mathbf{F}$ and $G/B \in \mathbf{F}$ imply that $G/(A \bigcap B) \in \mathbf{F}$. A class $\mathbf{F}$ is called *closed under normal subgroups*, or briefly, *normal hereditary* if $G \in \mathbf{F}$ and $N \triangleleft G$ imply that $N \in \mathbf{F}$. A normal hereditary class $\mathbf{F}$ is called *a Fitting class* if $\mathbf{F}$ is closed under products of normal $\mathbf{F}$-subgroups, i.e. G = AB where $A \triangleleft G$, $B \triangleleft G$, A, B $\in \mathbf{F}$ imply that $G \in \mathbf{F}$. A class $\mathbf{F}$ is called *a Fitting formation* if $\mathbf{F}$ is a formation and $\mathbf{F}$ is a Fitting class.

Let $\mathbf{F}$ be a non-empty Fitting formation. Then $G^{\mathbf{F}}$ and $G_{\mathbf{F}}$ are respectively an $\mathbf{F}$-*coradical* of the group G (i.e. it is the smallest normal subgroup of G quotient on which belongs to $\mathbf{F}$) and an $\mathbf{F}$-*radical* of G (i.e. the largest normal subgroup of G belonging to $\mathbf{F}$) (see [2, 14], for instance). Henceforth ω stands for a non-empty subset of the set $\mathbf{P}$ of all primes; $\mathbf{F}_{\omega}$ is a set of all ω-groups belonging to a class $\mathbf{F}$; $O_{\omega}(G)$ is an $\mathbf{E}_{\omega}$-radical of a group G where $\mathbf{E}$ is a class of all finite groups. Let f : $\omega \bigcup \{\omega'\} \rightarrow$ {formations of groups} where $f(\omega') \neq \varnothing$, h: $\mathbf{P} \rightarrow$ {formations of groups}, δ: $\mathbf{P} \rightarrow$ {non-empty Fitting formations} are functions which are called respectively an ωF-function, an **P**F-function and an **P**FR-function. A formation $\mathbf{F}$ = (G: $G/O_{\omega}(G) \in f(\omega')$ and $G/G_{\delta(p)} \in f(p)$ for any $p \in \omega \bigcap \pi(G)$) is called ω-*fibered* with the ω-*satellite* f and with the direction δ; a formation $\mathbf{H}$ = (G: $G/G_{\delta(p)} \in h(p)$ for any $p \in \pi(G)$) is called *fibered* with the *satellite* h and with the direction δ [19]. A fibered (ω-fibered) formation with the direction δ is called *local* (ω-*local*) if $\delta(p) = \mathbf{E}_{p'}\mathbf{N}_p$ for every $p \in \mathbf{P}$, where $\mathbf{E}_{p'} \mathbf{N}_p$ is the class of all finite p-nilpotent groups.

Remark 1. Every local formation is ω-local for any ω. If $\pi(\mathbf{F}) \subseteq \omega$ then an ω-local formation $\mathbf{F}$ is local (see corollaries 3.2 and 4.2 [19], for instance). A class $\mathbf{F}$ is called *saturated* (ω-*saturated*) if for any $N \lhd G$ such that $N \leq \Phi(G)$ (respectively $N \leq \Phi(G) \cap O_\omega(G)$) the following property is fulfilled: $G/N \in \mathbf{F}$ implies that $G \in \mathbf{F}$ (see [2, 17], for instance).

Remark 2. According to Gaschutz-Lubezeder-Shmidt Theorem, a non-empty formation is saturated if and only if it is local (see Theorem IV, 4.6 [2]). A.N. Skiba and L.A. Shemetkov established equivalency of the concepts of an ω-saturated and an ω-local formations (Theorem 1 [17]).

Let $\mathbf{F}$ and $\mathbf{X}$ are non-empty classes of groups, $\mathbf{F} \subseteq \mathbf{X}$. The class $\mathbf{F}$ is called *primitively closed in* $\mathbf{X}$, or briefly, P-*closed in* $\mathbf{X}$ if for any group $G \in \mathbf{X}$: $G/Core_G(M) \in \mathbf{F}$ for every $M < \cdot\, G$ implies that $G \in \mathbf{F}$ (see [2], for instance). A class $\mathbf{F}$ is called ω-*primitively closed in* $\mathbf{X}$, or briefly, ωP-*closed in* $\mathbf{X}$ if for any group $G \in \mathbf{X}$: $G/Core_G(M) \cap O_\omega(G) \in \mathbf{F}$ for every $M < \cdot\, G$ implies that $G \in \mathbf{F}$ (Definition 2.5 [20]). A class $\mathbf{F}$ is called P-*closed* (ωP-*closed*) if $\mathbf{F}$ is P-closed (ωP-closed) in $\mathbf{E}$.

Remark 3. According to Lemma 2.2 [20], every P-closed in $\mathbf{X}$ homomorph is ωP-closed in $\mathbf{X}$ for any ω. If $\omega = \pi(\mathbf{F})$ then ωP-closed in $\mathbf{X}$ class is P-closed in $\mathbf{X}$ (Remark 2.3 [20]). In the article [20] it has been established that a non-empty formation $\mathbf{F}$ is ωP-closed if and only if it is ω-saturated (see Lemma 2.4 [20]).

Let $\mathbf{F}$ be a non-empty class of groups. A subgroup H of a group G is called an $\mathbf{F}^\omega$*-covering subgroup* of G if $H \in \mathbf{F}$ and the property $H \leq U \leq G$, V is a normal ω-subgroup of U with $U/V \in \mathbf{F}$ implies that U = HV (Definition 3.2 [20]). A subgroup H of a group G is called an $\mathbf{F}^\omega$*-projector* of G if for every normal ω-subgroup N of G a subgroup HN/N is an $\mathbf{F}$-maximal subgroup of G/N (Definition 3.1 [20]).

Remark 4. Every $\mathbf{F}$-covering subgroup ($\mathbf{F}$-projector) of the group G is its an $\mathbf{F}^\omega$-covering subgroup (an $\mathbf{F}^\omega$-projector) of G for any ω. If ω = π(G) then an $\mathbf{F}^\omega$-covering subgroup (an $\mathbf{F}^\omega$-projector) of G is an $\mathbf{F}$-covering subgroup (an $\mathbf{F}$-projector) of G (Remarks 3.1 and 3.2 [20]).

Let $\mathbf{F}$ be a non-empty formation. A normal subgroup R of a group G is called an $\mathbf{F}^\omega$*-limit subgroup* of G if $R \leq G^{\mathbf{F}}$ and $R/(R \cap \Phi(G) \cap O_\omega(G))$ is a chief factor of the group G. A maximal subgroup M of a group G is called $\mathbf{F}^\omega$*-critical* in G if G = MR for an $\mathbf{F}^\omega$-limit subgroup R of G. An $\mathbf{F}$-subgroup H of a group G is called an $\mathbf{F}^\omega$*-normalizer* of G if there exists a chain $H = H_t \subset H_{t-1} \subset \ldots \subset H_1 \subset H_0 = G$ where $t \geq 0$ such that H_i is an $\mathbf{F}^\omega$-critical subgroup of H_{i-1} for every $i \in \{1, 2, \ldots, t\}$ (Definition 3.1 [21]).

Lemma 1 (Theorem 2 [22]). Assume that $\mathbf{F}$ is an ω-local formation, an $\mathbf{F}$-coradical $G^{\mathbf{F}}$ of a group G is a $\pi(\mathbf{F})$-selected ω-group. Then the group G has at least one $\mathbf{F}^\omega$-covering subgroup ($\mathbf{F}^\omega$-projector) and any two $\mathbf{F}^\omega$-covering subgroups (any two $\mathbf{F}^\omega$-projectors) of G are conjugate in G.

Lemma 2 (Theorem 3.4 [20]). Assume that $\mathbf{X}$ is a hereditary homomorph, $\mathbf{F}$ is a non-empty ωP-closed in $\mathbf{X}$ homomorph, $G \in \mathbf{X}$ and N is a nilpotent normal ω-subgroup of G. If H is an $\mathbf{F}$-subgroup of G such that G = HN then H is contained into an $\mathbf{F}^\omega$-covering subgroup of G. Particularly, if H is an $\mathbf{F}$-maximal subgroup of G then H is an $\mathbf{F}^\omega$-covering subgroup of G.

Lemma 3 (Lemma 3.4 [20]). Let $\mathbf{F}$ be a homomorph and G be a group. Then the following statements are true:

(1) If H is an $\mathbf{F}^\omega$-projector of the group G and $H < \cdot\, G$ then H is an $\mathbf{F}^\omega$-covering subgroup of G;

(2) If H is an $\mathbf{F}^\omega$-covering subgroup of G and $H \leq K \leq G$ then H is an $\mathbf{F}^\omega$-covering subgroup of K;

(3) If H is an $\mathbf{F}^\omega$-covering subgroup of G and N is a normal ω-subgroup of G then HN/N is an $\mathbf{F}^\omega$-covering subgroup of G/N;

(4) If N is a normal ω-subgroup of G and H/N is an $\mathbf{F}^\omega$-covering subgroup of G/N then every $\mathbf{F}^\omega$-covering subgroup of H is an $\mathbf{F}^\omega$-covering subgroup of G.

3. The Main Result

In Theorem 3.4 [20] it has been established conditions under which an $\mathbf{F}$-subgroup H of a group G is contained into some its $\mathbf{F}^\omega$-covering subgroup. In the following theorem we obtain conditions under which H is contained only into one $\mathbf{F}^\omega$-covering subgroup of the group G.

Theorem 1. *Assume that* $\mathbf{F}$ *is an* ω*-local formation,* G *is a group,* N *is a nilpotent normal* ω*-subgroup of the group* G, H *is an* $\mathbf{F}$*-subgroup of* G *such that* G = HN. *If* $\mathbf{N} \subseteq \mathbf{F}$ *then the following statements are true*:

(1) *A normalizer* $N_G(H)$ *is contained into a* $\mathbf{F}^\omega$*-covering subgroup of the group* G;

(2) H *is contained only into one* $\mathbf{F}^\omega$*-covering subgroup of* G.

Proof. Assume that $\mathbf{N} \subseteq \mathbf{F}$. Prove the statement (1). Induct on the order of the group G. If $G \in \mathbf{F}$ then G is an $\mathbf{F}^\omega$-covering subgroup of G and the statement (1) is true. Suppose that $G \notin \mathbf{F}$. Then from G = HN we infer that $N \neq 1$. Assume that K is a minimal normal subgroup of the group G contained into N. Then K is a nilpotent ω-group. Show that a quotient G/K satisfies the hypotheses of the theorem. Indeed since $\mathbf{F}$ is a formation and $H \in \mathbf{F}$ then $HK/K \in \mathbf{F}$. Moreover, $G/K = HK/K \cdot N/K$ and N/K is a nilpotent normal ω-subgroup of G/K. Since |G/K|<|G| then by induction $N_{G/K}(HK/K) \subseteq L/K$ where L/K is an $\mathbf{F}^\omega$-covering subgroup of the group G/K. From $N_G(H)K/K \subseteq N_{G/K}(HK/K)$ it follows that $N_G(H) \subseteq L$.

Since, according to Remarks 2 and 3, the formation $\mathbf{F}$ is an ωP-closed homomorph in $\mathbf{E}$ then by Lemma 2 we infer that $H \subseteq T$ where T is an $\mathbf{F}^\omega$-covering subgroup of the group G. Then by Lemma 3 (3) a quotient TK/K is an

$\mathbf{F}^{\omega}$-covering subgroup of G/K. From G/N = HN/N $\cong$ H/H $\bigcap$ N $\in \mathbf{F}$ we infer that $G^{\mathbf{F}} \subseteq N$ and, hence, $G^{\mathbf{F}}$ is a nilpotent ω-group. Since Lemma 1.2 (1) [14] implies that $(G/K)^{\mathbf{F}} = G^{\mathbf{F}}K/K$. Consequently, $(G/K)^{\mathbf{F}}$ is nilpotent and, so, $(G/K)^{\mathbf{F}}$ is a $\pi(\mathbf{F})$-selected ω-group. Then by Lemma 1 L/K and TK/K are conjugate in G/K. Therefore, there exists an element $x \in G$ such that L/K = $(TK/K)^{xK}$ From this we conclude that L = T_1K where $T_1 := T^x$. Lemma 1 yields that a subgroup T_1 is an $\mathbf{F}^{\omega}$-covering subgroup of G and by Lemma 3 (2) T_1 is an $\mathbf{F}^{\omega}$-covering subgroup of the group L.

1. Consider the case L $\neq$ G. Since H $\subseteq$ N_{G}(H) $\subseteq$ L then L = L$\bigcap$G = L$\bigcap$HN = H (L$\bigcap$N). Since L$\bigcap$N is a nilpotent normal ω-subgroup of the group L then by induction we infer that $N_L(H) \subseteq R$ where R is an $\mathbf{F}^{\omega}$-covering subgroup of the group L. Since L/K = $T_1K/K \cong T_1/T_1 \bigcap K \in \mathbf{F}$ then $L^{\mathbf{F}} \subseteq K$ and, therefore, $L^{\mathbf{F}}$ is a nilpotent ω-group. According to Lemma 1, R and T_1 are conjugate in L. Thus, R = $T_1^{\,y}$ for some $y \in L$. Since T_1 is an $\mathbf{F}^{\omega}$-covering subgroup of G then by Lemma 1 R is an $\mathbf{F}^{\omega}$-covering subgroup of G. The inclusion $N_G(H) \subseteq L$ implies that $N_G(H) = N_G(H) \bigcap L = N_L(H)$ and, hence, $N_G(H) \subseteq R$. Thus, if L $\neq$ G then $N_G(H)$ is contained into the $\mathbf{F}^{\omega}$-covering subgroup R of G.

2. Suppose that L = G. Verify that $N_1 \lhd G$ where $N_1 := T_1 \bigcap N$. Indeed, since N is a nilpotent normal subgroup of G and K $\cong$ K/1 is a chief factor of G then Corollary 4.1.1 [14] yields N $\subseteq$ F(G) $\subseteq$ $C_G(K)$ and, therefore, K $\subseteq$ $N_G(N_1)$. Since $N_1 \lhd T_1$ then G = $T_1K \subseteq N_G(N_1)$ and, hence, $N_1 \lhd G$.

2.1. Assume that $N_1 \neq 1$. Since G/N_1 = HN_1/N_1 · N/N_1 and N/N_1 is a nilpotent normal ω-subgroup of the group G/N_1 then by induction we infer that $N_{G/N_1}(HN_1/N_1) \subseteq S/N_1$ where S/N_1 is an $\mathbf{F}^{\omega}$-covering subgroup of G/N_1. Since $N_G(H)N_1/N_1 \subseteq N_{G/N_1}(HN_1/N_1)$ then it follows that $N_G(H) \subseteq S$. Lemma 3 (3) implies that the subgroup T_1/N_1 is an $\mathbf{F}^{\omega}$-covering subgroup of G/N_1. By Lemma 1.2 (1) [14] $(G/N_1)^{\mathbf{F}}$ is a nilpotent ω-group. Then, according Lemma 1, we infer that T_1/N_1 and S/N_1 are conjugate in G/N_1. Hence, $(T_1/N_1)^{aN_1}$ = S/N_1 for some element $a \in G$. Consequently, $T_1^{\,a}$ = S and Lemma 1 implies that S is an $\mathbf{F}^{\omega}$-covering subgroup of G. Apart from that, as we have shown above, $N_G(H)$ is contained into S.

2.2. Suppose that N_1 = 1. In this case prove that N = K. Indeed, since K $\subseteq$ N and $T_1 \bigcap N = 1$ then $T_1 \bigcap K = 1$. Therefore, G = L = T_1[K]. On the other hand, G = T_1[N]. Thus, N = K and from G = HN we infer that G = HK. Then G = H^xK. The equalities $H^x \subseteq T^x = T_1$ and $T_1 \bigcap K$ = 1 imply that G = H^x[K]. Then $H^x \lessdot G$ and by Lemma 3.17 (2) [11] we infer that $H \lessdot G$. Since $G \notin \mathbf{F}$ then H is an $\mathbf{F}$-maximal subgroup of G and, according Lemma 2, it follows that H is an $\mathbf{F}^{\omega}$-covering subgroup of G. If $N_G(H) \neq H$ then $N_G(H)$ = G. Consequently, $N_G(H^x)$ = $(N_G(H))^x$ = G and, hence, $H^x \lhd G$. From this we conclude that G = $H^x \times K$. Since $H^x \in \mathbf{F}$, K $\in \mathbf{N} \subseteq \mathbf{F}$ and $\mathbf{F}$ is a formation then G $\in \mathbf{F}$ which is contradiction. Thus, $N_G(H)$ = H. The statement (1) is proved.

Prove the statement (2). Induct by the order of G. As above, we can assume that $G \notin \mathbf{F}$ and N $\neq$ 1. Suppose that K is a minimal normal subgroup of the group G such that K$\subseteq$N. By Lemma 2 H is contained into an $\mathbf{F}^{\omega}$-covering subgroup of G. Put M := { T_1, … , T_m } is a set of all $\mathbf{F}^{\omega}$-covering subgroups of the group G containing H. Show that |M| = 1.

Assume that i, j $\in$ {1, 2, …, m}, i $\neq$ j. Since T_i and T_j are $\mathbf{F}^{\omega}$-covering subgroups of the group G containing H, and K is a normal ω-subgroup of G, Lemma 3 (3) implies that T_iK/K and T_jK/K are $\mathbf{F}^{\omega}$-covering subgroups of the group G/K, and, furthermore, HK/K $\subseteq$ T_iK/K and HK/K $\subseteq$ T_jK/K. Since the group G/K satisfies the hypotheses then by induction we infer that T_iK/K = T_jK/K and, hence, T_iK = T_jK := D.

a) Consider the case D $\neq$ G. By Lemma 3 (2) subgroups T_i and T_j are $\mathbf{F}^{\omega}$-covering subgroups of the group D containing H. Moreover, D = G$\bigcap$D = HN$\bigcap$D = H (N$\bigcap$D) and N$\bigcap$D is a nilpotent normal ω-subgroup of the group D. Then by induction we have T_i = T_j and, therefore, in this case we conclude that |M| = 1.

b) Assume that D = G. If $T_i \bigcap K \neq 1$ then from that a subgroup $T_i \bigcap K$ is normal in G and K is a minimal normal subgroup of G it follows that $T_i \bigcap K$ = K. Then K $\subseteq$ T_i and G = T_iK = T_i. Thus, in this case we have T_i = G = T_j. If $T_j \bigcap K \neq 1$ then we obtain the same conclusion.

Suppose that $T_i \bigcap K = 1 = T_j \bigcap K$. Then G = T_i[K] = T_j[K] and, hence, $T_i \lessdot G$, $T_j \lessdot G$. By Lemma 1 subgroups T_i and T_j are conjugate in G. Therefore, $T_i^k = T_j$ for some $k \in K$. Since $HK \cap T_i = H(K \cap T_i) = H$ and $HK \cap T_j = H(K \cap T_j) = H$ then $H^k = (HK \cap T_i)^k \subseteq (HK)^k \cap T_i^k = HK \cap T_j = H$. Consequently, $k \in N_G(H)$. By the statement (1) of this theorem we infer that $N_G(H) \subseteq T$ where T is an $\mathbf{F}^{\omega}$-covering subgroup of G. Hence, $k \in T$. Thus, we have established that for any i, j $\in \{1, 2, \ldots, m\}$ the following equality is true $T_i^k = T_j$ where $k \in T$. Since $H \subseteq N_G(H)$ then $T \in M$ and, therefore, $T = T_s$ for some s where $1 \le s \le m$. Assume that $r \in \{1,2,\ldots,m\}$. Since $T_r, T_s \in M$ then, by proved above, we infer that $(T_s)^l = T_r$ where $l \in T = T_s$. Consequently, $T_r = T$ for any $r \in \{1, 2, \ldots, m\}$. Thus, we conclude that |M| = 1. The statement (2) is proved. The theorem is proved.

Conclusion. In view of Remarks 1 and 4, Theorem 1 directly implies the following well-known result for local formations.

Corollary 1.1 (Carter, Hawkes, Theorems 5.8 and 5.9 [1], see also Theorem 15.9 [14]). *Assume that* $\mathbf{F}$ *is a local formation*, G *is a group with the nilpotent* $\mathbf{F}$*-coradical. Let* H *be an* $\mathbf{F}$*-subgroup of* G *such that* G = HF(G). *If* $\mathbf{N} \subseteq \mathbf{F}$ *then the following statements are true*:

(1) $N_G(H)$ *is contained into an* $\mathbf{F}$*-covering subgroup of* G;

(2) H *is contained only into one* $\mathbf{F}$*-covering subgroup of* G.

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