CURRENT ISSUES IN ENSURING THE QUALITY OF MATHEMATICAL EDUCATION

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The Authors:
I. Akulenko, D. Bobyliev, T. Bodnenko, M. Burda, S. Cernajeva, O. Chernobai, O. Chumak, V. Kirman, O. Kondratyeva, T. Krylova, Z. Kravchenko, L. Kulyk, I. Lovyanova, I. Malova, O. Plysiuk, Z. Serdiuk, I. Sitak, N. Tarasenkova, D. Tinkova, K. Vlasenko, I. Volodko, A. Yakovenko, & O. Zydkov

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CONTENTS

PREFACE

It is well known that the issue of the quality of training mathematics at school has been and remains the cornerstone for the effective functioning of industries, businesses and banks, the development of science, culture, education and society in general. School-leavers and University graduates should be competitive in the job market. This means that they should not only get a good education, but also be willing and able to further self-improvement. This monograph brings into dialogue the authors to address the following aspects: learning mathematics at secondary & vocational schools; mathematical training at the university; math teacher training in graduate and postgraduate education.

Mathematics as an educational subject is studied at all levels of the full secondary education. The course of mathematics at elementary school is based on the following semantic lines: the numerals and operations with them; seizes; mathematical expressions; sums with a plot; space ratio, and geometric figures. The basis of mathematics study at elementary school is arithmetic of integral numerals and seizes measuring. As propaedeutics to studying algebra and geometry some geometric shapes and their properties, numerical equalities, inequalities and so on are studied. Contents of mathematical education at basic school is determined by the following contextual lines: numbers, expressions, equations and inequalities, functions, elements of combinatorics, theory of probability and statistics, geometric shapes, geometric seizes. Each is developed considering the objectives of mathematical study at this level of schooling, which distinguishes two main stages: 5 – 6 forms (10 – 12 years) and 7 – 9 forms (12 – 16 years). Educational objectives on the first stage are implemented in the study of a single course on mathematics, on the second – two courses: algebra and geometry. Senior school mainly functions as a profile school, based on students and staff's needs and material facilities of the school. The study of mathematics at senior school in Ukraine is streamed into two levels: the level of standard and the specialized (profiled) level.

The process of subject-specialization in terms of modern senior school has both – objective and natural character and a long-term history. It is worthy of mentioning that the present times are characterized by the progress of subject-specialized school features, traced at different stages of subject-specialized training idea existing in domestic and foreign pedagogy, namely: the choice of the type of school with bifurcation or polifurcation; accent on differentiation as the principle of training; introduction of optional courses at the students' choice; the consideration of students' propensities and capacities, as well as the national demands for experts of different areas.

The monograph goes through the problems of: screening the contents of the math tutorials for lyceums; reports the findings of a study on the implementation of competence approach in solving physical problems by graphical method; comments on the issue of inter-subject relations and the development of distance concepts; defines didactic principles of tailored instruction in school mathematics; emphasizes high school specifics of the development of math concepts; argues in favor of using modeling elements when teaching math to engineering students; offers a review of the local experience with the implementation of STEM-education technologies.

The changes caused by joining the Bologna Process increased the interest in the theoretical and practical experience of the countries that have already implemented or are only carrying out the reform of their own education systems. Ukrainian math training at universities continues to operate with two-stage education system (bachelor and master). The BSc is still four years long; the training of general subjects is a semester in the number of lessons. The MSc training can be a year and a half or two years long. More and more attention is paid to the teaching of optional subjects. Similar process are under way in the Baltic countries. Nevertheless, unlike the Latvian educational reform, the Ukrainian higher education reform has not ended yet.

Under the umbrella of the scientific basis of ensuring the quality of math education at high school there are the issues of testing and monitoring the high school and University students' knowledge of mathematics and applying algorithms in teaching probability theory.

The success of the modernization of math education largely depends on the quality of teacher training which should meet current social and economic challenges. The monograph discusses such issues as training models and project-based learning as the means of enhancing math teacher training and retraining.

One should understand that the papers presented in the monograph do not exhaust all aspects of the problem. We hope to continue the discussion the information field of which is extremely broad. We believe that every outcome of the studies (both – theoretical and experimental) allows of improving the training of specialists in math education and gradually raising the quality of math education to the society's today's demands.

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CHAPTER 1 LEARNING MATHEMATICS AT SECONDARY & VOCATIONAL SCHOOLS

1.1 Conceptual Bases for the Selection of Textbooks on Mathematics for Lyceums*

M. Burda, N. Tarasenkova

*The article is published in the author's translation

The problem statement

The content selection for textbooks on Mathematics gained special significance as a response to a new social demand concerning the goals and tasks of the school education. Today the educative leitmotif is: the formation of the mathematical competence and the key competences necessary for a person's successful self-realization; the education content reorientation towards a person, the fostering of education subject's active cognition; the education organization basing on the student's environment interaction experience; the education orientation onto young person's abilities, spiritual and creative potential realization as well as onto stable self-education and self-development mechanisms elaboration. The content of teaching Mathematics is affected by its role in economy, technology, management, social processes as well as by the profiling, competence, activity- and personality-oriented approaches wide implementation in schools. The abovementioned factors presuppose development and employment of such conceptual bases for lyceum-oriented Mathematics textbooks content selection that could provide the high-quality mathematical education.

The latest research analysis

The problem of mathematics content at high school and its representation in textbooks was studied by researchers, methodologists and teachers of mathematics (Burda, Tarasenkova, Vasylieva, & Vashulenko[1], 2018; Bevz[2], 1989;

[1] Burda, M., Tarasenkova, N., Vasilieva, D., & Vashulenko, O. (2018). Kontseptsiya matematychnoyi osvity 12-richnoyi shkoly [The Concept of Mathematical Education in a 12-year School]. *Mathematics in Native School*, No. 7-8, 2-8. [In Ukr].

[2] Bevz, G. (1989). *Metodyka vykladannya matematyky [Mathematics Teaching Methods]*. Kyiv: Vyscha shkola. [In Ukr].

Bevz and Vasylieva[3], 2018; Dubinchuk[4], 1992; Malyovany[5], 2015; Slepkan[6], 2000; etc.). However, what still retains relevance is the study of maths textbook selection for lyceum based on the competence approach according to which the result of the mastered subject is the formation of a certain competence as the student's ability to act effectively in academic and existential situations.

The purpose of the article lies in revealing the conceptual bases for the selection of teaching texts, the mathematics textbooks system of tasks and methodology apparatus enhancing the high school student quality mathematical education.

Theoretical framework

The humanistic values of education predetermine the transition from the knowledge oriented educational paradigm to the competence-centered personality-oriented one. The *personality oriented approach* in education is seen as the enhancement of personal interaction in the educational process, of students' personal progress, capability development, self-identification conditions. The chief goal of the personality oriented learning is the formation of the student personality's I-concept as the system of acknowledged and unrealized (unconscious) self-images on which (s)he bases his/her behavior. The personality oriented education's central task is the formation of student personality's positive I-concept as the system of acknowledged and unrealized (unconscious) self-images that serve the basis for his/her behavior. In this regard, the modelling of success situations

[3] Bevz, V. and Vasylieva, D. (2018). The Particularities of Mathematics Curriculum in the New Ukrainian School Concept. *School Curriculum: Problems and Options for Development*. Kishynyov, Moldova. [in Rus.].

[4] Dubynchuk, O. (1992). The Differentiation of Mathematical Profiled College and Professional School Education Content : *Profiled Teaching in Professional Technical Educational Establishments: Research.-method. selection*, 32-37.

[5] Maliovanyi, Yu. (2015). Kontseptual′ni pidkhody do formuvannya zovnishn′oyi struktury profil′noyi osvity [Conceptual Approaches to the Formation of Profile Education External Structure]. *Ukrainian Pedagogical Journal, 1*. 77-84. [In Ukr.]

[6] Slepkan, Z. (2000). *Metodyka navchannya matematyky: pidruchnyk dlya studentiv [Mathematics Teaching Methods: a textbook for stud.*]. Kyiv, *Ukraine*: Zodiak-Eco. [In Ukr.]

– subjective psychic states of student's feeling of pleasure resulting from cognitive activity accomplishments is of primary importance. The success experienced by a student mobilizes his/her hidden abilities, fosters the emotively evaluative attitude to the objects of cognition, the realization of intellectual effort. And the greatest role here belongs to textbooks as the main learning means.

The personality's subjectivity as well as students' individuality reveals itself in the world cognition selectiveness as for the content, kind and form of its presentation, the selectiveness stability, the ways of learning material mastering, the emotionally evaluative treatment of cognition objects. In the personality oriented learning the content, methods, means and organizational forms are to be focused upon every students' experience discovery and use, upon the assistance in working out personally significant cognitive ways by organizing the cognitive activity. In the educative process the student's mastering of the social and historic experience should be accomplished not by ousting his/her individual experience, but through their coordination and through the use of everything accumulated by the student in his/her own life. This principle should serve the basis for working out teaching texts and other constituents of the assimilation organization apparatus.

In the process of mathematical knowledge, habits and skills assimilation as well as at the stage of their application the objective prerequisites for the enrichment of not only mathematical but universal cultural students' potential are laid out, ample possibilities are created for the formation and development of students' thinking, memory, ideas and imagination, their scientific outlook, algorithmic, informational and visual culture, their ability to determine causal-consequential relations between separate facts, to substantiate statements and mathematize situations. The employment of didactically weighed Mathematics teaching organization enables a considerable effect on the students' intellectual development, the positive personal features formation, their mental activity, cognitive independence, self-regulation, learning creativity upgrade. So the textbooks content realizes the personality -oriented learning model and is personality -centered, enhancing the learning model aiming at mathematical education as well as education by means of mathematics, the formation of mental qualities necessary for adapting and functioning at full value in modern society, the assimilation of the mathematical apparatus as a means of modern life problems formulation and solution.

Of primary importance in the textbook's content selection is the consideration of students' specificity in verbal and non-verbal data perception and procession organization, namely: the cognitive stimuli impact is characterized by instability and high degree of activation processes lability; the simultaneous involvement of cerebral cortex different zones at all stages of data perception and procession (sensory analysis, information synthesis, the stimulus categorization) are observed; higher speeds of data procession by the right cerebral hemisphere are revealed; the visual figurative thinking that is approximating the image categorizing operation is predominant, more so when the verbal logical thinking isn't perfect yet, but is in the stage of formation. Thus, the consideration of the logical and the visual beginnings unity in the student's mathematical training, timely detection of the logical and the visual beginnings' hypothetical conflicts, and the didactically balanced selection of the ways to minimize the gap are of primary importance. In this regard, the textbooks combine logical rigor and visual representation, so that deduction and the learning material abstract nature should be based on visuality and students' mathematical intuition.

In the textbook creation the idea of the opportunities *to conduct mathematics training is regarded in two areas* – direct learning and background training.

The latter is represented by propaedeutic and indirect learning. In the course of students' background training the powerful resources of the unconscious are involved – the experience of visual recognition is enriched, the certain intuitive prior knowledge is accumulated, and the experience in the individual subject-practical actions performance is gained. The subsequent expansion of the influences system becomes possible due to a specially construed visual range of training and a system of exercises aimed at preventive formation of students' abilities to perform certain activities. In general, the semiotic space, which is built in the course of study, should provide conditions for the students' free, psychologically comfortable life in the school Mathematics conditionality dimension, promote their active educational and cognitive activity, as well as encourage them to succeed independently. And this is only possible if the textbooks content meets the specificity of the students' educational and cognitive activity, and textbooks volume corresponds the learning time. Therefore, as we note[7] (Burda & Tarasenkova, 2016), there is currently a decrease in the volume of Mathematics courses that is achieved by avoiding excessive rigor in presentation, by reducing cumbersome calculations and transformations, by excluding the material that is used neither for the course logical deployment, nor problem solution and has no practical application.

The textbook content (textbooks, exercises and methodological apparatus) is aimed at *developing competencies* in students (Tarasenkova & Kirman[8], 2008; Tarasenkova[9], 2016), namely – mathematical, super-mathematical (interdisciplinary and specialist) and key competences. To students' mathematical competencies we refer: *content* or *informative* (has a notion of the ideas and methods of Mathematics, its role in the cognition of reality; possesses formally logical (definition, qualities, features of mathematical objects) and operational (methods, techniques, means of activities) knowledge; understands mathematical formulas and models as such that allow to describe the

[7] Burda, M. & Tarasenkova, N. (2016). Teoretyko-metodychni vymohy do zmistu shkil'nykh pidruchnykiv z matematyky [Theoretical and Methodological Requirements for the School Mathematics Textbook Contents]: *Problems of a Modern Textbook*: *a collection of scientific works*, *16*. 43-51. [In Ukr.].

[8] Tarasenkova, N. & Kirman, V. (2008). Zmist i struktura matematychnoyi kompetentnosti uchniv zahal'noosvitnikh navchal'nykh zakladiv [The Content and Structure of the Secondary School Students' Mathematical Competency]. *Mathematics in School*, *6*. 3-9. [In Ukr.].

[9] Tarasenkova, N. (2016). Kompetentnisnyy pidkhid u navchanni matematyky: teoretychnyy aspekt [Competency Approach in Teaching Mathematics: Theoretical Aspect]. *Mathematics in the Native School*, *11 (179)*. 26-30. [In Ukr.].

properties of objects, processes and phenomena); *procedural-operational* (depicts mathematical objects, finds out and substantiates their qualities; classifies them by their features; substantiates mathematical statements; uses mathematical objects' definitions, qualities and features to solve problems; measures and calculates the values; applies mathematical methods, techniques and ways of acting in the process of solving purely mathematical or practical problems); *research* or *investigative* (proposes and tests the hypotheses, sets out the program of activities, provides its results, makes decisions in the conditions of incomplete, excess, accurate and probabilistic information, evaluates the correctness and rationality of the tasks solved, interprets the results, taking into account the specific conditions and the objectives of the research); *informational and technological* (uses information and communication technologies in educational activity; finds and develops mathematical information (textbooks, directories, Internet resources), evaluates the obtained information, systematizes and generalizes it, makes the right conclusions). Depending on the goals, objectives and the level of learning mathematics, the degree of these competences mastering is different.

Taking into account Mathematics teaching goal, its role in the study of other subjects, the important task is to develop intersubjective mathematical competencies – interdisciplinary (Geometry, Algebra and Calculus, Mathematics and other subjects) and specializing (as an element of vocational training).

These competences imply that the student: understands the importance of Mathematics for other disciplines successful learning, full-fledged activities in various spheres of public life, in particular in future professional activities; recognizes and formulates the problems that arise in the content of other subjects or in the field of prospective professional activities and can be solved by mathematical methods; applies mathematical models to the study of other school subjects (Physics, Computer Science, Astronomy, Chemistry, Biology, etc.) and to the situations associated with would-be professional activities.

In addition, the textbook content should contribute to the key competencies formation. They are aimed at increasing motivation, interest in learning, developing the ability to apply knowledge and skills in various spheres of real-life activity, as well as gaining experience, creating values and views that can be put into practice. Key competencies are acquired in the process of solving problems of practical kind – the problems arising beyond the boundaries of Mathematics, but solved with the mathematical apparatus employment. There are many tasks of the kind in the published textbooks. However, it is recommended to give preference to the tasks that relate to contemporary socio-economic challenges and values (Vasylieva[10], 2018). They tackle, primarily, energy saving problem (gas, water, electric light and heat are valuable resources requiring economy), the financial literacy (planning and rational spending of personal, family funds, proper cooperation with financial institutions), health and ecology (thrifty expenditure of natural resources, the environment cleanliness, the healthy lifestyle choice, food products quality and correct nutrition, position towards alcohol, nicotine, etc.).

That is, the tasks should be value-orienting, enhance the students' sober behavior concerning energy resources, health, finance, the environment, interpersonal relations, and promote understanding of the mathematical education importance for successful life in modern society.

In the conditions of competence-oriented training, *special requirements are put forward to the textbook's methodological apparatus*. A textbook of a standard or professional level should become not only attractive to students, but also immune from the teacher and his methodological preferences and skills, as a training provider of student's independent learning. First of all, the textbook's structure and all its constituents should be logical and understandable for students, because the structuring method creates the first,

[10] Vasylieva, D. (2018). Matematychni zadachi yak zasib formuvannya klyuchovykh kompetentnostey uchniv [Mathematical Problem as a Means of Forming the Key Competencies in Students]. *Problems of a Modern Textbook: a collection of scientific works*, *21*. 83-92. [In Ukr.].

structural level in the system of the textbook conventions, to which the student is to get used. And this system should be unified for all structural units of the textbook. For example, each section can begin with the section "In the paragraph you will learn", and be ended with control questions and test tasks.

We associate another aspect of the management function of the textbook with the creation of conditions for the development of students' cognitive needs. To this end, each paragraph of the textbook should comprise not only the training material that pupils must learn, but also additional data (for example, in the "Find Out More" section) that contain information on the origin of names and designations, historical, biographical references on outstanding compatriot and foreign mathematicians, etc., as it is done in our Geometry textbook[11] (Burda, Tarasenkova, Bogatyryova, Kolomiyets, & Serdiuk, 2013) and our Mathematics textbook[12] (Burda, Kolesnik, Malyovany, & Tarasenkova, 2018).

The training text unfolds according to a certain plan, which is identical for each paragraph of the textbook. The volume of each semantic text unit should correlate with the students' age, and the content deployment should follow the patterns of the mental process course. To this end, it is advisable to use problem questions, which serve a way of transition to a new idea, contribute to the rethinking of what was learned. The training text should be selected so as to involve students as actively as it is possible into self-dependent speculations, accustom them to posing questions and responding to them. Note that visual accents play an important role in it. Such textbook structure and design enables students' self-dependent work.

[11] Burda, M., Tarasenkova, N., Bogatyryova, I., Kolomiyets, O., & Serdiuk, Z. (2013). *Heometriya. Pidruchnyk dlya 11 klasiv zahal'noosvitnikh navchal'nykh zakladiv (akademichnyy ta profil'nyy rivni.)[Geometry Textbook for the 11th Grade of the Secondary School (academic and profile levels)].* Kyiv, Ukraine: Publishing House "Osvita". [In Ukr.].

[12] Burda, M., Kolesnyk, T., Malovany, Yu., & Tarasenkova, N. (2018). *Matematyka (alhebra i pochatky analizu ta heometriya, riven' standartu): pidruch. dlya 10 klasu zakladiv zahal'noyi seredn'oyi osvity [Mathematics (Algebra and Calculus, and Geometry, standard level): textbook for the 10th form of general secondary education establishments].* Kyiv, Ukraine: UEPC "Orion". [In Ukr.].

In accordance with the scientific foundations of the activity-oriented approach, specially organized substantive activities should serve both the purpose of learning, and its means. *The realization of the activity-oriented approach to mathematics learning in the textbook* implies: students' constant involvement into various types of educational and cognitive activity; assimilation of not only formal and logical but also operational knowledge (how to act in particular situations in order to achieve the set goal); assimilation of methods of reasoning employed in Mathematics; creation of educational situations that stimulate students' independent discovery of mathematical facts. In the textbook (where possible) it is advisable to give advice on how to act in a particular training situation which should be formulated in the form of rules or instructions. That is, the content of the textbook should ensure the presence of the activity-oriented component in any mathematical knowledge the students acquire – they should learn where and how to apply them.

The textbook content's scientificity is provided by the logically sequential placement of the training material, the correct formulation of concepts and theorems definitions, and a sufficient degree of evidence strictness. The logical ordering of the material and the sequence of its presentation must meet the didactic principles and the requirements of Mathematics as a science: modern, substantive, unambiguous terminology; concepts, formulas, qualities are formulated in a correct mathematical language; evidences of statements are at a necessary degree of strictness; the representation in the textbook content of the methods and ways of activities, meeting the mathematical logic of cognition. The mathematical concepts content is clearly distinguished (all essential features are listed) and their volume (the set of objects where the concept is applied) is indicated. The concepts' content is revealed in definitions and their volume is measured with the use of classifications (the division of concepts on a certain basis). The proofs of the theorems in the textbooks should be not only strict, concise, but also feasible, understandable to the students. Before the formulation of the theorem, it is proposed to conduct a small study, a reduced description of the theorem is given, and its proof is divided into semantic blocks.

The combination of Continuous and Discrete Mathematics is an important feature of its contemporary courses. The development of computerization, information networks puts forward specific requirements for the style of human thinking, and hence for the content of school Mathematics. One of them is connected with the necessity of inclusion in the school course of Discrete Mathematics elements (combinatorics, mathematical logic elements with regards to their application, numerical systems, the theory of graphs elements, etc.).

Accessibility of training texts for students, the possibility to process them independently is one of the features of the textbooks. It is achieved by combining logical and visual means. The training material, as a rule, is based on visuality, students' intuition, their life experience[13] (Burda, 2018); the presentation of mathematical facts, if possible, begins with the empirical material analysis (examples from the environment, models, graphs, drawings, facts from other educational subjects, etc.) or with the description of practical actions; visuality should assume not only the illustrative but also a heuristic role, promote the creation of a pre-emptive idea concerning the new learning material content essence, facilitate the learning material perception and understanding.

The coherence of content and requirements for students' educational material acquisition is implemented in its two functions – the compensatory and the prognostic ones (Tarasenkova[14], 2016). The compensatory function provides

[13] Burda, M. (2018). Metodychni vymohy do pidruchnyka z matematyky rivnya standartu [Methodological Requirements for a Standard Textbook on Mathematics]. *Problems of a Modern Textbook: a collection of scientific works, 21*. 64 - 72. [In Ukr.].

[14] Tarasenkova, N. (2016). Kompetentnisni zasady zabezpechennya nastupnosti navchannya matematyky v riznykh lankakh osvity [The Competency Forming Bases for Enhancing Coherence in Learning Mathematics at Different Education Levels]. Proceedings from the Implementation of continuity in mathematical education: Realities and Prospects: *Vseukrayins'ka naukovo-praktychna konferentsiya (Odesa, 15-16 veresnya 2016 roku) – All-Ukrainian research practical conference*. (pp. 108-110). Kharkiv, Ukraine: «Ranok». [In Ukr.].

a link of current learning process with the previous level of education (the content specification, expansion and deepening, the detection and alignment of disadvantages and gaps in the students' training). The prognostic function ensures the students' training adaptation to studying Mathematics at the next educational level.

The textbook content is aimed at students' creative development. The developmental effect is mostly based on the development of skills to prove assertions and to solve problems, to apply mathematical methods to solving problems of applied content, to the abstract mathematical constructions essence understanding, etc. Still, attention is also paid to students' acquaintance with the significance of Mathematics in present-day human activity, especially, in regard to the historical context. The textbooks, consequently, include materials related to value orientations: Mathematics history glimpses, mathematical theories and methods, some facts concerning the fates of scientists who created the branch, the terms and symbols' origin. The developmental learning function is also realized through the personalized presentation of the material, that is, the presentation, where possible, of mathematical facts in terms of their historical formation and development.

The textbooks are designed for *graded, self-paced learning* of Mathematics. Not only the exercises and tasks of varying complexity tend to be focused on it, but also examples of typical tasks solutions, problem questions and tasks, etc. For those who are interested in the subject and want to deepen their knowledge, the paragraph "Learn more" is assigned. A feature of the textbook's tasks is that high complexity level tasks include elements of medium and high level tasks, and the latter are elements of tasks at the entry level. In the problem settings the described situations vary in terms of personal participation: the problem is formulated by means of this age students' I-vocabulary; the task is formulated by means of the student's closest environment vocabulary; the problem is formulated in terms that are distant from this age students' personal experience.

The competencies formation involves the reinforcement *of the textbook contents pragmatic orientation*. The training material content should be corresponded to the three stages of Mathematics practical application.

The first stage. The study of a mathematical fact, if possible, should begin with the empirical material analysis (examples from the environment, models, graphs, drawings, examples from the field of prospective professional activities, facts borrowed from other educational subjects, etc.) or from the description of practical actions. This enables finding out of the concept's essential features, the mathematical object properties and, on their basis, the corresponding statement independent formulation.

The second stage. The essence of the mathematical fact is clarified and substantiated, then merely mathematical problems are solved. When substantiating mathematical statements, one should neither overindulge in the formal and logical strictness of proofs nor in spending a lot of time on cumbersome transformations and calculations. More attention should be paid to the understanding of concepts, properties, and ideas' meaning.

The third stage. Pragmatic application. Schoolchildren should acknowledge that the process of applying Mathematics to any practical problem solution includes: formalization (the transition from the situation described in the task, to the situation formal mathematical model, and from it, to a clearly formulated mathematical problem); solving the problem within the constructed model framework; interpretation (applying the solution to the original situation). In a separate block of tasks entitled "Apply in Practice" there are practice-oriented tasks, typical practical situations, where it is necessary to employ the material under study. These stages should be inherent in educational activities, since they foster students' creativity, activity and initiative development.

The enhancement of the course's practical application is facilitated by the students' familiarization with the concept of a mathematical model, with the method of mathematical modeling, the development of ideas about the method's role in the scientific cognition and practice, the simpler mathematical model building skills formation.

Among the most important requirements for the textbook content the systematization of concepts, properties, and methods for solving tasks (tables, diagrams, tasks according to tables, classification) and the content integration (reinforcing the links of Algebra and Calculus with Geometry) should be mentioned. This will enhance the integrity of knowledge and improve its application to solving problems, in particular, of practical content.

Conclusions

The general theoretical and methodological requirements for the selection of educational texts, the system of tasks and the methodological apparatus of Mathematics textbooks for lyceums are offered, in particular: the formation of student personality's positive self-concept, a stable motivation to study the subject; the enrichment of schoolchildren's mathematical as well as general cultural potential; the scientific rigor and accessibility of textbooks; the educational texts scientificity and accessibility; the continuity observed in its two functions – the compensatory and the prognostic ones; the priority of the developmental learning function; practical orientation of educational material; the content meeting the students age and cognitive characteristics; the educational material systematization and integration. The compliance of the above-mentioned requirements to the Mathematics textbooks content, and of the suggested ways of their realization enhances the high-school students' quality mathematical education.

1.2 Competence Approach to Mathematics Teaching and Solving Physical Problems Graphically*

L. Kulyk, Z. Serdiuk, T. Bodnenko

*The article is published in the author's translation

A competence approach was put as the basis for building the contents and organization of the teaching mathematics process in the basic school. According to this approach the final result of teaching this subject is some formed competences as a pupil's abilities to apply the knowledge in studying and real life situations etc.

The present curriculum in mathematics for the 5-9 grades says[1] (2017), "teaching mathematics should make some contribution to the formation of the key competences". Namely, among the key competences they separate the basic competences in the natural sciences and technologies, which imply formation of the pupils' skill to recognize the problems arising in the environment which can be solved by the mathematical means; to build and research the mathematical models of the natural phenomena and processes etc.

We understand the subject mathematical competence as the pupils' ability to act on the basis of the received knowledge not only in exceptionally mathematical situations but also in the real ones. Therefore, supporting N. Tarasenkova[2] (2016), we consider two manifestation levels of the subject mathematical competence. Thus there are also two levels of its formation – factual level (an ability to act exceptionally in mathematical situations) and praxeological one (an ability to act in practical situations, including transferring knowledge to the other subject branches). Meanwhile, the praxeological level of the mathematics competence received by the pupils

[1] *Mathematics. 5-9 classes. Educational program for general educational establishments*. (2017). Retrieved from: https://mon.gov.ua/storage/app/media/zagalna%20serednya/programy-5-9-klas/onovlennya-12-2017/5-programa-z-matematiki.doc. [In Ukr.]

[2] Tarasenkova, N. (2016). *Competence approach in teaching mathematics: theoretical aspect*. *Mathematics in native school. № 11* (179). 26-30. [In Ukr.]

on some mathematical contents not very seldom transforms into the key mathematical competence, which is necessary for studying other school subjects.

The present curriculum in physics for the 7-9th grades[3] (2017) says, that "the process of the teaching physics in the basic school is aimed at the development of a pupil's personality, formation of his/her scientific outlook and the corresponding way of thinking, formation of the subject, scientific-natural (as a branch competence) and the key competences". Thus, among the key competences they defined the mathematical competence which provides for the pupils' skills: to apply mathematical methods for description, research of the physical phenomena and processes, solving physical problems, working-out and evaluation of the experiment results; to understand and use the mathematical methods for the analysis and description of the real phenomena and processes physical models; realization of the importance of the mathematical apparatus for the description and solving physical problems and tasks. The teaching resources which should be mastered by the pupils include: tasks on making calculations, algebraic transformation, making plots and pictures, analysis and presentation for the results of the experiments and laboratory works, processing of the statistic information, information presented in the plot, table and analytical forms.

A bright and efficient means for the formation of the basic school pupils' the mentioned above competences is conducting binary lessons in mathematics and physics or, which is more realistic, usage of the physical problems corresponding to some studying theme at the lessons of mathematics. From the 7th grade pupils start studying physics as well as they start studying algebra and geometry separately. Thus in the 9th grade they have a sufficient base of knowledge in all these subjects. Therefore usage of the physical problems or the problems of the physical contents at the lessons of algebra or geometry is rather actual and efficient.

[3] *Physics. 7-9 classes. Educational program for general educational establishments*. (2017). Retrieved from: https://mon.gov.ua/storage/app/media/zagalna%20serednya/programy-5-9-klas/onovlennya-12-2017/7-fizika.doc. [In Ukr.]

Many Ukrainian scientists, who deal with competence approach implementation into the educational process of the secondary education institutions, focus their attention mainly on the theoretical and methodological grounds of the problem. Contents, structure, formation of the future specialists' competence were considered in the works of N. Bibik[4] (2004), A. Khutorsky[5] (2003) and others. Many works were fixed on the development problems of the school mathematical education in Ukraine. Namely, I. Akulenko[6] (2013) researched the theoretical bases of the future teachers of mathematics' competence oriented methodical preparedness in the profession-oriented school, I. Lovianova[7] (2014) researched profession oriented teaching mathematics in the profession-oriented school, N. Tarasenkova, I. Bogatyriova, O. Kolomiets and Z. Serdiuk[8] (2015) received the considerable results in the development of the means for checking mathematical competence in the basic school etc. T. Zasyekina[9] (2016), O. Pinchuk[10] (2011) and M. Sadovyi[11] (2015) presented some methodical aspects of renovation the contents, methods, organizational forms and means of the pupils' active activity according to the competence approach in the process of teaching physics.

[4] Bibik, N. (2004). Competency Approach: Reflexive Analysis. *Competency Approach in Modern Education: World Experience and Ukrainian Perspectives: Library on Educational Policy.* Kiev. 45–50. [In Ukr.]

[5] Khutorskoy, A. (2003). Key competencies as a component of a personally oriented education paradigm. *People's Education. № 2.* 58–64. [In Ukr.]

[6] Akulenko, I. (2013). *Competency-oriented methodical preparation of the future teacher of mathematics of the profile school (theoretical aspect). Monograph.* Cherkasy: Publisher Chabanenko Yu. [In Ukr.]

[7] Lovyanova, I. (2014). *Professional training of mathematics in profile school: theoretical aspect. Monograph.* Cherkasy: Publisher Chabanenko Yu.A. [In Ukr.]

[8] Tarasenkova, N., Bogatyreva, I., Kolomiyets, O., Serdiuk, Z. (2015). Means of checking mathematical competence in the basic school. *Science and Education a New Dimension, III (26), Issue 71.* 21-25. [in Ukr.]

[9] Zasekina, T. (2016). *Formation of key and subject competences of students in the process of teaching physics.* Chisinau: Institutul de Stiinte ale Educatiel. 228-230. [In Ukr.]

[10] Pinchuk, O. (2011). Formation of the subject competences of primary school students in the process of teaching physical means of multimedia technologies: *Candidate's thesis.* Kiev [In Ukr.]

[11] Sadovyi, M. (2015). Method of formation of experimental competencies of senior pupils by means of modern experimental kits in physics. *Pedagogical sciences: theory, history, innovative technologies. Sumy.* 268-279. [In Ukr.]

It is still an acute problem in school practice to teach pupils studying mathematics to properly apply their knowledge for solving problems in other branches, namely physics, chemistry, biology, geography etc. in spite of the wide range of the pedagogical, psychological, and methodological researches (according to the questionnaire results of the pupils, teachers and students; according to the results of the state final examination and external independent testing in mathematics; according to the results of TIMSS and PISA). A teacher should correctly from the didactic point of view combine the formation of the key mathematical competences as a result of training the basic abilities and skills and the skill to use them while solving applied problems or even competence tasks. It is necessary to do this work at the lessons of mathematics, namely at the binary lessons of mathematics and physics, mathematics and biology etc. as well as during out-of-classes activity, for example, involving the contents of the optional courses, electives and mathematical circles.

According to the curriculum[12] (2017) a functional line is one of the main notions in the course of algebra in the 7-9th grades, besides it is developing in the close connection with the identical transformations, equations and inequations. As a rule the properties of the functions are defined according to their plots, i.e. on the basis of the visual imaginations, and only some properties are substantiated analytically. During teaching functions a leading place is given to the formation of the abilities to make and analyze the plots of the functions, according to the functions to characterize the processes described by them, to the ability to understand a function as a certain mathematical model of the real process. For training the pupils' knowledge, skills and abilities one should use physical problems, where real physical processes are described.

A graphical method in physics provides for the usage of plots for the description and explanation of the real processes and conformities, and it is a powerful means

[12] *Mathematics. 5-9 classes. Educational program for general educational establishments.* (2017). Retrieved from: https://mon.gov.ua/storage/app/media/zagalna% 20serednya/programy-5-9-klas/onovlennya-12-2017/5-programa-z-matematiki.doc. [In Ukr.]

for solving physical problems. It is closely connected with studying plots of functions in the course of mathematics. Studying of one and the same material in different courses mutually enriches academic subjects and fills mathematical examples with the concrete contents. This method should be used from the first lessons of the course called “The Bases of Cinematics”. Graphical image of the rectilinear motion laws and the analysis of the plots allow to teach the pupils to define the motion character and numerical meanings of the path, displacement, speed and acceleration according to the plot; to visually depict functional dependences of the cinematic values; to compare the plots of motions, according to which cinematic values can be defined; to teach the pupils to solve the problems in encounter motions of the bodies (to define time and place of the encounter, speed at the moment of the encounter etc.).

Problems using plots can be divided into two kinds:

- problems, where receiving an answer is possible as a result of making a plot;
- problems, in which receiving of an answer is possible as a result of plot analysis.

An example of the first kind of problems can be the following problem.

Problem 1

A ball is feely falling down from some height. Considering its hit on the ground absolutely resilient, make a dependence of the ball motion speed against time.

Solution

We are doing an imaginary experiment. Let’s image, that a ball is at some height and its initial speed equals zero. As a result of its free falling down at the moment of touching the ground, the ball had some speed ϑ. Due to absolutely resilient hit the direction of the ball speed changes into the opposite one, and it module remains the same. This happens during a very short period of time, which can approximately be considered instantaneous. The next movement of the ball is rectilinear, uniformly retarded and its final speed equals

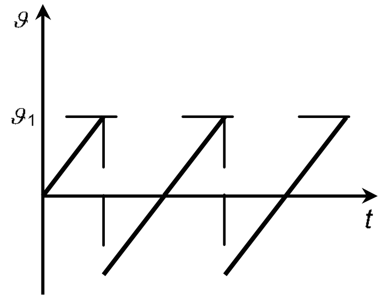

Figure 1.2.1

zero. Making a plot for dependence of the ball motion speed against time is better to draw on the blackboard by one of the students with active discussion of the audience. The plot is presented on the Figure 1.2.1.

Usage of plots contributes to the pupils' visual and deeper understanding of the physical process, it teaches them to express the functional dependence graphically, it gives an opportunity to imagine the given task, and also its solution. Since schoolchildren, even those who make plots very well, do not always see the connection of the real processes with the functional dependence.

The preference of the graphical method usage as compared with the other methods can be demonstrated by the following problem.

Problem 2

A car drove the distance between two settlements with the average speed ϑ_c = 54 *km/h* during the time t = 16 *min*. Speedup and braking lasted t_1 = 8 *min*, and the car moved equally for the rest of the way. What speed ϑ did the car have during equable motion?

Solution

Let's depict the plot for the dependence of the car speed against time (Figure 1.2.2). The section *OA* corresponds to the speedup of the car, the section *AB* corresponds to the equable motion, and the section *BC* corresponds to its braking. The path driven by the car is numerically equal to the area of the trapezium formed by the plot of speed:

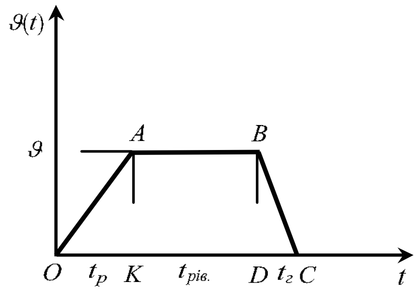

Figure 1.2.2

$$S = \frac{AB + OC}{2} AK \text{ or } S = \frac{2t_{equal} + t_1}{2} \vartheta .$$

On the other hand, from the definition of the average speed

we have: $S = \vartheta_c t$. Thus, $\vartheta = \dfrac{2\vartheta_c t}{2t - t_1}$.

In order to prepare pupils for solving the data of the physical problems it is desirable to revise with them the following:

1 Linear function, its properties and plot;
2 Areas of the main geometric figures (triangle, trapezium, parallelogram, rectangle, rhombus etc.) depending on what exactly geometric figures will be used in the offered tasks.

To prepare the pupils to solving the mentioned physical tasks it is recommended to offer them the following questions for actualization the basic knowledge:

- What function is called a linear one?
- What is the plot of the linear function?
- How many meanings of the linear function is it necessary to know for making its plot?
- How is it possible to make a plot of the linear function?
- With what meanings of k the linear function $y = kx + b$ is increasing (decreasing; stable)?
- How does number b characterizes the linear function?
- How is it possible to find zeroes of the function according to the plot?

- How can we find points of intersection the plot of the function with the coordinate axes?
- How can one define the intervals of the function increasing (decreasing) according to the plot?
- According to what formulae do they calculate an area of a random triangle?
- According to what formula do they calculate an area of an equilateral (rectangular) triangle?
- How can one prove equation of two uneven figures areas?
- According to what formula do they calculate an area of a trapezium?
- How is it possible to find an area of a trapezium, knowing its altitude and middle line?

Except traditional tasks, offered to the pupils at the lessons of mathematics for making plots of the simpler linear functions like: $y = 0{,}6x - 1{,}2$; $y = -3x + 1{,}5$, it is recommended to offer the schoolchildren to make plots of the more complicated functions[13] (2015):

$$1)\ y = |x| + x;\ 2)\ y = |x| - x;\ 3)\ y = \frac{|x|}{x};$$

$$4)\ y = \begin{cases} x-2, \text{ if } x \le -1, \\ 3x, \text{ if } -1 \le x < 2, \\ 2x+4, \text{ if } x \ge 2; \end{cases} \quad 5)\ y = \begin{cases} 5, \text{ if } x \le -2, \\ -x+3, \text{ if } -2 < x < 2, \\ 0{,}5x, \text{ if } x \ge 2. \end{cases}$$

Later pupils can be offered example problems, for instance, from the textbook[14] (2017).

Problem 3

Dependence of the distance driven by a biker against time is presented by the formula:

$$s = \begin{cases} 20t, \text{ if } 0 \le t < 1, \\ 20, \text{ if } 1 \le t < 3, \\ 50-10t, \text{ if } 3 \le t \le 4. \end{cases}$$

[13] Tarasenkova, N., Bogatyreva, I., Kolomiets, O., Serdiuk, Z. (2015). *Algebra: textbook for the 7th form of general education institutions.* Kiev. [In Ukr.]

[14] Tarasenkova, N., Bogatyreva, I., Kolomiets, O., Serdiuk, Z. (2017). *Algebra: textbook for the 9th form of general education institutions.* Kiev. [In Ukr.]

Make a plot of the given function and define its properties according to the plot.

Problem 4

The temperature of the piece of ice is -3°C, it was heated. In 5 minutes ice melted. 10 minutes later the temperature of the melted water began changing from 0°C to 3°C. Make a plot of the dependence of temperature against time. Define the properties of the function.

The problems of the second type contribute to the development of the pupils' basic mental actions, namely analysis and synthesis, generalization and systematization, comparison etc. and formation of their certain subject competences, since analysis and making plots result in problem solution in a general form, which enables to use this approach for solving other problems of this series. The given problems can be offered to the pupils at the binary lessons in physics and mathematics, for example, in the 9^{th} grade, when the pupils have already mastered all the knowledge, skills and abilities, which are necessary for solving the following problems. Of course, binary lessons should not be conducted very often, they require a thorough and long-lasting preparation of both teachers and pupils. But sometimes such lessons are rather efficient. Since problem 5 is offered to solve in two ways, – this can be done in different groups of pupils, then the representatives of every group will explain their solution way to the rest of the pupils, and after that all the pupils will discuss together the possibilities of using one or the other way to solve the different problems.

Problem 5

At what maximum distance l_{max} can a person be, if he is running equally with the speed ϑ in the direction of the bus, moving in the same direction with acceleration a, to be able to catch the bus?

Solution

1^{st} way. We will depict the plots of coordinate dependence against the time for a bus (1) and for a human (2, 3, 4)

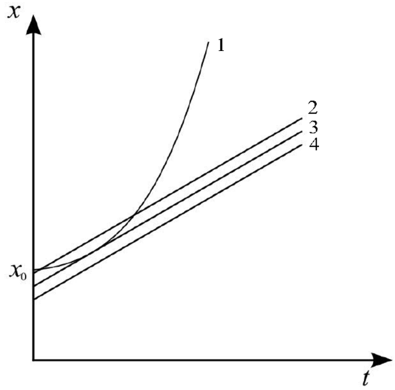

Figure 1.2.3

(Figure 1.2.3). If these plots do not intersect (1, 4), then the human will never catch the bus, two mutual points on the plot (1, 2) do not also satisfy the problem condition (the human must only catch the bus), thus the case fits, when the plot of the coordinate dependence against the time for the human is tangent to the plot of the coordinate dependence against time for the bus (1, 3). In the last case the speed of the bus equals with the human's speed at the moment of encounter.

For a deeper analysis of the given task it is necessary to make plots of the speed dependence against time for the human and for the bus (Figure 1.2.4, Figure 1.2.5).

In the Figure 1.2.4 we analyze dependence l_{max} against the human's speed change ($\vartheta_{л2} \rangle \vartheta_{л1}$). We choose the points *B* and B_1, which correspond to the equation of the human's speed and the speed of the bus. Since l_{max} equals the area of the triangles *OAB* and OA_1B_1 , we make a conclusion, that the bigger the human's speed is the bigger is l_{max}.

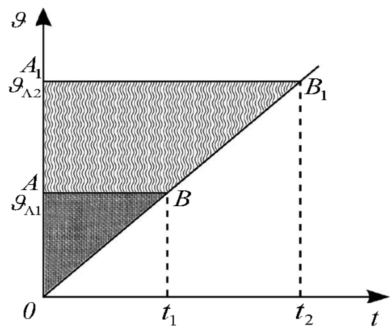

Figure 1.2.4

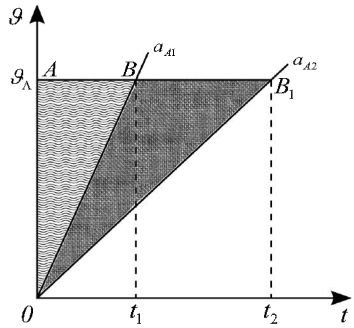

Figure 1.2.5

In the Figure 1.2.5 we consider the situation for different meanings of the bus acceleration ($a_{A1} > a_{A2}$). Comparing the areas of the triangles OAB and OAB_1 we come to the conclusion, that the bigger the acceleration of the bus is, the less is the distance l_{max}. In the Figure 1.2.4 we see, that $l_{\max} = S_{AOB} = \frac{1}{2}\vartheta_л t$. On the other hand, acceleration of the bus is $a_A = \frac{\vartheta_л}{t}$. As a result we get $l_{\max} = \frac{\vartheta^2}{2a}$. Thus, the given problem can be solved by making and analyzing the plots of motion.

While solving a problem the main attention should be paid not only to the solution of the concrete problem, but also to the general approach to solving. An ability to analyze the problem and see its solution in general helps the pupils to consciously find the necessary values, instead of random search for the correct solution. All this requires a pupil's ability to analyze, to generalize, to make an imaginary experiment, to imagine physical processes, which, in their turn, contributes to forming these or those subject competences.

With the purpose of developing flexibility of thinking pupils can also be offered to solve the same problem in another way, in an algebraic one.

Solution

2nd way. We will write down the condition of the human and bus encounter: $x_1 = x_2$ or $\vartheta t = x_0 + \frac{at^2}{2} \Rightarrow at^2 - 2\vartheta t + 2x_0 = 0$.

The condition of the single solution is that a discriminant equals zero. From this condition we find the searched distance: $D = 4(\vartheta^2 - 2ax_0) = 0 \Rightarrow x_0 = \frac{\vartheta^2}{2a}$.

If the discriminant equals zero, x_0 has the maximum meaning, and the time when the human will catch the bus equals $t = \frac{\vartheta}{a}$.

The detailed analysis of different ways to solve the problems will allow the pupils to master one or the other approach and further use it in solving essentially new problems. For training the offered approach to solving problems which provides for the analysis of the plots the schoolchildren are given the following problem (we recommend to give it as the home-task).

Problem 6

Points *A* and *B* are located at the distance $s = 4$ *km* from one another. From the point *A* in the direction of the point *B* started a car, which moved all the time with equal speed. Simultaneously, another car started in its direction from the point *B* with the initial speed $v_0 = 32$ *m/s*. It moved with the constant acceleration $a = 0{,}2$ m/s^2, directed all the time in the same way as the speed of the first car. It is known that the cars twice overran each other. Within what limits is the speed of the first car?

Solution

The plot of the second car motion is a parabola (Figure 1.2.6). It is evident that the speed of the first car cannot be very big, as otherwise overtaking would happen only once (point *B*, while the point *A* corresponds to the encounter of the cars). The speed cannot also be very small (*OC*), because in this case the cars cannot happen to be side by side at all. It means that the equation expressing equality of the cars coordinates: $\upsilon_1 t = s - \upsilon_0 t + \frac{at^2}{2}$, should have two true solutions, besides, both of them correspond to the later moments of time, than the moment of stopping (instantaneous) of the second car: $0 = -\upsilon_0 + at$.

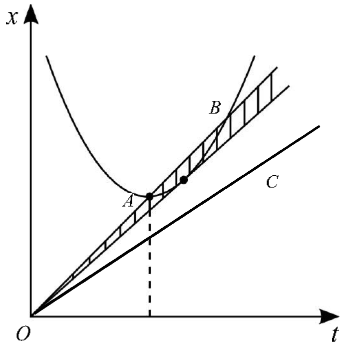

Figure 1.2.6

$$\frac{at^2}{2}-(\upsilon_1+\upsilon_0)t+s=0; \quad t^2-\frac{2(\upsilon_1+\upsilon_0)}{a}t+\frac{2s}{a}=0;$$

$$D=\left[\frac{2(\upsilon_1+\upsilon_0)}{a}\right]^2-\frac{8s}{a}>0; \quad \frac{2(\upsilon_1+\upsilon_0)}{a}>\sqrt{\frac{8s}{a}};$$

$$2(\upsilon_1+\upsilon_0)>2\sqrt{2sa}; \quad \upsilon_1>\sqrt{2sa}-\upsilon_0.$$

Taking into consideration that this speed should not be big to make a overtaking twice, consider the stopping moment of the second car $t=\frac{\upsilon_0}{a}$. Substituting this time in the equation of the motion, we have: $\upsilon_1\frac{\upsilon_0}{a}=s-\upsilon_0\frac{\upsilon_0}{a}+\frac{a}{2}\frac{\upsilon_0^2}{a}$;

$$\upsilon_1=\frac{a}{\upsilon_0}s-\upsilon_0+\frac{a}{2}\frac{\upsilon_0}{a}; \quad \upsilon_1=\frac{as}{\upsilon_0}-\frac{\upsilon_0}{2}.$$

Thus, $\sqrt{2sa}-\upsilon_0<\upsilon_1<\frac{as}{\upsilon_0}-\frac{\upsilon_0}{2}$.

In general, an important basis for the pupils' formation of the subject competences (mathematical ones at the lessons of physics or physical ones at the lessons of mathematics) is making didactically balanced system of the integrated competence tasks (in mathematics and physics) for the pupils of the basic and later senior profession-oriented school. That will take into account the main tasks for studying mathematics and physics at school and will contribute to the multi-sided development of pupils.

We consider that application of such instrument as a binary lesson in related academic disciplines (physics and mathematics, mathematics and biology, mathematics and chemistry, mathematics and informatics, mathematics and geography) is rather efficient for the formation of the subject competences, mentioned in the present curricula in the mentioned academic disciplines. Making such lessons can be the subject matter of our further scientific research.

1.3 Development of Distance Concepts through Metric Spaces as the Implementation of Inter-subject Relations (7th – 9th high school grades)*

I. Lovianova, D. Bobyliev

*The article is published in the author's translation

Introduction

The formation of mathematical concepts and in particular the concept of distance is one of the most important problems in the methodology of mathematics. The concept of magnitude, distance, metric space occupies a fundamental place in the system of mathematical knowledge.

Such geometric magnitudes as lengths, areas, volumes, angle measures are studied in a systematic course of geometry. In the methodology of mathematics, the problem of pupils mastering the measurement of geometric quantities is one of the most difficult tasks, both in theoretical and in methodological sense. Pupils come insufficiently prepared to study the systematic course of geometry. Information and ideas about the values and measure of the values are not sufficiently systematized, generalized and little clarified to pupils' mind.

Analysis of sources

The concept of distance has historically been subjected to multiple generalizations, which was also reflected in the content of the school mathematics course. In the methodology of mathematics, there was paid great attention to the problems of formation among pupils of the concept of distance (Aleksandrov[1], 1980; Markushevich[2], 1952;

[1] Aleksandrov, A. D. (1980). O geometrii [About geometry]. *Matematika v shkole, 3*, 5. [In Rus.]

[2] Aleksandrov, P. S., Markushevich, A. I., & Khinchin, A. YA. (1952). *Entsiklopediya elementarnoy matematiki [Encyclopedia of elementary mathematics]. (Vols 1 – 5).* Moskva-Leningrad: GITTL. [In Rus.]

Kolmogorov[3], 1971; Slepkan[4], 2006; Froydental[5], 1983; Bevz[6], 1989; and others).

In order to facilitate the assimilation of this material in school in recent years, changes are made to school textbooks and teaching aids, concepts are clarified, information is supplemented, study sequences are changed, material is shortened, etc. Until 1960, the school course had a special section "Concrete numbers and operations on the numbers".

Later, due to the modernization of school mathematics, the term "concrete numbers" was replaced by the term "value". Some mathematicians proposed to exclude the concept of magnitude from the school course of mathematics altogether since mathematics can do without it. But the school course of mathematics cannot be limited only to "pure" mathematics and for applied mathematics the concept of magnitude is one of the most important. N. Vilenkin[7] (1973) notes: "The concept of magnitude is fundamental when it comes to applications of mathematics".

In school practice, the importance of this important concept is not given due attention, while the role of distance in the development of a system of knowledge in geometry is fundamental. Unfortunately, in schools, pupils get a vague idea of the distance; most of the important questions are simply omitted while teaching.

The purpose of the study is to develop a method of studying the concept of distance and scientifically correct method for pupils in a course of geometry of the VII-IX grades.

Exposition of the main material

In the process of studying, it is important to achieve understanding by pupils of the fact that the mathematical

[3] Kolmogorov, A. N. (1971). O sisteme osnovnykh ponyatiy i oboznacheniy dlya shkol'nogo kursa matematiki [On the system of basic concepts and notation for the school course of mathematics]. *Matematika v shkole, 2*, 19. [In Rus.]

[4] Slepkan, Z. I. (2006). *Metodika navchannya matematiki* [Methodology of teaching mathematics]. Kiev: Vishcha shkola. [In Ukr.]

[5] Froydental, G. (1983). *Matematika kak pedagogicheskaya zadacha* [Mathematics as a pedagogical problem]. Matematika: Prosveshcheniye. (in Rus.)

[6] Bevz, G. P. (1989). *Metodika vikladannya matematiki* [Methodology of teaching mathematics]. Kiev: Vishcha shkola. [In Ukr.]

[7] Vilenkin, N. YA. (1973). O ponyatii velichiny [On the concept of magnitude]. *Matematika v shkole, 3*, 47. [In Rus.]

concepts with which they operate on the lessons are abstractions of real phenomena and processes of the surrounding world. Pupils' perception of the relationship between geometry and the surrounding world is achieved by combining theoretical and contemporary applied aspects of the school course of geometry. This is facilitated by the fact that the program and the training manuals reflect intra-subject and interdisciplinary communications.

Through the concept of magnitude, the real properties of objects and phenomena are described, the cognition of the surrounding reality occurs; familiarity with the dependencies between the values helps to create a holistic view of the pupils' world; the study of the process of measuring values contributes to the acquesition of practical skills necessary for a person in his or her daily activities.

Speaking of geometrical magnitude, one should clearly distinguish the geometrical figure itself, the magnitude, and the numerical value of this quantity. For example:

1 A geometric figure A________B;
2 The value is the length of the segment AB : $AB = 4$ cm;
3 The value of the magnitude is the numerical value of the length of the segment AB : 4.

The difference of the length of the segment from the numerical value of the length is that the first remains unchanged, and the second depends on the selected unit of measurement.

While studying the concept of length in school, it is important that pupils have a fairly complete and at the same time accessible idea of how to measure length; the role and place of length in the knowledge of nature; length properties (Kurant[8], 1947). Understanding these issues contributes to the formation of pupils' scientific outlook.

The study of the concepts of length allows one to learn not only the qualitative connections of various aspects of objective reality, but also to evaluate them quantitatively.

The first ideas about the length, as a property of objects, occur in children' worldview long before school. From the first days of school, the task is to clarify the spatial concepts

[8] Kurant, R (1947). *Chto takoye matematika* [*What is mathematics*]. Moskva: Gostekhizdat. [In Rus.]

of children. An important step in the formation of this concept is acquaintance with a straight line and a segment, as a "carrier" of linear extent, which is essentially devoid of other properties.

Starting a systematic study of algebra and geometry in the VII grade, pupils have already certain knowledge of length and distance, about a geometric figure as a non-empty set of points, about a coordinate line and a coordinate plane. In the VII-IX grades, pupils get information about the two-dimensional Euclidean space and the method of coordinates. Such knowledge about length and distance creates objective prerequisites for the consideration of modern concepts of distance and metric space. The axioms of distance (metric space) are included in the axioms system of geometry, on the basis of which pupils of the VII-IX grades explore the metric two-dimensional Euclidean space with the substantial-deductive method. So, you can gradually introduce the concept of metric space and create conditions to understand the structural unity of various topics in a school mathematics course for pupils within the framework of a unified concept of metric space.

The concept of distance had studied in the V-VI grades is replicated, systematized and combined in a course of algebra and geometry of VII grade but there is still lack of new material and therefore the focus is on the deductive research method that is already known. Distance is the basic concept, which is given an axiomatic definition based on the concept of scalar quantity.

However, as practice has shown, most pupils realize the distance in geometry as a number (identifies the value with a number). During the proof of the theorem, without explaining the corresponding properties of homogeneous scalar observation, only the illusion of logical proof remains. Therefore, pupils first need to find out those properties of scalar observation on which he or she will rely on, considering the first concepts and theorems of the systematic course of geometry.

For example, such a sequence of heuristic questions and tasks is possible.

1 Remember, with what values you have already faced in the lessons of mathematics in V-VI grades? (Length, distance, time, temperature).
2 What properties of these quantities do you know? (Homogeneous values can be added and compared).
3 Compare the lengths of the two segments and find the measures of their length, measure their length in centimeters and millimeters. (Pupils perform this task on individual cards). a). Has the length (magnitude) of each segment changed after measurement? What determines the numerical value of the length (magnitude) of the segment? b) Will the length of the segment change or the numerical value of its length and how much, if you reduce (increase) the unit of measurement of the length of the segment 10, 20, 50 times?
4 The lengths of the segments are 50 cm, 500 mm, 5 dm, 0.5 m. Are they equal to each other? Why?
5 Remember, what the value was taken as distance between two points in the V-VI grades.
6 Give examples of two unequal homogeneous (scalar) values. (The length of the segments is 5 m and 7 m, time: 2 hours and 5 hours, etc.). Write down the relationship between them using the inequality sign. Will the inequality break, if in each of its parts add or subtract the same value that is homogeneous to the data?

After completing this work, pupils should come to the conclusion:

1 Distance and length are values;
2 Homogeneous quantities can be added, subtracted, compared (put the sign “equal”, “more”, “less”);
3 By choosing a quantity that is homogeneous with a given one, it is possible to measure this quantity with it and find its numerical value;
4 The value does not depend on the unit of measurement;
5 If from each of two unequal values you get (add) a value equal to the data, then the inequality is not violated. (Pupils should familiarize themselves with other characteristics in the lessons of algebra and physics).

Then pupils perform a special laboratory work and formulate the basic properties (axioms) of distance:

1 $|AB| > 0 \Leftrightarrow A \neq B$ and $|AB| = 0 \Leftrightarrow A = B$;
2 $|AB| = |BA|$;
3 $|AB| \leq |AC| + |CB|$.

It is most difficult for pupils to understand that each of these properties is necessary, and together they are sufficient for an axiomatic definition of the concept of distance. Such an understanding should be formed gradually, having first mastered the understanding of each individual property in the exercises. In order to establish connections between verbal statements, their symbolic recording and a graphic image, pupils perform special exercises according to the scheme (in direct or reverse order): the drawing is a verbal expression of the dependence depicted on it – a symbolic recording.

In the VII grades there is an opportunity to review more detailed the issue of the distance from a point to a figure and from one figure to another. Pupils of the V-VI grades have already done such exercises by inductive-practical method. After studying the topic "Distance from a point to a straight line" it became quite easier for pupils to do such exercises.

Exercise 1

Find, by building, the distance from point C to segment AB of a straight line.

This task can be given a certain practical meaning.

Exercise 1.a

The plumbing on the street of the city (village) is shown by a segment AB on the plan. How to conduct water to the house C by the shortest way? It is advisable for pupils to distribute two graphically defined conditions according to which two cases need to be considered: a) the base of the perpendicular CO is the point of the segment AB; b) the basis of this perpendicular does not belong to the segment AB.

The solution is analyzed and summarized by using tables. This helps pupils to develop observation, to do the simplest rationale, contributes to the development of their thinking.

Exercise 1.b

Calculate how many meters of pipe you need to supply water to the house *C* (indicating the scale of the scheme).

Solving this task based on the solution of the previous one that leads pupils to the conclusion that distance can be expressed in numbers.

Exercise 2

Find the construction of the distance from point *D* to the set of points of the triangle *ABC* (various locations).

Exercise 3

Find by constructing the distance from the segment to the segment.

You have to use the following algorithm to find the distance from a given point to a given convex polygon using the construction: 1) find the sides of the polygon (there are no more than two), such that the straight lines, which are determined using them, divide the plane into two half-planes, and this polygon and this point lies in different half-planes; 2) from this point we draw perpendiculars to the marked sides; 3) if the basis of one of the perpendiculars constructed belongs to the side of the polygon, then the length of the segment of this perpendicular is taken as the required distance; 4) if the base of none of the perpendiculars does not belong to the sides of the polygon, then we take the segment connecting the given point with the vertex of the polygon for the required distance. This algorithm is given in tasks 1-3.

Exercise 4

Find the distance from the point to the polygon.

Exercise 5

Find the distance from the circle to the given polygon.

Exercise 6

Find the distance from the polygon to the polygon.

While solving tasks of building a figure as a set of points with certain metric properties, the role of the pattern also grows. The task system should provide a gradual complication of them.

In order for pupils to clarify the metric properties of a parabola, they have to do the following tasks:

1 Find the set of points equally distant from this point and the x-axis.
2 Build several points of this set using drawing tools.
3 Find the set of points equally distant from the given point and ray.

To solve the first task, pupils should remember how the distance between two points on a plane is analytically expressed, and be able to select a full square.

The challenges of moving tasks have a significant role in the formation of metric concepts. Such tasks help pupils to realize that geometric concepts and theorems reflect the spatial properties and relationships of real objects. Pupils will be able to solve such tasks successfully only after completing the exercises that form the ideas and methods of the movements considered. The idea of constructing the law of displacement for an arbitrary point is the most difficult to assimilate for pupils. In the application of this law for the characteristic (defining) points of the figure to be moved, pupils also encounter certain difficulties. Therefore, it would be better for pupils to follow such sequence of operations:

1 Repeat the rules by which an arbitrary point of the plane moves (This operation at the beginning of such exercises is performed in verbal form. Next, pupils perform it in their minds).
2 Find the characteristic points of the shape you want to move, apply this rule to them (using drawing tools).
3 On the constructed characteristic points of the shape, build its image.

Pupils should be required to justify maintaining the distance between pairs of characteristic points before and after moving.

The task of finding the distances between the figures, which are solved by the method of displacements, it is advisable to provide with a certain practical content. Then the concept of distance (and other concepts) is concretized through the objects of the surrounding reality, which creates in pupils a positive emotional attitude towards mathematics. Here reciprocal problems are useful: to mathematize a specific situation and for a given mathematical task to find the corresponding life concretization.

It is important to build models of abstract concepts which is an effective means of finding contradictions in false theoretical statements and is preparing for solving the problems of consistency of axiomatically constructed mathematical theories. In addition, the theoretical position can be specified, which causes interest among pupils (Ryzhik[9], 2003).

The very important question is the relationship between distance and its numerical value, consideration of which it is advisable to begin when considering the ratio of numbers and quantities. Exercises for determining and comparing distances between pairs of points on different numerical line models should be considered. By comparing the lengths of the segments and their numerical values, the pupils conclude that the ratio of the distances does not depend on the choice of the unit of measurement, therefore, the comparison of distances can be replaced by comparing their numerical values with the same unit of measurement (its dimension does not indicate during the clarification of theoretical questions).

The teacher should gradually explain to the pupils why the distance as a geometric quantity should be replaced (with a fixed unit of measurement) with its numerical value: this makes it possible to establish a close connection between studying algebra and geometry, replacing in many cases the approximate distance measurement with an exact calculation. In the process of solving metric problems by the method of coordinates, the teacher must draw the pupils' attention to the fact that the coordinates are used as an aid, the geometric properties of the figures do not depend on the choice of the coordinate system. However, the type of equation that is used to determine the geometric shape

[9] Ryzhik, V. I. (2003). *3000 urokov matematiki* [*3000 math lessons*]. Moskva: Prosveshcheniye. [In Rus.]

analytically depends on the choice of the coordinate system. Pupils should be taught to choose such an arrangement of the coordinate system relative to a given shape, so that the coordinates of these points and the equations of the lines in question have the optimal form (contain the minimum number of parameters).

The attention of pupils purposefully should be directed to the fact that the school course of mathematics, geometry is replete with particulars. So, in many geometry textbooks, different kinds of distances are mentioned: from a point to a straight line and between two straight lines. But even this sorting of various distances does not cover what one sometimes encounters in practice or in problems from the same geometry textbooks.

In such situations, I want more generality, naturally the desire to have a definition of a distance that would work in all cases. Thus, we will follow the well-known scientific tendency: to formulate the concept in the maximum generality in order to then apply it in a particular case. In most textbooks, the distance between two figures is defined as the length of the shortest segment connecting the points of these figures. In other words, the distance between two figures is the distance between the nearest points of the figures. If the figures have a common point, the distance between them is considered to be equal to zero. Zero distance between the figures indicates their contact. Note also that if one of the figures is a point, then we will talk about the distance between the figure and the point, and if both figures are points, then we have the distance between the points. Thus, our definition is consistent with the axiom of distance.

As a propaedeutics of the notion of distance in space in the VII-IX grades, it makes sense to consider the distance over the surface, say in problems where we are talking about the development of figures. A well-known example: a spider is on one side of a cube, and a fly is on another; it is required to find the shortest path along the surface of the cube from the spider to the fly (provided that the data in the problem is sufficient) (Natanson[10], 1950).

[10] Natanson, I. P. (1950). *Prosteyshiye zadachi na maksimum i minimum* [*The simplest tasks for maximum and minimum*]. Moskva: Nauka. [In Rus.]

In accordance with this definition, finding the distance between figures is the task of finding the smallest value between points belonging to different figures, i.e., the problem of "extremum". Such tasks can be considered as optimization tasks. "Extreme problems", and in particular, to search for the smallest value, are found in geometry. With this general approach, finding the distance between the figures becomes the task of finding extremum for the figures.

It should be noted that the search for the distance between the figures is as problematic as finding the extreme values of the function. The definition of the distance between the figures as the length of the shortest segment between their points does not imply the existence of such a segment. And if it is, then the question arises: how many such shortest segments can be constructed, how many times can the shortest length be achieved? Both must be clarified specifically.

It happens that there is no distance between two figures – this is when they do not have the nearest points, for example, if the given figures are circles without borders and have no common points. To put it mathematically, if two "open" circles are given, then in this case it is impossible to indicate the nearest points of the figures. In a school course, geometries with similar figures do not have a case, but beyond its limits such situations exist, and in this case one can introduce the concept of the distance between two figures as an exact lower bound of all possible distances between points of these figures.

The length of the shortest segment between two figures is not always easy to find, even if there is enough data in the problem. Sometimes this requires a lot of computational work. It is necessary to caution pupils from verbal confusion that sometimes occurs. You can hear or read about the "shortest distance between the figures" as if there are many such distances and you need to find only the smallest of them. The distance between two figures, if any, is unequivocally determined, although it can be achieved more than once. And if something exists in one instance, then there is simply nothing to choose from.

Let us consider how to use the general definition of the distance between figures in space in the simplest cases. The simplest figures in space are the point (A, B, ...) and the line (a, b, ...). Accordingly, one can consider questions about the distance p between:

1 Two points $|A, B|$;
2 A point and a straight line $|A, b|$;
3 Two straight lines $|a, b|$.

The study and subsequent application of these questions about the distances between the simplest figures usually do not cause difficulties for pupils, with the exception of the latter case.

Conclusion

The teacher should develop (by simple examples, starting from the seventh grade) the thesis about the existence of non-Euclidean spaces studying metric concepts for pupils. After all, the fact of the existence of geometries other than Euclidean is of fundamental importance: it provides an opportunity to find out the role of idealization in scientific natural science, to show geometry as an abstract mathematical discipline. The clarification of these issues has a positive effect on the formation of pupils' worldview.

1.4 Pace-of-learning-adjusted High School Mathematics: Didactic Principles of Differentiated Instruction*

O. Plysiuk

*The article is published in the author's translation

Introduction

In order to provide high-quality educational services for students teacher must considerate the individual educational trajectory, which should be realized through the free choice of the types, forms and pace of education, educational programs and the level of their complexity, methods and means of teaching of the mathematics lessons. One of the directions towards solving this problem is the differentiation of learning based on the pace of learning, or rather, based on the difference in the rate of learning among students of the same class. The problem of learning the material by students on mathematics lessons is quite acute due to the large amount of information that students receive at school, from the media, from the environment, in the family. Practice shows that considering the intellectual capabilities of students, introducing a differentiated approach on the mathematics lessons leads to an increase of the learning rate of the new material. The task is to construct a learning process for students based on a deeply differentiated basis, which is one of the ways to considering the difference in the learning pace.

The concept of the learning pace is not new in the theory and practice of study. Many researchers who analyzed the problems of internal differentiation during study, understood and understand the learning pace of material differently, therefore, they put their understanding into it. By way of using the learned information, there are two types of activities, one of them is reproductive, the other is productive. Proceeding from the essence of each of these types it can be affirmed that the primary is reproductive activity, and productive one is a superstructure over it. During reproductive activity, the learned material is only reproduced by the subject of learning in various connections and combinations – from a literal copy to any of its reconstructive reproduction and application in typical situations that are conditioned and determined by learning.

Review of the latest researches by the subject

The scientific and theoretical basis includes the works of well-known teachers, where the idea of differentiated learning based on students' abilities was studied in such aspects as learning the forms, levels and types of differentiation (Malafiyk[1], 2015; Babanskyi[2], 1977; Butuzov[3], 1972; Bespalko[4], 1991; Selevko[5], 1998; Zahvyazinsky[6], 1990; Unt[7], 1990; Pasichnik[8], 1990; Deinichenko[9], 2006; Sikorskyi[10], 2000; De Groot[11], 1995; Firsov[12], 1994; etc.); the study of differentiation as a mean of failure preventing

[1] Malafiik, I. V. (2015). *Didactic of the new school: tutorial*. Kyiv, Ukraine: Publishing House "Slovo". [In Ukr.].

[2] Babansky, Yu. K. (1977). *Optimization of the study process: the general educational aspect*. Moscow, Russia: Pedagogika. [In Rus.].

[3] Butuzov, I. D. (1972). *Differentiated approach to teaching students during a modern lesson*. Novgorod, Russia. [In Rus.].

[4] Bespalko, V. P. (1991). *The components of educational technology*. Moscow, Russia: Pedagogika. [In Rus.].

[5] Selevko, G. K. (1998). *Modern educational technologies: textbook*. Moscow, Russia: Nar. obrazovanie. [In Rus.].

[6] Zagvyazinsky, V.I. (1990). *Innovative processes in education:* [collection of scientific papers]. V. I. Zagvyazinsky (Ed). Tyumen: Tyum. state University, Russia. [In Rus.].

[7] Unt, I. E. (1990). *Individualization and differentiation of learning*. Moscow, Russia: Pedagogika. [In Rus.].

[8] Pasichnyk, I. D. (1990). *Operational structures of systematization in the process of mastering the school course of mathematics*. Rivne, Ukraine: Oblpoligraphizdat. [In Ukr.].

[9] Deinichenko, T. I. (2006). Differentiation of study in the process of the group form of its organization (On the example of natural-mathematical cycle: *Extended abstract of candidate's thesis*. Kharkiv, Ukraine. [In Ukr.].

[10] Sikorsky, P. I. (2000). *Theory and methodology of differentiated study*. Lviv, Ukraine: Spolom. [In Ukr.].

[11] Groot R., de (1995). Differentiation in education. *School Principal. №1*. 2-6. [In Rus.].

[12] Firsov, V. V. (1994). *Differentiation of teaching based on mandator learning outcomes*. Moscow, Russia. [In Rus.].

(Budarny[13], 1965); the development of cognitive abilities (Kirsanov[14], 1980; etc.).

Differentiation of education process is carried out with the help of different educational tools and on different levels, in different organizational forms. The level differentiation is based on the learning outcomes planning. T. Deynichenko[15] (2006) believes that the student, considering his/her interests and abilities, has the opportunity to choose the amount of material, to vary his/her workload. Dutch scientist Ronald de Groot[16] (1995) distinguishes three levels of differentiation. Micro level involves the use of different approaches to separate groups of children of a class. I. Butuzov[17] (1968) believes that the basic meaning of the differentiated approach is that, knowing and considering the individual differences of students in the study, determine the most rational nature of work in the classroom of each of them.

The purpose of the study is to substantiate the feasibility of using differentiated study of the mathematics for students of the high school, considering the difference in the learning rate at all stages of studying the material.

Main material

As the theory and practice of teaching show, the process of learning is proceeding stepwise. Psychologists and teachers distinguish 5 levels of assimilation: understanding, recognition, reproductive, productive and creative. Sometimes the notion of levels of knowledge is used. It is worth to note that in many scientific and pedagogical publications there is a division into three levels of assimilation: low, medium and high. Moreover, it is believed that such classification of

[13] Budarny, A. A. (1965). Individual approach to learning. *Soviet Pedagogy*. 7. 51-56. [In Rus.].

[14] Kirsanov, A. A. (1980). *Individualization of educational activities of the pupils*. Kazan: Tat. Publishing House. [In Rus.].

[15] Deinichenko, T. I. (2006). Differentiation Of Study In The Process Of The Group Form Of Its Organization (On The Example Of Natural-Mathematical Cycle: *Extended abstract of candidate's thesis*. Kharkiv, Ukraine. [In Ukr.].

[16] Groot R., de (1995). Differentiation in education. *School Principal*, 2-6. [In Rus.].

[17] Butuzov, I. D. (1972). *Differentiated approach to teaching students during a modern lesson*. Novgorod, Russia. [In Rus.].

levels is suitable for all cases of life. I think that equality is a fundamental feature of not only the process, but also its outcome, and therefore each level of assimilation differs from each other by its internal content, its psychological and pedagogical content. The notions of “low”, “medium”, “high” – have a domestic meaning, and they are not interpreted in science. Each of these levels of assimilation has its own, that belongs only to it, method of diagnostics, which allows us to affirm: the student has mastered knowledge at this level or has not. The learning of the material by the student begins with mastering the zero level – the level of understanding, that is the essence of the material. Further, the movement continues in the direction of reproductive, productive and creative learning. Following the law of the hierarchy, we can assert that the process of learning goes from the simplest level to the most complex one.

Mathematics is one of the disciplines that needs knowledge gained from the study of the previous material. Teaching according to the program, every teacher expects his students to master the material at a productive or creative level. But unfortunately, not all children learn the material at these levels. In my opinion, it is possible to teach everyone, but everyone at different level, depending on the characteristics of the student. The school curriculum is steadily growing, which causes insufficient time to study and students’ exhaust. Students with average abilities are tired more than others. Considering that, teachers reduce the pace and depth of the material. This approach brings the learning process closer to the optimal one, but simultaneously worsens the conditions for children with higher abilities. These children became not interested in studying, their interest disappears. Differentiation of study solves problems of school by creating a new methodological system for teaching of students.

The differentiation of learning by level involves managing the knowledge process, which considerate the individual capabilities of students. Each of the levels of learning: understanding, recognition, reproductive, productive or creative, has its own requirements for diagnostics of learning. Rate of material learning is the speed of learning from lower to higher level of assimilation. Comparing the learning pace of students of the class, each student can be classified to a group with different learning rate (Malafiik & Plysiuk, 2015[18], 2016[19]). And this is a sufficient basis for the implementation of the internal differentiation. Students' differentiation should correspond to the conditions of the frontal, group, paired and individual forms of study, which would create opportunities for activating their educational and cognitive activities. The use of different types of activities must be determined by the teacher considering: the didactic and educational goals; the specifics and complexity of the educational material; previous training and abilities; teacher's qualification. The knowledge of the students' mental differences for the formation of typological groups, which are based on different levels of knowledge, is essential here. Groups for differentiated learning should be formed according to specific study conditions. Even on different stages of the lesson, differentiation can be made depending on the content of the study material. Considering individual possibilities, it is possible to distinguish four types of students depending on the learning pace of the educational material. Allocated groups are not homogeneous, because they include students with different levels of abilities.

Students with the highest potential, with the highest learning rate of educational material with great interest work on cognitive tasks. High autonomy can be traced in their activity. Such students can work without constant supervision of the teacher. Some students in this group need increased complexity of exercises, tasks, where they can show their creativity.

Second group students also have high educational capabilities, but sometimes they need a teacher support.

[18] Malafiik, I. V., Plisyuk, O. R. (2015). About the new understanding of the "Learning Pace of the Material". *Thematic issue "Higher education of Ukraine in the context of integration into the European educational space". Annex 1 to Issue 36, volume VI (66).* [In Ukr.].

[19] Malafiik, I. V., Plisyuk, O. R. (2016). Methodical aspects of the process of math. *Thematic issue "Higher education of Ukraine in the context of integration into the European educational space". Annex 1 to Issue 37, volume III (71).* [In Ukr.].

Such students have skills of independent work, the learning rate is almost the same as the students of the first group, but sometimes they are lack of the focus, sometimes – the ability to work and patience to perform tasks of increased complexity. Therefore, students in this group need periodic monitoring and assistance.

The third group students have average educational opportunities. Such students are not sufficiently acknowledged with the training material required for further study of the subject. Of course, such students need the help of a teacher, they are not always ready to work independently, their learning pace is slower than in the previous two groups of students. They cannot always accomplish the task, and especially the exercises of the increased complexity.

Characterizing the fourth group of students, it is worth saying that work with such group of students is the hardest. These are students with the low learning ability, with a low learning rate of the material. While studying a new educational material, such students face difficulties, as they have no background knowledge from previous topics.

It should be noted that each of these groups can be conditionally divided into three subgroups, as such groups contain students with different learning rates. Therefore, teachers should pay attention to this when preparing the tasks for the lesson. These tasks should be sufficient and they must be different not only by the level of assimilation, but also the individual characteristics of students of subgroups must be considered.

While working at the Ostroh Regional Lyceum-boarding school with an intensive military-physical training of the Rivne Regional Council, I used methods of differentiated mathematics teaching in the process of changing the pace of learning as a way to provide an individual educational trajectory. The main goal of the introduction of differentiated learning was the change in the pace of student learning and the good passing of the external independent evaluation, as in our lyceum study students of 10-11th year of study. Our students enter the 10th class with different levels of knowledge, and the teaching material of the high school is more difficult than the material of secondary school, so the use of differentiated training methods is necessary to considerate the individual characteristics of students. The achievement of compulsory learning outcomes in the system of level differentiation becomes the main criterion on the basis of which the content of the educational work of students is adjusted to achieve compulsory results, or to master the educational material at a higher level. Many years of practice have shown that groups of students are formed primarily on the basis of the criterion of achieving compulsory results, due to which I have a reference point for determining the content of the differentiation of learning in the process of changing the learning pace. At the beginning of the study, diagnostic control work is carried out to determine the level of knowledge of students, and in the future students are divided into groups and subgroups according to the learning rate and considering their IQ. As the student remains at each level of learning for some time, therefore to diagnose the learning pace students must have 5 levels of tasks. Each task should diagnose only one level, the one for which diagnosis was made. In other words, all tasks must be valid relatively to their respective levels of learning.

The main stages of technology, built on the above considerations, are as follows:

Stage 1. Examine the class, as deep as possible knowledge about each particular student.

Stage 2. Thorough preparation for testing for the purpose of determining the IQ coefficient based on Eysenck's methodology or some other practices.

Stage 3. Determination of the coefficient of IQ and acquisition (virtual) students into the groups: high, low and medium.

Stage 4. Establishing on the basis of the curriculum the basic concepts, rules, dependencies and regularities.

Stage 5. Determining the time (number of lessons) that the curriculum establishes for studying the topic and determining the number of lessons for the first, main, basic study of the subject matter.

Stage 6. Structuring the material of the training topic, planning and forecasting the expected result at the end of the basic training.

Stage 7. Organization of study of the material mainly in the form of different types. At the end of the basic period the set of the groups is so that representatives of different cognitive groups were represented.

Stage 8. Organization of group work. Creating conditions so that stronger students explain for the weaker ones. Interviews of stronger and weaker students about the learning of the new material. The explanation of weaker students for stronger ones. Assessment of educational activities by each student.

Stage 9. The first guiding (or test) work. Based on this work, the learning rate is set.

Stage 10. Completion of the groups based on the combination of the IQ coefficient and the learning rate of the material.

Stage 11. Organization of work of different groups:

a High-speed group: the movement of learning knowledge on the basis of system-centered (creative level);
b Medium-speed group: work on practical-productive assimilation (productive);
c Low-speed group: the direction of understanding (reproductive assimilation).

Stage 12. Preparation for the final control work.

Stage 13. Conducting the final control work.

Stage 14. Final approval of evaluations and tasks for the long-term perspective (Malafiyk & Plysiuk[20], 2015).

Allowing levels of students' learning opportunities, considering the difficulties that students can meet, has made it possible to organize properly the work and support of students in the class. The analysis of the results of the students' diagnostics, their capabilities, allowed me to highlight the areas in which it was necessary to work with the subgroups, namely: motivation of students, their

[20] Malafiik, I. V., Plisyuk, O. R. (2015). About the new understanding of the "Learning Pace of the Material". *Thematic issue "Higher education of Ukraine in the context of integration into the European educational space". Annex 1 to Issue 36, volume VI (66).* [In Ukr.].

desire to enter higher education institutions, formation of their stable interest in studying mathematics, organization of work on the elimination of gaps in knowledge, skills of students, creation of successful learning situations.

The work with the gifted students was important as well. To their individual educational trajectory it was necessary to include such activities as studying additional topics not included to the program, participation in competitions, competitions of the Small Academy of Sciences of Ukraine, writing and presenting their research works, that is, to provide them with the opportunity to acquire research skills. Such work had a positive result. Students who agreed to engage in research activities became winners in the section "Applied Mathematics" in the competition of the Small Academy of Sciences of Ukraine. Therefore, I believe that it is important to control the teacher, his/her adjusting the student's educational activities and to help develop student's cognitive interest in learning mathematics and improving their knowledge.

The experimental work confirmed the hypothesis of the study: the introduction of differentiated mathematics training in the process of changing the learning pace by high school students enables the provision of an individual educational trajectory to each student, regardless of his/her preferences and abilities, as well as to master the content of education, to comprehend student's life goals. The dynamics of changing the level of real learning opportunities of students are reflected in the diagrams. The experimental data indicates a significant impact of the developed technology on the reduction of students with low and medium learning level. As a result of the work, the number of pupils whose level of educational opportunities has become high or sufficient has increased. Figure 1.4.1 illustrates the change in the number of students regarding the levels and pace of learning at the beginning and end of the experiment.

The differentiation at different stages of one lesson, depending on the tasks and content of the learned material is real. A differentiated approach is appropriate when students are interviewed, during updating the knowledge, studying

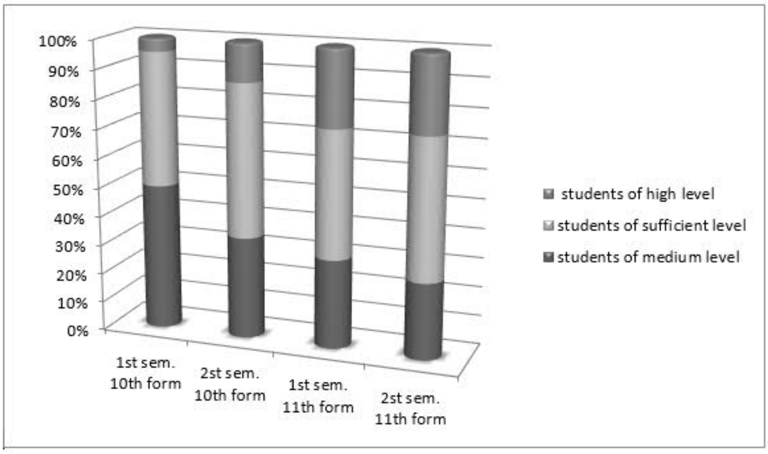

Figure 1.4.1 Changes in students' knowledge quality depending on the experiment period (%).

the new material, as well as explaining the new material, during self- and control work, as well as when compiling a homework that can be structured. It is very important in the process of explaining the new material, to support the weak student in order to teach the basis of the new material. An example of help can be a triple explanation. In this case, it is necessary to provide each student with a complete training in mathematics. Students who have the inclination and ability to math should have the opportunity to master it more deeply.

Level differentiation is made not due to the fact that one student gives less material, and other gets more, but due to the fact that students are offered the same amount of educational material, but different requirements for its assimilation are set. Therefore, there should be tasks of different levels in textbooks. The teacher must ensure the sequence of passing by a student from one level to another. In order to determine what form of activity is chosen by a teacher, he/she needs to analyze the content of the training material, compile a list of basic knowledge and skills, prepare a task framework for collective, group and individual pupils' work, tasks for checking students' knowledge.

Another method of considering the learning pace is the self-work with the elements "lessons on the inside."

The teacher in advance gives the students the opportunity to work out the theoretical material from the textbook, everything that is not clear to them, they must write in the notebook, and then in the class work over the material and perform the level tasks together with those who understood the material.

For selection in subgroups you can apply specially developed by psychologists tests of intellectual development. Tests represent a series of different tasks that a child must perform over a suitable period of time. Each test consists of many different tasks, where the level of difficulty gradually increases. Among them, there are test tasks for logical and spatial thinking, as well as other types of tasks. According to the test results IQ is calculated. It is noticed that the more variants of the test passes the subject, the better results he/she demonstrates.

The most famous test is the Eysenck Test (Gubenko[21], 2016). More specific are Tests by Wexler, Raven, Amthauer (Dmitriev & Drozdov[22], 2000).

To the date, there is no single standard for IQ tests. Tests are divided into age groups and show the development of a person, according to his/her age. That is, a 10-year-old child and graduate may have the same IQ, as the development of each of them corresponds to their age group. The Eysenck Test is designed for the age group from 18 and provides a maximum IQ level of 180 points. But the Eysenck Test, which consists of 30 tasks, is designed for students.

Consequently, the learning pace is the speed of movement of learning from the lowest to the highest level of assimilation. Determining the learning pace of material creates the basis for building the appropriate learning technology and expands the possibilities for organizing internal differentiation in the learning process. During the lessons of mathematics in the Lyceum, while practicing, the validity of the idea of the learning level considering different rates of assimilation for

[21] Hubenko, O. V. (2016). *Psychological diagnosis and activization in senior pupils and young artists of scientific and technological strength: scientific and methodical counselor*. Kyiv, Ukraine: Pedagogichna dumka. [In Ukr.].

[22] Dmitriev, V. Yu., Drozdov, O. Yu. (2000). *Elements of practical psychology: theory and methods of psychodiagnostics: Tutorial, part 3*. Chernigiv, Ukraine. [In Ukr.].

the completion of groups of students and the implementation of internal differentiation in learning was tested.

For the implementation of internal differentiation, keeping in mind the coefficient of mental development, an experimental 10th class was selected, in which 24 students were enrolled. According to the test, the students were divided into three groups: high (3 groups), middle (2 groups) and low (1 group) level of IQ.

During the practice of pedagogical researches the situations happens when it is necessary to assess the presence of connection of statistical characteristics. This can be done through correlation analysis. For the needs of my research to determine the tightness of the links between quantitative and qualitative features, if their values are ranked, the Spirmen rank correlation coefficient is used, which is calculated by the formula:

$$r_s = 1 - \frac{6\sum_{i=1}^{n}(x_i - y_i)^2}{n(n^2 - 1)}, \qquad (1)$$

where n – the volume of objects; $(x_i - y_i)$ – the difference between the ranks of the i-object (Rudenko[23], 2012).

The coefficient r_s takes values in the range from –1 to 1. After calculating the correlation coefficient r_s using the above formula, we evaluate the significance of this coefficient.

Table 1.4.1 contains data on the success of students in mathematics with a high rate of assimilation (group 3).

The correlation coefficient between the success rate and the coefficient of intellectual development of students with a high rate of material mastering is high. This means that in the process of studying the subject, the teacher "saw" these students, kept in mind their intellectual capabilities, and therefore carried out internal differentiation.

$$r_s = 1 - \frac{6(0+4+36+4+9+1)}{24(576-1)} = 1 - \frac{6 \cdot 54}{24 \cdot 575} = 1 - \frac{324}{13800} = 1 - 0.023 = 0.977.$$

[23] Rudenko, V. M. (2012). *Mathematical statistics: Tutorial*. Kyiv, Ukraine: Center for Educational Literature. [In Ukr.].

		x_i		y_i	$x_i - y_i$
№	y	r_y	IQ	r_{IQ}	$r_y - r_{IQ}$
5	10	2	121	2	0
7	11	1	114	3	-2
11	11	1	108	7	-6
14	10	2	113	4	-2
17	9	3	110	6	-3
22	11	1	128	1	0
23	8	4	112	5	-1

Table 1.4.1 The success of students with a high learning rate.

Table 1.4.2 contains data on students with an average rate of assimilation (Group 2).

The correlation coefficient between the success rate and the coefficient of intellectual development of students with a high rate of material mastering is high. Thus, during the assimilation of the whole topic, the IQ rate of students was considered.

$$r_s = 1 - \frac{6(9+1+9+9+1+36+9+0+16+16+1+1)}{24(576-1)} =$$

$$= 1 - \frac{6 \cdot 108}{24 \cdot 576} = 1 - \frac{648}{13800} = 1 - 0.047 \approx 0.953.$$

		x_i		y_i	$x_i - y_i$
№	y	r_y	IQ	r_{IQ}	$r_y - r_{IQ}$
1	9	2	112	5	-3
2	9	2	116	3	-1
3	9	2	112	5	-3
4	10	1	115	4	-3
6	8	3	115	4	-1
8	8	3	104	9	-6
13	8	3	110	6	-3
15	10	1	122	1	0
18	8	3	108	7	-4
19	9	2	110	6	-4
20	10	1	120	2	-1
24	7	4	112	5	-1

Table 1.4.2 The success of students with an average learning rate.

		x_i		y_i	$x_i - y_i$
№	y	r_y	IQ	r_{IQ}	$r_y - r_{IQ}$
9	7	4	112	2	2
10	8	3	110	3	0
12	9	2	108	4	-2
16	10	1	103	5	-4
21	7	4	116	1	3

Table 1.4.3 The success of students with a low learning rate.

$$r_s = 1 - \frac{6(4+0+4+16+9)}{24(576-1)} = 1 - \frac{6 \cdot 33}{24 \cdot 576} = 1 - \frac{198}{13800} = 1 - 0.014 \approx 0.99.$$

Table 1.4.3 contains data on students with a low learning rate (group 1).

Consequently, students who had a low learning pace were in the teacher's field of view while studying this topic.

It is worth to make the following remark. As a result of the fact that the ranks are repeated in the calculation formula for the correlation, it is necessary to make the correction: where *a, b* are the volumes of each group of identical ranks, respectively, for the results of measuring of two variables. These corrections are zero if the measurement results are

$$T_y = \frac{\sum (a^3 - a)}{12}, \quad T_{IQ} = \frac{\sum (b^3 - b)}{12},$$

different. Then formula (1) for determining the correlation coefficient will look like:

The calculations with the correction showed that the numerical values of the coefficient do not substantially change with allowance for the correction.

$$r_s = 1 - \frac{6\sum_{i=1}^{n}(x_i - y_i)^2 + T_y + T_{IQ}}{n(n^2 - 1)}. \qquad (2)$$

In the control class, where the educational activity was carried out in the usual way, i.e. the IQ coefficient was not implemented, the correlation coefficient between the learning outcomes and the IQ of the class students is 0.65. This means that there is the link, but not significant. The average value of this coefficient for students in the experimental class is:

This is substantially higher than 0.65, while the cognitive abilities of the pupils of both classes did not differ significantly. As it can be seen from the above results, there

$$r_{sc} = \frac{0.977 + 0.953 + 0.99}{3} = 0.973 \approx 0.97.$$

are all grounds for introducing the notion of coefficient of differentiation, which will characterize the consideration of intellectual capabilities of students, the differentiation of the educational process and considering the learning pace of material by students.

If we compare the success of each student with the IQ level of these students separately, we can conclude that the level of academic achievement almost coincides with the level of IQ. But there are some cases where the IQ level is higher than the level of academic achievement. I believe that such students do not work at full strength, that is, they did not use all their capabilities and could receive better grades than they had. There are cases where, on the contrary, the level of IQ is lower than the level of academic achievement. Experience shows that such students are diligent, seek to get a higher grade and achieve it by their work.

Conclusions

The implementation of the differentiated mathematics study in high school in the process of changing the learning pace is one of the means to provide an individual educational trajectory. Determining the learning rate of the material provides the basis for building the appropriate technology of learning and thereby expands the capacity for organizing teaching for high school students using internal differentiation in the learning process.

This article substantiates the expediency of using differentiated mathematics study in the process of changing the learning pace. This technique requires the diagnosis of student learning opportunities and their grouping based on their individual characteristics and the learning pace. The experimental data obtained indicate that the introduction of differentiated mathematics training in the process of changing the pace of knowledge acquisition by high school students positively influenced their level of knowledge, increased the interest in learning mathematics and increasing the pace of study and the learning pace activities.

The prospects for further research are in creating educational and methodological materials for the implementation of differentiated mathematics study, which will allow to build the individual educational trajectories and choose tasks, considering the individual characteristics of students.

1.5 Development of Mathematical Concepts in High School: Specifics*

Z. Kravchenko

*The article is published in the author's translation

Introduction

Modern trends in the development of education worldwide imply the introduction of a new educational paradigm, emphasizing not only the content or process, but also envisaging the final result. This is confirmed by a competent approach ubiquitously applied to educational processes, since modern education heavily relies on strengthening self-dependent and cognitive activities.

Thus, the aim of modern education varies from what it used to be, and just mastering mathematics is not sufficient. What is needed is the development of personality by means of mathematics. At the forefront are the developmental functions of subject mastering, in particular, through mastering the concepts of the subject.

The prior methodological problem of teaching mathematics is the formation of concepts, since it affects the quality of mastering mathematics by the students and the development of their cognitive abilities.

It should be noted that scientists focus on identifying the main stages of formation of concepts, but do not consider the issue of steps to be taken at every stage.

Thus, the paper focuses on the analysis of theoretical background and methodological issues of forming mathematical concepts.

Conceptual framework

The concept is the subject area of both philosophy and logic. In particular, the concept or notion was described by a German philosopher Hegel in his dialectical logic, where he had identified three forms of thinking, including categories, reflexive definitions and concepts. Hegel was the first to separate concept and imagination. He interpreted the concept as the basis of being, but supposed that cognition began with the perception of the exterior of the subject rather than its essence.

The review of domestic and foreign literature on psychology and pedagogics shows that at present there is no single view on the definition of the concept. It may be interpreted as a form of thinking that plays an important role in cognition. One of the most frequent definitions implies that the concept is a form of thought, which reflects the essential properties of objects and phenomena.

The meaning of the concept was under close scrutiny of numerous studies by psychologists (Vygotsky[1], 2018; Galperin[2], 1995; Davydov[3], 1995; Talyzina[4], 1984; et al.). Some scientists (Menchinskaya[5], 2004; Rubinstein[6], 1962) do not separate but consider the processes of forming and assimilating scientific concepts as a single whole.

Presumably, there are two ways of concept assimilation, namely if one follows from specific knowledge to general or from matterless and abstract knowledge to specific, and then to truly abstract knowledge N. Menchinskaya[7] (1955).

The concept may be defined as the thought of the subject of research, as part of the elementary thinking system V. Svetlov[8] (2018). This particular definition of the concept emphasizes the necessity to produce an image of the object rather than its worded definition.

[1] Vygotsky, L. (1991). *Pedagogicheskaya psihologiya [Pedagogical Psychology]*. Moscow, Russia : Pedagogika. 480. [In Rus.].

[2] Galperin, P. (1995). *Psihologiya kak ob'ektivnaya nauka [Psychology as objective science]*. Moscow, Russia. [In Rus.].

[3] Davydov, V. (1995). *O ponyatii razvivayushchego obucheniya [On developing education]*. Tomsk, Russia : Peleng. [In Rus.].

[4] Talyzina, N. (1984). *Upravlenie processom usvoeniya znanij (psihologicheskij aspekt) [Knowledge assimilation process (psychological aspect)]*. Moscow, Russia : MGU. [In Rus.].

[5] Menchinskaya, N. (2004). *Problemy obucheniya, vospitaniya i psihicheskogo razvitiya rebenka [Problems of education, upbringing and psychic development of a child]*. Voronezh, Russia : MPSI. [In Rus.].

[6] Rubinstein, S. (1962). *Sein und Bewusstsein*. Berlin: VEB [In Germ.].

[7] Menchinskaya, N. (1955). *Puti povysheniya uspevaemosti po matematike [Ways of performance improvement on math]*. Moscow, Russia : PH APN RSFSR. [In Rus.].

[8] Svetlov, V. (2018) *Osnovy filosofii [Fundamentals of philosophy]*. Moscow, Russia: Jurait. [In Rus.].

To operate the concepts students are expected to apply active thinking, because only in this case they will be able to clearly understand the essence of the subjects being studied and the phenomena of the theory reflected in the framework of scientific concepts. The ability to think includes the ability to operate the concepts.

Formation of concepts in mathematics teaching has great theoretical background (Akulenko[9], 2014; Dalinger[10], 2006, Sarantsev[11], 2001; Slepkan[12], 2000; Tarasenkova[13], 2003; et al.).

The concept may be defined as a form of scientific cognition, which reflects the essential in the objects being studied and fixes the acquired knowledge with special terms, thus producing a mental image of the object N. Metelsky[14] (1982).

A review of the existing interpretations of the concept shows that, firstly, there is no universal understanding of what the concept is, and, secondly, different definitions of the concept make its functions vary, thus obscuring its role in the educational process.

We agree with the opinion of Z. Slepkan[15] (2000) and presume that the concept is a form of thinking, which reflects the general essential and distinct specific properties

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[9] Akulenko, I. (2014). Aktualni tendentsii ta vplyvy v teorii ta praktytsi profesiinoi pidhotovky maibutnoho vchytelia matematyky [Current trends and influences in the theory and practice of future teacher training in mathematics]. *Molod' i rynok, 8.* 10–13. [In Ukr.].

[10] Dalinger, V. (2006). *Metodika obucheniya uchashchihsya dokazatel'stvu matematicheskih predlozhenij: kniga dlya uchitelya [Methods of teaching students the proof of mathematical sentences: a book for teachers]*. Moscow, Russia: Prosveshchenie. [In Rus.].

[11] Sarantsev, G. (2001). *Metodologiya i metodika obucheniya matematiki [Methodology and methods of teaching mathematics]*. Saransk, Russia. [In Rus.].

[12] Slepkan, Z. (2000). *Methodology of teaching mathematics*. Kiev, Ukraine: Zodiac-Eco. [In Ukr].

[13] Tarasenkova, N. (2003). Theoretic-methodical principles of the using of the sign and symbolic mean in teaching mathematics of the basic school students *Doctor's thesis*, Cherkasy, Ukraine. [In Ukr.].

[14] Metelsky, N. (1982). *Didaktika matematiki: obshchaya metodika i ee problemy [Didactic mathematics: general guidelines and problems]*. Misnk, Belarus: BGU. [In Rus].

[15] Slepkan, Z. (2000). *Methodyka navchannya matematyky.* Kiev, Ukraine: Zodiac-Eco. [In Ukr].

and features of real objects or phenomena. Correspondently, in our thinking mathematical concepts reflect spatial forms and quantitative relations of reality, abstracting from real situations.

The algorithm of concept formation

The literature review shows that scientists have developed general issues of the theory of concepts (its content, scope, definition, and classification), as well as some issues of concept assimilations. However, there is still no comprehensive methodological framework explaining the formation of mathematical concepts in classrooms, taking into account the conditions for its implementation.

Formation of concepts implies a set of requirements for the teaching process, the observance of which will ensure high-quality assimilation of basic mathematical concepts by the students.

According to V. Dalinger[16] (2006), G. Sarantsev[17] (2001) formation of mathematical concepts for students to assimilate should be done gradually. In particular, the process can be presented as follows:

1 The students are expected to record definitions in a short schematic form;
2 The teachers are expected to progressively check the students for their ability to recognize the objects referred to the concept definition and make relevant conclusions;
3 The students are expected to recognize the objects referred to the concept definition and make relevant conclusions without teachers' assistance.

V. Dalinger[18] (2006) identifies the stages in formation of mathematical concepts as follows:

[16] Dalinger, V. (2006). *Metodika obucheniya uchashchihsya dokazatel'stvu matematicheskih predlozhenij: kniga dlya uchitelya [Methods of teaching students the proof of mathematical sentences: a book for teachers]*. Moscow, Russia: Prosveshchenie. [In Rus.].

[17] Sarantsev, G. (2001). *Metodologiya i metodika obucheniya matematiki [Methodology and methods of teaching mathematics]*. Saransk, Russia. [In Rus.].

[18] Dalinger, V. (2006). *Metodika obucheniya uchashchihsya dokazatel'stvu matematicheskih predlozhenij: kniga dlya uchitelya [Methods of teaching students the proof of mathematical sentences: a book for teachers]*. Moscow, Russia: Prosveshchenie. [In Rus.].

1) Consideration of examples of objects within the scope of the concept; 2) The introduction of a term denoting the concept; 3) Consideration of examples beyond the scope of the concept; 4) The wording of the concept definition; 5) Providing additional data, in particular non-essential properties of the concept; 6) Systematization of the knowledge provided V. Dalinger[19] (2006).

It is well worth noting that one lesson may be insufficient to convey the content of some mathematical concepts. In the process of studying mathematics the knowledge of their content and scope is regularly deepened. In this case, the meaning of the concept is revealed gradually, through establishing the connections and relations of the given concept with others.

Thus, P. Galperin[20] (1995), N. Talyzina[21] (1984) believe that the formation of concepts should not be extended in time, but be done in one go, when the content of the new concept is assimilated in full within a specific time slot.

Unlike others, N. Talyzina[22] (1984) insists that the process of concept formation should be viewed on account of activity, that is, steps related to the formation and functioning of the concept.

We believe that formation of concepts occurs as follows:

1) Motivation for introducing the concept; 2) Identification of its essential properties and introduction of the appropriate terms with their definitions, mastering the patterns of object recognition, making conclusions about the reference of the object to the concept, and assimilation of the concept definition; 3) Application of the concept; 4) Systematization of concepts relative to the system of previously studied

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[19] Dalinger, V. (2006). *Metodika obucheniya uchashchihsya dokazatel'stvu matematicheskih predlozhenij: kniga dlya uchitelya [Methods of teaching students the proof of mathematical sentences: a book for teachers]*. Moscow, Russia: Prosveshchenie. [In Rus.].

[20] Galperin, P. (1995). *Psihologiya kak ob'ektivnaya nauka [Psychology as objective science]*. Moscow, Russia. [In Rus.]

[21] Talyzina, N. (1984). *Upravlenie processom usvoeniya znanij (psihologicheskij aspekt) [Knowledge assimilation process (psychological aspect)]*. Moscow, Russia : MGU. [In Rus.].

[22] Talyzina, N. (1984). *Upravlenie processom usvoeniya znanij (psihologicheskij aspekt) [Knowledge assimilation process (psychological aspect)]*. Moscow, Russia : MGU. [In Rus.].

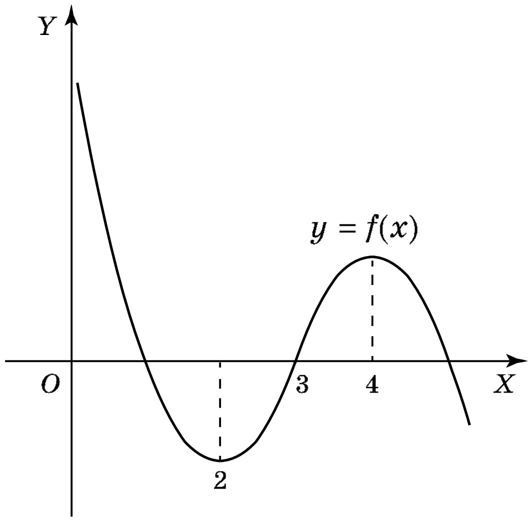

Figure 1.5.1 The Graph of $y = f(x)$.

concepts; 5) Check and correction of the level of concept assimilation.

Obviously, studying theoretical material and its practical application essentially depends on the curriculum which students should master (standard or field-specific level). Hence, the level and complexity of curriculum affect the educational goals and requirements for the student activities and performance. At the same time, we can propose some general methodological guidelines that can be used at each of the above-mentioned levels.

To exemplify, let us scrutinize the formation of such concepts as *maximum* and *minimum values* of the function. Thus, when introducing the appropriate terms at a standard level, we offer students to describe the shape of the function (ref. Figure 1.5.1) around the particular point near the axis of x equal to 4.

For this purpose, let us ask students to consider some neighborhood of 4 and compare the values of the function at point 4 and the neighboring points (left and right of 4). We should note that for all $x \neq 4$ in the neighborhood of 4, $f(x) < f(4)$ is true. This means that the value of the function at point 4 proved to be the greatest among all other values in the neighborhood of point 4. Thus, we may use the term *maximum* that means 'the greatest' from Latin and introduce the concept of *maximum value* of the function. Now we can proceed to the worded definition. Point x_0 of the domain of $f(x)$ is called the maximum value of the above function if there is such δ-neighborhood $(x_0-\delta; x_0+\delta)$ of x_0, that for all $x \neq x_0$ from this neighborhood $f(x) < f(x_0)$ is true.

Although x_0 is called the *maximum value*, the function is not supposed to reach its maximum value at this point (e.g. the graph in Figure 1.5.1 shows that if $x < 0$ function reaches greater value than $f(4)$). In the same way, analyzing the value of function in the neighborhood of point 2, we explain the *minimum value* of the function to the students.

Point x_0 of the domain of $f(x)$ is called the *minimum value* of the function if there is such δ-neighborhood $(x_0-\delta; x_0+\delta)$ of x_0, that for all $x \neq x_0$ from this neighborhood $f(x) < f(x_0)$ is true.

It would be also relevant to inform students that the maximum and minimum values are called extremum values. With reference to Figure 1.5.1, we offer the students to find out whether they can use $\delta = 4$, that is, consider δ-neighborhood (0; 8). The students will find out that they cannot do it, as $f(0) > f(4)$. Indeed, in this neighborhood, the characteristic property is not true, but for example, in δ-neighborhood of point 4 when the interval is (3;5), ($\delta = 1$), the characteristic property is true. After that, studying calculus the students should learn that this value is called *local maximum*.

The teacher should draw students' attention to the fact that when finding the relevant δ-neighborhood of point 4 (or point 2), it is sufficient to draw a direct parallel line *Ox* across the relevant point on the graph and choose such functional arguments as a δ-neighborhood that are symmetrical to the value of the given x_0 where the points on the graph below the direct line are for the maximum values and above the direct line – for the minimum values.

With definitions having been formulated, the students are offered to look at function graphs in Figure 1.5.2

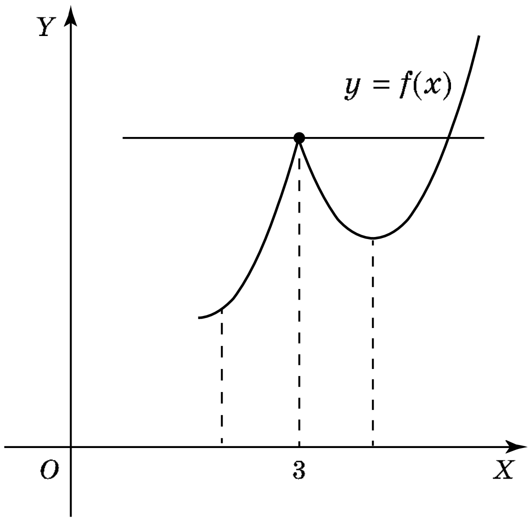

Figure 1.5.2 The Graph of $y = f(x)$.

and Figure 1.5.3 and determine whether $x_0 = 3$ is the maximum value.

To motivate and indulge students in research activity, they should be offered to look at Figure 1.5.4 and determine whether point $x_0 = 3$ is maximum or minimum value of the function.

Students who obtain field-specific education are offered to complete these tasks without teachers' assistance, for their

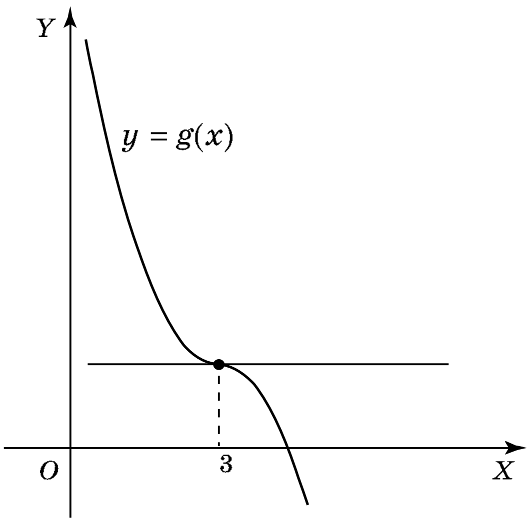

Figure 1.5.3 The Graph of $y = g(x)$.

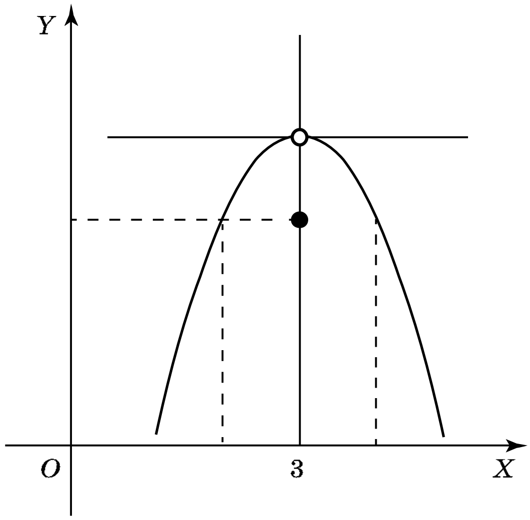

Figure 1.5.4 The Graph of $y = f(x)$ with a jump at point $x_0 = 3$.

own self-study. However, those educating with standard research requirements are expected to complete these tasks under their teachers' supervision. In cases when students differ greatly by their general level of preparation, it is appropriate to apply various means of clarification and exemplification more actively, which will improve the understanding of the concepts by all students. At that, students with better performance should be engaged in more analytical explanations, and those with mediocre skills should pay their attention to the analysis of examples and illustrations.

It turns out that in this case the characteristic property is not true for the maximum value. Then we offer students to check the characteristic property of the minimum value, and it turns out to be true if they us a small δ -neighborhood. So, point $x_0 = 3$ is the minimum value of the function. The teacher should mention that the minimum value of continuous function is at its lowest point, but the function having a jump can reach its minimum value within some neighborhood of point x_0 at an isolated point on the graph of this function.

To memorize and consolidate the mathematical concepts students should practice operating and applying concepts in different situations. We get quite good results from frontal questioning of students. They make special exercises where they need to give the appropriate definitions and assertions in different situations, and to quickly understand the task. It is much more useful than a simple reproduction of the worded definitions of the concepts.

To summarize, after introducing the concepts of minimum and maximum values of the function, the students should be asked to indicate the minimum and maximum values of the functions shown in figures and explain why they think so.

It should be noted that dealing with mathematical concepts, students are to be taught how to operate the converse assertions, when solving the tasks. For example, the increasing function implies that if $x_1 > x_2$, then $f(x_1) > f(x_2)$. So can it be asserted that when the function is increasing and $f(x_1) > f(x_2)$, then $x_1 > x_2$? To justify their answer, the students look at the graph of the increasing function in the figure and make a conclusion that this is true. Let us suppose that x_1 does not exceed x_2, i.e. $x_1 \le x_2$. Thus, we can assume that if $x_1 \le x_2$ and $f(x)$ is increasing, then $f(x_1) \le f(x_2)$, that contradicts the inequation $f(x_1) > f(x_2)$. This means that our assumption is wrong and if $f(x_1) > f(x_2)$, then $x_1 > x_2$.

While studying algebra and the fundamentals of calculus, students quite often encounter difficulties of general and personal character. Certain difficulties are associated with the inability to organize self-studying activity, as well as the complexity of the material being studied. Personal difficulties are associated with insufficient motivation or when teachers do not appropriately encourage creative activities and train research skills in the course of learning. Inability to overcome difficulties impedes the learning progress. The course of algebra and calculus requires quite intensive educational efforts from students. The difficulties may be related to the complexity of the basic concepts, high abstractness and dynamism of the studied variables. Specifically, when students study derivative and its application in calculus, the first and most difficult concept they come across is *the limit*.

Thus, field-specific education implies that students will learn the limit of the function using ε, δ (ref. Cauchy). Number B is called the limit of function $f(x)$ at point a (at x that tends to a), if for any ε there is δ, that for every $x \neq a$ that comply with $|x - a| < \delta$, inequation $|f(x) - B| < \varepsilon$ is true.

To study this concept properly, students should be given a task to find the limit, after what they are asked to reproduce the definition with reference to the result obtained. The teacher should mention that when defining the limit of $f(x)$, $x \to a$ all values $x \neq a$ (with δ-neighborhood of a) are taken into account. Studying this, students might be confused either with the concept of the limit itself, or with the sense of ε-neighborhood, δ-neighborhood and $|f(x) - B| < \varepsilon$.

To teach the students to operate this concept in proving $\lim\limits_{x \to a} f(x) = B$, the teacher may give the following prompts:

1. Analyze $|f(x) - B| < \varepsilon$ for any arbitrary ε;
2 For any $x \neq a$ from some neighborhood of a $|x - a| < \delta$ is true;
3 Explain (using either inequation changes or its properties) that δ (written through ε) obtained from $|x - a| < \delta$ (at $x \neq a$) leads to $|f(x) - B| < \varepsilon$;
4 Use the definition of the function limit in point a and justify that $\lim\limits_{x \to a} f(x) = B$.

When calculating the limit of the function with students, the teacher should draw their attention to the fact that all elementary functions are continuous at each point of their domain, i.e. $\lim\limits_{x \to a} f(x) = f(a)$.

Therefore, in the simplest cases, to determine the limit of a continuous function students can use $f(a)$ (if it exists), but when studying at a more profound level it is necessary to draw students' attention to the fact that for functions that are not continuous at point a, $\lim\limits_{x \to a} f(x)$ is not equal to $f(a)$ .To better understand the concept of the function limit at a certain point, students should work with illustrations of the function graphs.

The process of concept formation is quite often narrowed to learning its definition, therefore to identify the properties of the concept it is necessary to use counterexamples.

A counterexample may consist in some task that helps to identify and eliminate false associations, formed by the students. These tasks may contain incomplete or contradictory statements or data that may serve as counterexamples.

This technique is used in cases when the student must be convinced that he or she is wrong in his thinking. In order to convince the student of the falsity of a general statement, it is often enough to make and analyze at least one counterexample.

Students should take into account the fact that even one word missing may lead to misinterpretation of the concept. For example, if a student may assert that the function graph is a set of points in the plane with $(x;\ f(x))$ coordinates, then $f(x) = x^2$ can be represented as a graph if $x \in Z$. By analyzing this counterexample, the student will arrive at the conclusion that the word *all* is missing in the definition.

When using counterexamples, it is necessary to remember that the tasks given to students should be solved in a class with immediate teacher's assistance and error analysis.

Conclusions

Thus, concept formation requires holistic study of all its constituent and essential properties. To make the process of concept formation and assimilation efficient, teachers are expected to give correct definitions of the concepts, distinguish between essential and non-essential concept properties, understand the inter-relation between the essential properties, realize and systemize the approaches to task solutions, namely compliance with the concept definitions and classification, and interconnection of the concepts.

It is important that students should be provided with comprehensive tasks that reflect systemic interrelation between the concepts. Of rather great significance is the use of counterexamples that helps students get the proper understanding of the concepts under study.

1.6 Modeling Elements and the Development **of the Engineering Students' Mathematical** Competence*

D. Tinkova

*The article is published in the author's translation

Introduction

Reorganization of the current Ukrainian education takes its course under the slogan "education for life". Such a reorientation of the education system, including the vocational and technical one, is carried out in the direction of youth training for practical activities.

An important role in training the students of institutions of vocational (vocational and technical) education (IV(VT)E), including those in the machine-building professional field, before the application of acquired knowledge in practice, is played by the study of the mathematics course, and in particular stereometry topics. In the educational process, the problems of practical orientation, which are based on the method of mathematical modeling, become relevant. Therefore, we consider it expedient to increase attention on solving the students' practical problems contributing to the formation of mathematical competence in them.

Background

The methodology (techniques) of teaching mathematics in institutions of vocational education is considered in the writings of I. Hyrylovska[1] (2013), O. Volianska[2] (1999),

[1] Hyrylovska, I. (2013). Formuvannia v uchniv profesiino-tekhnichnykh navchalnykh zakladiv umin rozviazuvaty stereometrychni zadachi na pobudovu [Formation of ability to solve stereometric problems of construction in students of vocational and technical educational institutions]. *Extended abstract of candidate's thesis.* Cherkasy, Ukraine. [In Ukr.]

[2] Volianska, O. (1999). Vyvchennia alhebry i pochatkiv analizu v profesiino-tekhnichnykh uchylyshchakh v umovakh vprovadzhennia osvitnoho standartu [Study of algebra and principles of analysis in vocational schools in the conditions of implementation of the educational standard]. *Extended abstract of candidate's thesis.* Kiev, Ukraine. [In Ukr.]

Ya. Chernenko[3] (2018), and others. The issues of mathematical competence development in students were studied by M. Holovan[4] (2012), N. Tarasenkova & V. Kirman[5] (2008), S. Rakov[6] (2005), and others. The works of V. Shvets & A. Prus[7] (2007), and others are devoted to the issues of mathematical modeling in the school education system. However, the question of the mathematical competence formation through modeling of figures by students of IV(VT)E in the machine-building professional field are not sufficiently explored, which led to the choice of this topic.

Theoretical framework

Competency approach is directivity of the educational process to form and develop the subject and key competencies of the individual.

In the State standard of basic and complete general secondary education[8] (2011) approved by the resolution of the Cabinet of Ministers of Ukraine dated November 23, 2011 No. 1392, the *key competencies are*:

[3] Chernenko, Ya. (2018). Formuvannia heometrychnykh umin uchniv profesiino-tekhnichnykh navchalnykh zakladiv [Formation of geometric skills in students of vocational and technical schools]. *Extended abstract of candidate's thesis*. Cherkasy, Ukraine. [In Ukr.]

[4] Holovan, M. (2012). Matematychni kompetentnosti chy matematychna kompetentnist? [Mathematical competence or mathematical competence?] Development of intellectual skills and creative abilities in school students and higher education students in the process of teaching disciplines of the natural and mathematical cycle "ITM*plus 2012": mizhnarodna naukovo-metodychna konferentsia (6-7 hrudnia 2012 r.) - *International scientific and methodological conference*. (pp. 36-38). Sumy, Ukraine: Vyrobnycho-vydavnyche pidpryiemstvo "Mriia" [In Ukr.]

[5] Tarasenkova, N., & Kirman, V. (2008). Zmist i struktura matematychnoi kompetentnosti uchniv zahalnoosvitnikh navchalnykh zakladiv [Content and structure of mathematical competence of students of general educational institutions]. *Matematyka v shkoli – Mathematics at school*, *6*, 3-9. [In Ukr.]

[6] Rakov, S. (2005). Formuvannia matematychnykh kompetentnostei vypusknyka shkoly yak misiia matematychnoi osvity [Formation of mathematical competences of a school graduate as a mission of mathematical education]. *Matematyka v shkoli – Mathematics at school*, *5*, 2-7. [In Ukr.]

[7] Prus, A. & Shvets, V. (2007). *Prykladna spriamovanist stereometrii: 10—11 kl [Applied orientation of stereometry: 10-11 gr.]*. Kyiv, Ukraine: Shkilnyi svit [In Ukr.]

[8] Derzhavnyi standart bazovoi i povnoi zahalnoi serednoi osvity [State standard of basic and complete general secondary education]. (n.d.). *kmu.gov.ua*. Retrieved from: https://www.kmu.gov.ua/ua/npas/244862959 [In Ukr.]

- ability to study;
- skills to communicate the state, native and foreign languages;
- mathematical and basic competencies in the field of natural science and technology;
- information and communication competency;
- social competency;
- civic competency;
- general cultural competency;
- entrepreneurial competency;
- health preserving competency.

Objective (branch) competencies include communicative, literary, artistic, interdisciplinary, aesthetic, natural sciences, mathematical, design and technological, information and communication, social sciences, historical and health preserving.

The Law of Ukraine 'On Education' dated May 09, 2017 No. 2145-VIII[9] (2017) also defines the *key competencies* required by each person for a successful life-sustaining activity. This is in particular:

- fluent state language;
- ability to communicate one's native (in case it is not the state language) and foreign languages;
- mathematical competence;
- competence in the field of natural sciences, engineering and technology;
- innovativeness;
- environmental competence;
- information and communication competence;
- lifelong learning;
- civil and social competences related to the ideas of democracy, justice, equality, human rights, well-being and a healthy lifestyle, with awareness of equal rights and opportunities;
- cultural competence;
- entrepreneurship and financial literacy;

[9] Zakon Ukrainy «Pro osvitu» vid 05.09.2017 № 2145 VIII [The Law of Ukraine "On Education" dated September 05, 2017, No. 2145 VIII]. (n.d.). *zakon.rada.gov.ua*. Retrieved from:: https://zakon.rada.gov.ua/laws/show/2145-19 [In Ukr.]

- other competences provided by the standard of education.

The mathematical competence of the IV(VT)E students of the machine-building professional field implies the acquired characteristics of the individual, which combines mathematical knowledge, abilities, skills, personal qualities that determine motives, readiness and ability to solve professional tasks, aptitude to understand the essence of the mathematical modeling method and the ability to apply it in the professional field, realizing the whole result of the activity.

Components of mathematical competence of students of vocational (vocational and technical) education:

- motivational component is system of motives, goals, needs and aspirations for the study of mathematics, improvement of knowledge, skills and experience of mathematical activity for better mastering the cycles of general professional and vocational and theoretical training;
- cognitive component is a set of mathematical knowledge, skills and experience of a theoretical and practical nature to use in other subjects study;
- activity component is a complex of mathematical skills (analytical, computational, algorithmic, functional, geometric, stochastic, probabilistic, mathematical modeling) for solving typical practical problems using mathematical methods;
- reflexive and value component is adequate introspection (self-reflexive analysis) and self-evaluation of one’s mathematical activity results, a desire to improve the results of one’s activities, understanding the role of mathematical competence as one of the leading social values;
- personality component is ability to volitional stress, perseverance, endurance, restraint, etc.

An important role is given to the study of stereometry in the life of a modern graduate of the IV(VT)E and is stipulated by the need of think spatially, make calculations, and model in the process of the most life and professional problems solving.

Social and personal significance	Cognitive component	Active component
PARALLELISM OF STRAIGHT LINES AND PLANES IN SPACE		
The need to use spatial ideas for orientation in the environment; The need to recognize spatial figures in objects of the real world; Use of knowledge about the construction of geometric shapes in building the real objects	The concept of the reciprocal placement of objects in space (parallel, immersive, within one plane, in different); Imagining and picturing of spatial figures on paper; Technics for building spatial figures from cardboard; Concept of the geometrical figure model	Describe the reciprocal placement of objects in space; Use the axioms of stereometry to solve tasks; Recognize, name and depict spatial figures: prism, pyramid; Associate real objects with models of spatial figures
PERPENDICULARITY OF STRAIGHT LINES AND PLANES IN SPACE		
The need to use spatial ideas for orientation in the environment; Finding the distance between real objects of the surrounding area; Finding angles between objects of the real world	The concept of the mutual placement of objects in space; The concept of distance between objects; The notion of the angle as a geometric value; The concept of a two-cornered corner as a geometric figure	Describe the reciprocal placement of objects in space; Calculate distances in space; Calculate angles in space
POLYHEDRONS		
The need to recognize polyhedrons in objects of the environment; Use of knowledge about construction of polyhedrons when constructing models or real objects; The need to measure and calculate quantities associated with polyhedra in everyday life	The concept of polyhedron; Polyhedron elements; The concept of the correct polyhedron; Techniques for building polyhedron from cardboard; The concept of the area of the side surface of the prism, the pyramid; The concept of the area of the full surface of the prism, the pyramid	Recognize, name and depict polyhedra: prism, parallelepiped, pyramid; Recognize and name the correct polyhedra: tetrahedron, octahedron, icosahedron, hexahedron, dodecahedron; Build models of cardboard polyhedral; Calculate the basic elements of polyhedral; Calculate the area of the lateral surface and the area of the full surface of the prism and pyramid

SOLIDS OF REVOLUTION		
Need for recognition of solids of revolution in household objects; Use of knowledge about the construction of solids of revolution when constructing models or real objects; The need to measure the values associated with the solids of revolution in everyday life	The concept of the solid of revolution; Elements of the solids of revolution; Techniques for constructing rotational bodies from cardboard	Recognize, name and depict the solids of revolution: a cylinder, cone, globe, sphere; Build models of solids of revolution from cardboard; Calculate the basic elements of the solids of revolution
VOLUMES AND AREAS OF GEOMETRICAL FIGURES SURFACES		
Need to find the area and volume of environmental objects	Concept of the area of the lateral surface of geometric bodies; Concept of the total area of geometric bodies; The concept of the volume of geometric bodies	Calculate volumes of parallelepipeds, prisms, pyramids, cylinders, cones, globes, spheres; Calculate the area of the lateral and full surfaces of the cylinder, cone, globe, sphere

Table 1.6.1

Therefore, one needs to develop the mathematical competence forming from the first year of study in the the IV(VT)E.

A description of the approximate content of mathematical competence in the study of topics in stereometry by students of the IV(VT)E in the machine-building professional field is carried out in Table 1.6.1 in accordance with the sections of programmed topics, “Parallel lines and planes in space”, “Perpendicularity of right lines and planes in space”, “Polyhedra” “Solids of revolution”, “Area and volume of geometric shapes”.

The approximate content of the mathematical competence of IV(VT)E students in studying the topics of stereometry

Methodology, results and discussion

Cognitive and active components formation of the mathematical competence mainly takes place in the training activities using Mathematics teaching techniques.

To test the level of mathematical competence, we conducted a research in which 208 students of IV(VT)E participated from different regions of Ukraine in the specialties Electric and gas welder, Layout designer of multiple types of equipment, Locksmith.

At present, the components of mathematical competence have to be built on the basis of a competent approach in thematic planning of each lesson in stereometry for institutions of general secondary education. We are presents a directed lesson by lesson thematic planning on the example of "Polyhedrons" topic.

1 *Lesson topic:* Polyhedra, their elements. Correct polyhedra

Concepts that are introduced for the first time, and concepts that are developing: The student considers the terms 'polyhedron', 'correct polyhedron', 'polyhedron revolution'.

Components of the student's mathematical competence:
- *Cognitive:*
  Names the concept of polyhedra, regular polyhedron.
  Gives examples of polyhedral from the environment.
  Names elements of polyhedra, regular polyhedra.
  Shows elements of polyhedron.
  Gives examples of polyhedra in the daily grind.
- *Active:*
  Distinguishes regular and irregular polyhedra.
  Depicts polyhedra on paper.
  Produces models of regular polyhedra made of cardboard.
- *Reflexive and value:*
  Evaluates the constructed model of polyhedron and image of the polyhedron on paper.

2 *Lesson topic:* Prism, right and regular prism

Concepts that are introduced for the first time, and concepts that are developing: The student considers the terms 'prism', 'right prism', 'regular prism'.

Components of the student's mathematical competence:

- *Cognitive:*

  Names the definition of prism, right prism, regular prism.

  Gives examples of types of prisms from the surrounding reality.

  Names elements of prism.

  Shows elements of prism.

- *Active:*

  Distinguishes the prism, right and regular prism.

  Represents varieties of prisms on paper.

  Produces models of cardboard prisms.

  Calculates the elements of the prism.

- *Reflexive and value:*

  Evaluates: the built prism model; print of prisms on paper.

  Justifies the correctness of solving the task.

3 *Lesson topic:* Parallelepiped. Rectangular parallelepiped. Cube

Concepts that are introduced for the first time, and concepts that are developing: The student considers the terms 'parallelepiped', 'rectangular parallelepiped', 'cube'.

Components of the student's mathematical competence:

- *Cognitive:*

  Names the definition of a parallelepiped, a rectangular parallelepiped, a cube.

  Provides examples of a parallelepiped, a cube from the surrounding reality.

  Names elements of a parallelepiped, a cube.

  Shows elements of a parallelepiped, a cube.

- *Active:*

  Distinguishes a parallelepiped and a rectangular parallelepiped.

  Depicts a parallelepiped, a cube on paper.

  Makes models of parallelepiped and cardboard cube.

  Calculates the elements of a parallelepiped, a cube.

- *Reflexive and value:*

Evaluates the built model of a parallelepiped, a cube; image of a parallelepiped, a cube on a paper.
Justifies the correctness of solving the task.
Corrects mistakes in calculations.

4 *Lesson topic:* Solving tasks

Concepts that are introduced for the first time, and concepts that are developing: The student operates the terms "polyhedron", "regular polyhedron", "prism", "parallelepiped", "cube".

Components of the student's mathematical competence:

- *Cognitive:*
  Formulates the definition of a polyhedron, a regular polyhedron, a prism, a straight prism, a regular prism, a parallelepiped, a rectangular parallelepiped, and a cube.
- *Active:*
  Solves the problem of finding the elements of polyhedra.
  Represents polyhedra on paper.
  Builds a model of polyhedron made of cardboard.
  Measures models of polyhedra.
- *Reflexive and value:*
  Realizes the difference among a polyhedron, a regular polyhedron, a prism, a straight prism, a regular prism, a parallelepiped, a rectangular parallelepiped, and a cube.
  Estimates: the constructed model of a polyhedron; a polyhedron image on paper.
  Justifies the correctness of the task.
  Fixes errors when calculating.

5 *Lesson topic:* Pyramid. Regular pyramid. Concave pyramid

Concepts that are introduced for the first time, and concepts that are developing: The student considers the terms "pyramid", " regular pyramid", "concave pyramid".

Components of the student's mathematical competence:

- *Cognitive:*
  Names the definitions of pyramid, regular pyramid, concave pyramid.

Gives examples of a pyramid from the surroundings.
Names the pyramid elements.
Shows the pyramid elements.

- *Active:*
  Distinguishes the regular and irregular pyramids, pyramid and concave pyramid.
  Depicts types of pyramid on paper.
  Makes pyramid models from cardboard.
  Calculates elements of the pyramid.
- *Reflexive and value:*
  Estimates: the constructed pyramid model; image of a pyramid on paper.
  Justifies the correctness problems solution.
  Fixes errors when calculating.

6 *Lesson topic:* Construction of polyhedron cross sections

Concepts that are introduced for the first time, and concepts that are developing: The student considers the term "trace method".

Components of the student's mathematical competence:

- *Cognitive:*
  Names the concept of cross section.
- *Active:*
  Represents polyhedra on paper.
  Builds the cross sections of the polyhedron using the trace method.

Oriented lesson by lesson thematic planning for ***"Polyhedrons" topic in IV(VT)E***

According to our observations, during the study of the topic "Polyhedrons", students only depict polyhedrons in notebooks, while not using the construction of models of geometric shapes made of cardboard. Thus, with the proper organization of educational activities, the students acquire the cognitive component through the concept of the polyhedron image, and the activity component is formed through the ability to build an image.

In determining the level of mathematical competence we rely on the above components of mathematical competence: motivational, cognitive, activity, reflexive and value, and personal.

During the research study, the students were offered a task to divide into groups and build a house, rocket, tank or robot model. Students began to depict the model figures in notebooks.

Criteria and indicators to assess the IV(VT)E students' results are presented in Table 1.6.2.

Component	*Criterion*	*Indicator*	*Detection method*
Motivational	The student's orientation	Interest in learning activities	Taking questionnaires
Cognitive	Techniques for constructing polyhedra	by yourself with the support of the help	Tasks
Active	Execution of polyhedra construction	Independently Relying on assistance Preservation of proportions Linear construction	Work on assignments
Reflexive and value	Attitude to oneself and to one's educational and cognitive activity	Self-determination self-esteem	Presentation of results
Personal	Student's personal qualities	Ability to control oneself, one's emotions	Observation

Table 1.6.2

Criteria and indicators of the mathematical competence development level in students when studying the ***"Polyhedrons" topic***

Our research revealed the following results.

- The questionnaire showed that 55% of students were working through external coercion. Students were forced to do the tasks by their teachers in the classroom. The results of the questionnaire are presented in Table 1.6.3.

Interestedness in learning activities	Number of students in percent
Getting high scores	25%
Self-development	3%
Necessity of knowledge for professional activity	17%
External coercion	55%

Table 1.6.3

- The research of the cognitive component of mathematical competence of students showed that 30% of students know the techniques of constructing polyhedra. The results of the completed task are presented in Table 1.6.4.

Techniques for constructing polyhedra	Number of students in percent
Independently	30%
Resting on the group mates' assistance	5%
Resting on the teacher' assistance	65%

Table 1.6.4

- The research of the active component of mathematical competence showed that 35% of students succeeded in maintaining the proportions of the proposed models of the rocket, tank and robot, and 35% of students succeeded in representing the models taking into account the linear constructive component. The data are presented in Table 1.6.5.

Execution of polyhedron construction	Number of students in percent
Preservation of proportions	35%
Linear construction	35%

Table 1.6.5

- The research of the reflexive and value component of the mathematical competence showed that 37% of students presented their work successfully, 63% of students made mistakes in their representations. The data are presented in Table 1.6.6.

Presentation of the results of work	Number of students in percent
Successfully	37%
With shortcomings (mistakes)	63%

Table 1.6.6

- According to the results of the observation, the personal component of the mathematical competence of students revealed in the fact that 90% of students behaved worthy, balanced, communicated politely, although 10% of students attracted attention with their negative behavior during the time of task completion. The teamwork was successful. The data are presented in Table 1.6.7.

Ability to manage one's emotions	Number of students in percent
Able	90%
Not able	10%

Table 1.6.7

After the students portrayed the models in the notebooks, they were offered to build the models they'd just drawn on paper, and the results were as follows: only 8% of the students were able to complete the proposed task, while 92% could not do it (Figure 1.6.1).

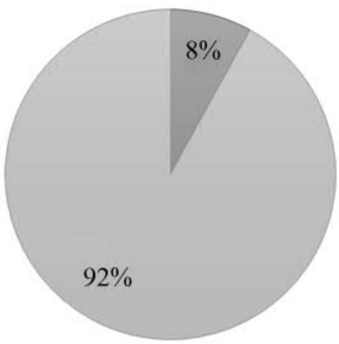

Figure 1.6.1 Results of models construction from paper.

An intermediate stage of the research study showed that the level of mathematical competence of students of IV(VT)E is low.

In order to improve the level of mathematical competence in students of the IV(VT)E of the machine-building professional field we've conducted a forming experiment, in which 182 students of IVTE of Zaporizhzhia and Cherkasy regions participated in the specialties of the machine-building professional field. Based on the fact that the students formed knowledge about the construction of geometric shapes, we set the task for the students to break into groups and build a model of a house, rocket, tank or robot. Most students were eager to make rocket (Figure 1.6.2) and house (Figure 1.6.3) models.

Figure 1.6.2 Rocket modeling by the IV(VT)E students.

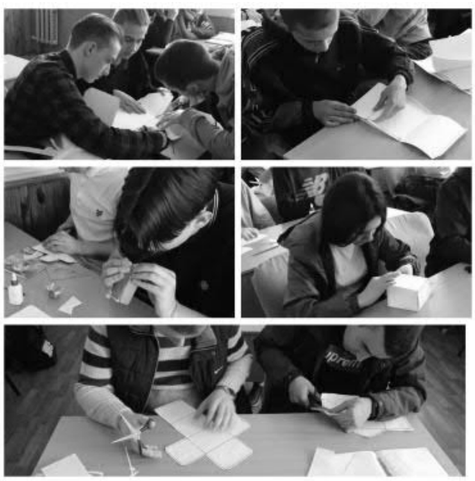

Figure 1.6.3 House modeling by the IV(VT)E students.

According to the results of the questionnaire, the motivational component (Figure 1.6.4) is composed of 65% of students that are motivated by the fact that they receive a high score for 'handcraft', 5% of students that make tasks for self-development, 28% make the assignment because of the need in knowledge for their professional activity, and 2% of students fulfill the task through external coercion.

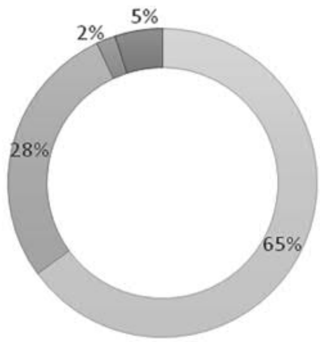

Figure 1.6.4 Results of the motivational component of mathematical competence.

Cognitive component (Figure 1.6.5) comprises 35% of students that independently drew the development of the head and body frame of the rocket model, 10% made it with the help of a group mate, and 55% drew it with the help of a teacher.

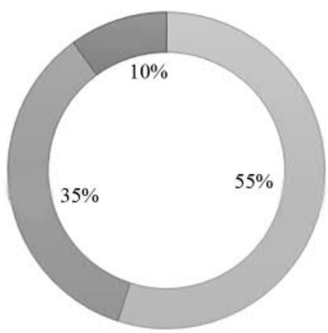

Figure 1.6.5 Results of the cognitive component of mathematical competence.

Active component (Figure 1.6.6) shows that 60% of students were able to maintain the proportions of the geometric figure, while the correct linear construction was made by 62% of the students.

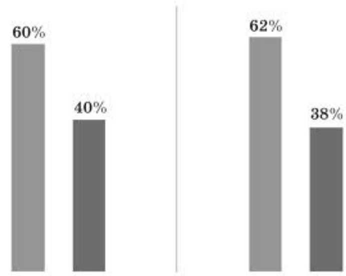

Figure 1.6.6 Results of the active component of mathematical competence.

Reflexive and value component (Figure 1.6.7): 70% of students successfully presented their models, 30% of students presented rockets with defects. One should note that in the process of manufacturing parts, 15% of students reworked their models, looking at the work of their group mates.

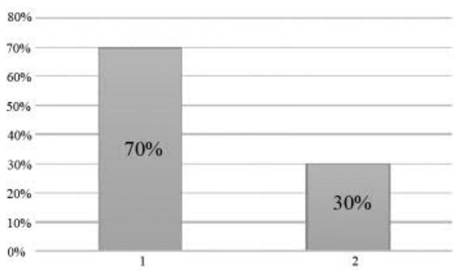

Figure 1.6.7 Results of reflexive and value component of the mathematical competence.

Personal component had 13% of students that do not know how to manage their emotions while constructing a model. According to our observations, the work in teams was carried out in a coordinated manner.

After completing the assignment, the students concluded that the skills acquired in the classroom will be useful to them in future work.

Comparing the results of two studies, we can draw the following conclusions:

- the motivational component in the construction of cardboard models has increased, external coercion has fallen to 2% (Figure 1.6.7).
- the cognitive component is that 35% of students can make the figure model (Figure 1.6.8).

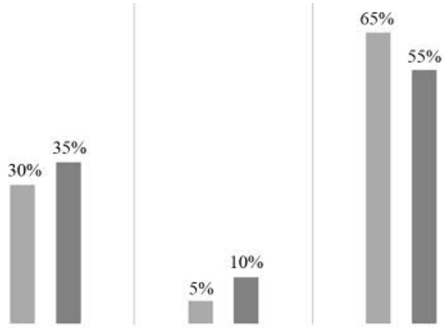

Figure 1.6.8 Comparison of the motivational component "before" and "after".

- the analysis of the active component showed that the ability to correctly linearly build figures increased up to 62%, while maintaining the proportions of the geometric figure increased to 60% (Figure 1.6.9).

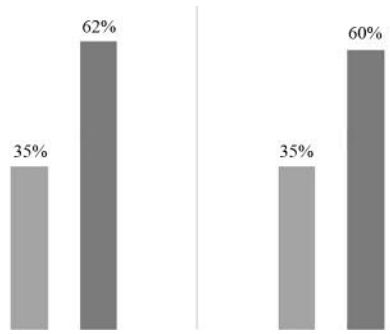

Figure 1.6.9 Comparison of the activity component "before" and "after".

- the analysis of the reflexive and value component showed that students are more successful in presenting their models, rather than drawings according to them (Figure 1.6.10).
- our observations have shown that during the research, the students worked in teams in a coordinated manner

and only 10% of the students attracted attention to their negative behavior.

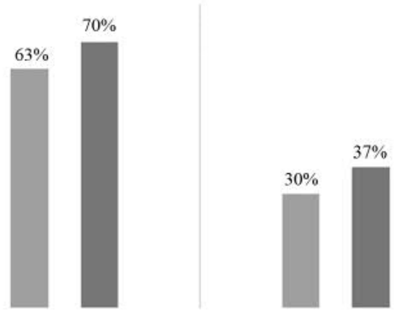

Figure 1.6.10 Comparison of the reflexive and value component “before” and “after”.

Based on the fact that the indicators of mathematical competence have improved, we can conclude that the level of the mathematical competence has increased.

Conclusion

Consequently, while studying the stereometry topics by the students of the IV(VT)E of the machine-building professional field the use of tasks to construct models of geometric shapes (including a house, a rocket, a robot, a tank, etc.) better develop mathematical competence than the usual tasks to depict figures in notebooks.

1.7 A Review of the Experience with the Implementation of STEM-education Technologies*

K. Vlasenko, I. Sitak, O. Chumak, O. Kondratyeva

*The article is published in the author's translation

The modern world assigns difficult tasks for the educational community: during their studies, students must apply their skills in practice, classes must take place in an interesting form, the result of teaching should be the competitiveness of specialists, their self-realization and high intelligence. For future professionals, this means getting paid jobs and satisfaction with their life. One of the ways of solving this problem for scientists is to consider the use of STEM education technologies, the introduction of which has become the basis for training specialists for high-tech industries in such countries as the USA, Australia, Great Britain, Germany, China, Israel, Korea, Singapore. In the countries are successfully implemented state programs in the field of STEM-education.

The STEM acronym was proposed for the first time by the National Science Foundation USA (NSF) as a designation of four specific components: science, technology, engineering, and mathematics. This definition was introduced first in 1997 in the text of the NSF project called STEMTEC (Science, Technology, Engineering, and Math Teacher Education Collaborative)[1] (2011). At the present time, there are about 100 public secondary schools in the United States that are implementing STEM disciplines. Educational institutions create groups of students with a shared interest in STEM, who are mastering the in-depth STEM content. In this context, it is taking into account the fact that young people have the opportunity to participate in research and discovery, students can try their hand at the actual production in

[1] STEM: Good Jobs Now and for the Future. U.S. Department of Commerce Economics and Statistics Administration. (2011). *ESA Issue Brief, July 2011*. 3-11.

the STEM area, students are learning the role models of STEM professions.

We adhere to the view of American scientists G. Gonzales and J. Kuoenzi[2] (2012), who have identified STEM education as covering the teaching and learning technologies in the field of natural sciences, technologies, engineering, and mathematics. According to scientists, STEM education should combine educational activities at all levels of education – both in pre-school education and at the level of obtaining a Doctorate Degree, both in formal education and in informal education.

STEM programs researchers A. Joyce[3] (2014), M. Guz, I. Heathway, J. Connings, and M. Vandwiya[4] (2013) point out the importance of providing communications between the teaching place of future specialists and the place of their future work through the organization of educational and research laboratories or professional research laboratories. The scientists argued that the effectiveness of the above process is achieved during the students` educational activities by means of involvement of the methods and experience of the professional solving the practical tasks, which they will inevitably face in future professional activities. When describing the general and specific features inherent in the educational activities of the students, who are mastering a future profession, the researchers showed that such activity results in the professional growth of the personality of the future specialist and the development of his or her professional abilities, the professional competence acquired.

Introduction of STEM education technologies is a long-standing need in Ukrainian education. In addition, we can already observe some attempts and directions of implementation of certain experience in higher education

[2] Gonzales, H., & Kuenzi, J. (2012). *Science, Technology, Engineering, and Mathematics (STEM) Education : A Primer : CRS Report for Congress*. Congressional Research Service.

[3] Joyce, A. (2014). Stimulating interest in STEM careers among students in Europe: Supporting career choice and giving a more realistic view of STEM at work. *European Schoolnet*. Retrieved from https://www.educationandemployers.org/wp-content/uploads/2014/06/joyce_-_stimulating_interest_in_stem_careers_among_students_in_europe.pdf.

[4] Goos, M., Hathaway, I., Konings, J., & Vandeweyer, M. (2013). High-Technology Employment in the European Union. *Discussion paper*, *4*, 23-31.

institutions and their promotion in the formation of students' competence in today's society. In this regard, an analysis of the use of STEM education experience and the improvement of selected approaches requires separate consideration.

The purpose of the study is to analyze the global experience of introducing STEM education technologies at the different levels of teaching and designing of certain approaches to the Ukrainian educational space, and to develop ways of improving of the STEM education system implementation through the experience of joining the laboratory on the basis of the East Ukrainian Volodymyr Dahl National University (EUNU).

Research methods: a systematic and comparative analysis of results of the scientific and methodological research, normative documents, synthesis, systematization, generalization of available theoretical positions, methods and practical results that facilitate the implementation of STEM education technologies in Ukraine.

According to the report, prepared by the group of experts for members of the US Congress (Kuenzi[5], 2008), there are from 105 to 252 STEM education programs or other similar activities in 13-15 federal agencies of the country. Annual federal allocations for STEM education typically range from $ 2.8 billion to $ 3.4 billion. All published land inventories indicate that as the key institutions in the development and implementation of STEM technologies appear the Ministry of Education, the National Science Foundation, the Health Service and social services. More than half of the federal funding for STEM education is designed to meet the needs of higher education institutions and students, the rest goes to programs at grades 11-12.

In order to implement STEM teaching more effectively, the United States introduced the Next Generation Science Standards (NGSS) for the K-12 level in early 2013. The objective of NGSS[6] (2012) was to increase the scientific

[5] Kuenzi, J. (2008). *CRS Report for Congress: Science, Technology, Engineering, and Mathematics (STEM) Education: Background, Federal Policy, and Legislative Action.* Congressional Research Service.

[6] *A Framework for K-12 Science Education: Practices, crosscutting concepts, and core ideas.* (2012). Committee on Conceptual Framework for the New K-12 Science Education Standards. National Research Council.

focus of student education, which is being implemented in eight classes of eight states of the country. The authors of the standard believed that scientific education K-12 should reflect the mutual relations of all branches of science, taking into account how it is practiced and occurs in the real world. According to new approaches, high school students are actively involved in scientific and engineering practice and use cross-cutting concepts to deepen understanding of key ideas in science and technology. This program requires significant investments to support teachers, who are providing STEM education, as well as support for STEM Innovation Networks. In addition, the US federal educational policy in STEM area focuses on issues related to STEM education in general – such as the federal efforts management and expanding the participation of the disadvantaged population, as well as those groups that are specific to STEM education at the primary, secondary, and average level.

However, despite significant efforts to implement STEM teaching in the United States, there is still a very low percentage of young people in the country interested in mastering STEM professions. Specialist in Education Policy Jeffrey J. Kuenzi notes that 28 percent of secondary school first year students are interested in STEM education, but 57 percent of these students will lose interest in technical specialties before graduation from secondary school. Moreover, the government's concern also focuses on the quality of teachers training, programs and standards. At the level of institutions of higher education, priority is given to efforts to attract and retain students in STEM specialties.

The problems of the United States are not unique. In England, the Royal Academy of Engineers emphasizes that the British will have to prepare 100.000 different STEM specialists each year by 2020 to meet demand.

According to the report by Rhys Morgan, Chris Kirby and Aleksandra Stamenkovic[7] (2016) (report for the Lloyd's Register Foundation from the Royal Academy of Engineering Education and Skills Committee), very few women and young

[7] Morgan, R., Kirby, C., & Stamenkovic, A. (2016). The UK STEM Education Landscape. *Royal Academy of Engineering*. Royal Academy of Engineering Prince Philip House, 3 Carlton House Terrace, London SW1Y 5DG.

people from ethnic minority groups are represented among young people, who choose engineering specialties.

Royal Academy conducted a detailed analysis and found more than 600 institutions that are to support engineering through the establishment of a system for continuing teachers' professional development, their direct interaction with pupils and students, the use of databases and web searches. According to scientists, in order to ensure young people's awareness of their careers and career opportunities in engineering, STEM education should start for young people aged 11-14.

Taking into account the disadvantages of engineering training, the Ministry of Education of the United Kingdom, among the key areas of its work, highlighted the increasing prestige of engineering education among young people through the improved support for STEM subjects by the teachers, the increased STEM education in primary schools, improved education and teaching in the vocational education sector, an increased access to education for the ethnic and gender groups, that are insufficient represented in STEM sphere, development of innovation teaching and increasing of involvement methods of employers to higher education, coordination of STEM-education development activities in order to reduce duplication.

The German higher education system also pays much attention to the introduction of STEM education (in the German version: the MINT Degree), but mostly in higher education. According to research[8] (2015), Germany lacks 210 thousand employees in mathematics, informatics, nature study and technology. To overcome this scarcity, the German companies are heavily investing in training MINT-specialists, hence engineers. Particular attention is paid to solving the issue of gender inequity in MINT education. In 2014, the National Pact for Women in STEM Careers program was started up, which is supported by more than 180 partners from various fields of policy, education, business, science and media.

[8] *International MARCH Workshop Lithuania*. (2015). Retrieved from https://sciencemarch.eu/index.php/events-mnu-uk/workshops-mnu-uk/intl-ws/intl-ws-lt.

Schools focusing on STEM education create an advanced nationwide network of best practice. The Association of Employers, which includes 31 institutions, is investing in the development of talents, funding of the educational programs for students and teachers (teaching, prizes, programs, research). The association also finances camps, training, conferences, cooperation with universities and joint research, expeditions, individual research.

The following technologies are used: "Flip the Classroom" – instead of getting information from a teacher and then doing a homework themselves, students watch the video tutorials at home and perform then exercises together at school; LEIFI Physik – a teacher analyzes a popular sequel (for example, "The Big Bang Theory") and uses video material to capture students with physical problems; QuantumLab Te idea – Students get acquainted with quantum physics by participating in interactive online experiments. The scientists of Germany note the importance of developing programs designed for junior pupils.

In Singapore, the introduction of STEM education begins as early as with pre-school education. In order to support STEM education, the Ministry of Education at the Singapore Research Center created a special for STEM inc. unit. Experienced professionals cooperate with teachers for joint development of STEM lessons, provide teacher training and the joint lessons teaching, aimed at developing youth's interest in STEM. According to the recommendations of the center's specialists, the typical STEM lesson has 4 components:

1 Determination of a real world problem;
2 Formulation and formulation of questions for the study of the problem;
3 Search and decision making;
4 Study of practical experience.

The program, developed by the Ministry of Education in Singapore, is designed for 50 years. Among the projects are the following ones: construction of green houses, creation of waterways connecting parks, design of the high-speed railway communication between Jurong East and Kuala Lumpur. Thus, the program encourages young people to acquire knowledge and skills in technology, engineering and design.

Among the problems in Singapore is being observed the lack of interest of young people to build a career in their homeland. It is noted that the most successful graduates of technical universities prefer to work in Europe or the United States. The same problem exists in Australia.

In 2012, the engineering movement "Robooky" arose in Australia, which was subsequently transformed into a network of engineering and robotechnics "Robooky" under the STEM system of Massachusetts University of Technology (USA). Having gained experience in international technology companies, having studied the experience of advanced countries in the field of engineering and robotechnics, the like-minded people have created for the younger generation one of the largest educational areas for robotechnics and engineering, which is presented in the USA, Russia, CIS and many other countries. Classes in Robooky help the schoolchildren to determine their future profession, to develop entrepreneurial skills and engineering thinking. Thanks to the STEM approach, children are taking a close look at the logic of the phenomena that occur, understand their interconnection, while studying the world systemically. During the classes, students learn the basics of management and self-presentation, which, in turn, provide them with a radically new level of development. Robooky Engineering and Robotechnics Schools combine various engineering modules (construction, marine, aerospace, industrial) and give children the opportunity to try themselves in completely different professions. And even if students do not choose an engineering profession in the future, the acquired knowledge and skills become their significant advantage. According to statistics, STEM-educated professionals have higher earnings even when they choose a profession that is not related to STEM. Unfortunately, the number of such schools is not sufficient, learning in them is not financially accessible to the majority of the population.

Not paying attention to certain difficulties, each of the above countries continues to develop STEM-education. This is due to the fact that the modern world needs high-tech industries that are in contact with natural sciences and creative industries. In addition, according to the study "A framework for STEM education"[9] (2016), the involvement of only 1% of the population in STEM-professions increases the gross domestic product of the country by $ 50 billion.

Thus, the introduction of STEM education technologies can change the economy of our country, make it more innovative and competitive. According to the methodological recommendations[10] (2017) on the implementation of STEM education in Ukraine, "the main goal of STEM education is to implement the state policy taking into account the new requirements of the Law of Ukraine "On Education" in regard to strengthening the development of the scientific and technical direction in the teaching and methodological activities at all educational levels; creation of the scientific and methodological basis for improving the creative potential of young people and the professional competence of scientific and pedagogical employees. The main key competences of the "New Ukrainian school" concept, namely: communication in the state and foreign languages, the mathematical literacy, competence in natural sciences and technologies, information and digital literacy, life skills, social and civil competencies, entrepreneurship, the general cultural, environmental literacy and the healthy life, harmoniously enter the system of STEM-education, creating the basis for the successful self-realization of an individual both as a specialist, and as a citizen".

To coordinate the introduction of STEM education technologies in native schools, a STEM education department

[9] Patrick, L., & Neill, T. (2016). Turning it up: A framework for STEM education. *OK Math*. Retrieved from http://okmathteachers.com/stemframework.

[10] Metodychni rekomendatsii shchodo vprovadzhennia STEM-osvity u zahalnoosvitnikh ta pozashkilnykh navchalnykh zakladakh Ukrainy na 2017/2018 navchalnyi rik. Lyst IMZO № 21.1/10-1470 vid 13.07.17 roku [Methodical recommendations on the implementation of STEM education in general and non-school educational institutions of Ukraine for the 2017/2018 academic year. IMZO letter № 21.1/10-1470 dd. 13.07.17] (n.d.). drive.google.com. Retrieved from https://*drive.google.com*/file/d/0B3m2TqBM0APKekwtZFdhWXJuODg/view [In Ukr.].

was set up at the Institute for the Modernization of Educational Content. The task of the department is to provide methodological assistance on the issues of scientific and theoretical aspects of STEM education, training of pedagogical personnel, adult education organizations, organization of thematic forums, presentations, seminars, workshops, STEM-studios and STEM-workshops.

Implementation of STEM education methodological principles in the educational process will allow students to formulate the most important characteristics that determine a competent specialist. According to the recommendations in Ukraine, the model of mixed teaching is being used, which combines traditional teaching in the classroom with online learning and practical learning activities.

The development of STEM education in Ukraine is taking place at all levels of formal education thanks to the enthusiasm and the initiative of teachers and lecturers. There are being created research laboratories, where pupils and students work together under the guidance of experienced specialists. Where for one reason or another it is impossible to create a real laboratory, there are used virtual ones. Software complexes allow to conduct experiments and to set up virtual experiments on Physics, Biology, Sociology, etc. Among such programs are to be mentioned the following ones: VirtuLab (allows to change the parameters of the course of experiments and observe the changes), Interactive Simulations (a program for simulation of experiments with the ability to change the parameters of the course and select tools), Yenka (a virtual laboratory for laboratory work in Physics, Chemistry, Programming, Design of 2D and 3D models), Virtual Chemistry Laboratory (a chemical Internet laboratory, which allows to select reagents and manipulate them). Most of these resources are English-language based ones, which greatly complicate their use by teachers in the educational process.

The Small Academy of Sciences, which is the center of the systematic out-of-school scientific and practical activity, makes a significant contribution to the development of STEM education in our country. Olympiads, conferences, competitions for the protection of scientific and practical works carried out by the specialists of the Small Academy of Sciences promote the attraction of students to the educational, practical and research activities, deepen students' knowledge of technical and natural sciences, form the cognitive interests of students, promote the professional self-determination of students. However, the lack of sufficient funding in the Small Academy of Sciences and the absence of its own research base is a significant barrier for introducing STEM teaching technologies.

The STEM Education Coalition was founded in 2016. The founders of the Coalition were the well-known domestic and foreign companies, including Ukrainian Nuclear Society, Energoatom, Kyivstar, Ericsson, Syngenta, United Minerals Group, Samsung, Microsoft Ukraine, etc. The Coalition has the task of developing recommendations for teaching STEM disciplines for schools, out-of-school and higher education institutions, organizing and conducting vocational guidance for young people, and training teachers of the innovative teaching technologies. Three main directions of their implementation are divided among the main ones as follows.

1 Development of the technological literacy of educational institutions. Technological literacy refers to the ability to use resources and tools, processes and tools that are responsible for accessing and evaluating information and the ability to use these resources to gaining new knowledge or making new products;
2 Increase of women's audience in STEM courses up to 30% ;
3 Development of "school-companies", "university-companies" connections.

The Coalition plans to create opportunities for experimental and research work in schools, conducting the scientific and technical competitions, olympiads, quests, hackathons, etc.

Unfortunately, all activities of the Coalition are mainly concentrated in the capital and several regional centers. Children from rural areas and small towns are not able to take part in appropriate activities, as a rule.

The development of STEM education is provided by the creation of various centers at higher education institutions. This contributes to increasing the interest of young people in the study of fundamental, engineering and natural sciences, enabling senior pupils and students to develop their research potential, encouraging students to enter a technical specialty of higher education institutions (HEI). Creating such cents raises the prestige of an educational institution, increases the competitiveness of its graduates, enables teachers to master modern technologies, promotes high-quality education of school pupils, broadens their knowledge and skills, adapts future students to the conditions of student's scientific activity.

Among the HEIs that have established STEM education centers, are to be mentioned the following ones:

- Borys Grinchenko Kyiv University (SMART Training Center - LEGO, Strawberry, Matrix, 3-D printer);
- Kryvy Rih State Pedagogical University (School of Robotechnics);
- Ternopil National Pedagogical University (STEM-center – LEGO, Arduino);
- Oles Gonchar Dnipro National University (Engineering School);
- K. D. Ushynsky South Ukrainian National Pedagogical University (Interuniversity Laboratory "Internet of Things");
- Kherson State University (STEM-school – LEGO, Arduino).

As you can see, there is no technical university in this list, because the creation of such centers requires significant funds, which technical universities often do not have.

One of the modern and budget methods for involving young people in STEM technologies is the creation of the Fabrication Laboratories, which are already around 1200 in the world. There are 7 such laboratories in Ukraine. The first laboratory appeared in 2014 in Kyiv. Fabrication Laboratories (FabLab) are the Modern laboratory factories for 3D modeling, 3D printing, prototyping and technical creativity. The activities in these laboratories provide students with free access to advanced equipment required for technical creativity – 3D printers, laser cutters, milling machines, Arduino kits and constructors, Raspberry boards and IoT (Internet of Things) sensors, other tools and electronics. The main tasks of the Fablabs are:

- development of engineering creativity and innovative thinking of the youth;
- teaching of students, students, graduate students and creative young people on the basics of microelectronics, robotechnics, digital prototyping, 3D modeling, 3D printing, the use of laser equipment and CNC machines;
- improving the quality of teaching and improving the ability to employ students by means of cooperation of universities, business and industry based on FABLABs;
- stimulation of youth entrepreneurship;
- involvement of students in the choice of engineering specialties;
- advanced training (or new competencies) of lecturers in the field of Internet of Things (IoT), 3D modeling, 3D printing and digital prototyping.

Since 2018, such Fabrication Laboratories began to function in Ukraine. In Kharkov, the laboratory operates on the basis of the Semen Kuznets Kharkiv National University of Economics, and in Odessa there is a laboratory based on the K. D. Ushinsky South Ukrainian National Pedagogical University and is called #MiRONAFT.

The authors of the study are participants in the project on the creation of a laboratory in Severodonetsk based on the Volodymyr Dahl EUNU. The laboratory started functioning in December 2018. This is a small workshop equipped with digital manufacturing tools: 3D printer, a laser and vinyl cutter, embroidery machines. On the basis of the laboratory pupils and students can create a variety of objects made of wood, plastic, metal or cardboard. Training and workshops, practical classes and IT forums are held by the teachers of V. Dahl EUNU on the basis of FabLab. This allows young people in the region to improve their creative abilities and realize their own projects. Community representatives, school teachers and parents are involved in the projects. Through involving in the project, we found out the difficulties of the project development and identified the ways of their overcoming.

Thus, the introduction of STEM-technologies has begun in Ukraine, but this process is not free of problems and difficulties. The main ones are, in our opinion, the following:

- activities on the implementation of STEM education is carried out only outside of school time and are not systemic;
- the implementation of interesting projects is hindered by the lack of necessary knowledge and skills among the participants;
- insufficient volume of hours for studying Mathematics, Physics at school and a constant reduction of hours for learning fundamental disciplines in higher technical school;
- insufficient quantity and capacity of laboratories for the implementation of STEM education.

Among the ways to improve the implementation of STEM education, are identified the following ones.

Integrated courses for the preschool, secondary and tertiary education must be developed. Robotechnics, communications, urban transport, healthcare, space exploration, environmental issues, spread and prevention of diseases should be included in the courses. When considering such topics, pupils and students should use Mathematics and other sciences for simulating, so the subject of informal events of STEM education should be coordinated with programs in Mathematics, Physics and Informatics.

In our opinion, involving employers in the creation of “University-factories” or “School-factories”, which will allow to develop the practical-oriented projects and implement them in real terms, may be effective. Availability of the laboratories and workshops equipped at the enterprises will allow to avoid spending extra funds for educational equipment.

In order to overcome the difficulties hampering the implementation of STEM education in Ukraine, a new culture of the science-based teaching should be created that shall summarize the differentiated teaching and learning experience in the various STEM fields. It is necessary to implement a state program of mass training of pedagogical employees for the realization of STEM education technologies.

We also propose to create the educational online platforms, which accumulate the experience of teachers and design teams that implement the effective STEM practices. Such platforms will enhance cooperation between educational institutions, research institutions, employers, government agencies and civil organizations in the field of STEM education.

CHAPTER 2

MATHEMATICAL TRAINING AT THE UNIVERSITY

2.1 Testing as a Method For Assessing Students' Knowledge*

I.Volodko, S.Cernajeva

*The article is published in the author's translation

Introduction

Nowadays, the computers and other IT technologies are increasingly entering not only our daily life but also education field. Use of electronic media and resources is a routine of today's youth, so the information they operate with is mainly found on the internet not in the books, consequently use of IT and communication technologies in education and study process also increase. Likewise, computers and IT technologies are used for assessment and monitoring of the students knowledge. A contemporary assessment system is being developed in parallel to the traditional students' knowledge assessment system in Latvia – information and communication technologies (ICT) based evaluation system where tests are used as means of pedagogical tool for measuring the students progress. Assessment of the students knowledge by means of tests provide wide possibilities for study process modernisation and optimisation and also significantly reduces work load of teaching stuff.

Review of literature

Contemporary literature offers many articles about use of information and communication technologies in the field of highest education and study process. Boom of ICT provides vast possibilities for development of new teaching aids like online consultations, video materials, tests for students' knowledge assessment. Development of computers and information technologies also influences mathematics tuition process[1]. The authors describe synergy of mathematics and computer science and also explore pupils' motivation to engage in mathematics study process with the aid of ICT.

[1] Galbraith, P. & Haines, C. (1998). Disentangling the nexus: Attitudes to mathematics and technology in a computer learning environment. *Educational Studies in Mathematics*, *36 (3)*, 275-290.

Pupils and students appreciate use of ICT at the universities. The article[2] explores the way the students utilize and how they perceive ICT at study process in a university.

The article[3] describes 10 main teaching methods in particular and compares their usefulness. Assessment of the pupils knowledge by means of tests is named as one of the main methods that has been recognized as the most efficient one.

Resources of literature offer several articles that describe assessment of the pupils and students knowledge with the aid of tests. For example J. Appleby, P. Samuels and T. Treasure-Jones[4] (1997) describe the test *Diaglosys* designed for assessment of the students' mathematics knowledge, the framework of which can be applied to the other disciplines. Another article[5] analyses correlation between types of test problems and assessment results. The authors conclude that students show better results in the tests that can be passed by recalling facts and algorithms, whereas in the tests that demand comprehensive understanding and in many cases the right answers can be found only in books, the results are worse. E. Seemann[6] (2015) analyses the matter of the test results evaluation. As mathematical problems can be quite complicated ones that comprise several sub-problems, the test evaluation system shall measure interim results and shall determine whether student uses appropriate approach for solution of the problem.

Knowledge assessment of prospective students prior to their studies allows spotting students with the weakest

[2] Breen, R., Lindsay, R., Jenkins, A. & Smith, P. (2001). The Role of Information and Communication Technologies in a University Learning Environment. *Studies in Higher Education*, *26 (1)*, 95-114.

[3] Dunlosky, J., Rawson, K.A., Marsh, E.J., Nathan, M.J. & Willingham, D.T. (2013). Improving students' learning with effective learning techniques: Promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest, Supplement*, *14 (1)*, 4-58.

[4] Appleby, J., Samuels, P. & Treasure-Jones, T. (1997). Diagnosys – A knowledge-based diagnostic test of basic mathematical skills. *Computers and Education*, *28 (2)*, 113-131.

[5] Boesen, J., Lithner, J. & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics*, *75 (1)*, 89-105.

[6] Seemann E. (2015). Unit testing Maths automated assessment of mathematic exercises. *Lecture Notes in Computer Science*, *Vol. 9307*, 530-534.

knowledge so they can be provided with duly assistance. Students of Singapore Technological Institute undergo mathematical tests three months ahead of commence of their studies[7]. The test results indicate, which of the prospective students shall undergo supplemental online courses to obtain essential mathematics knowledge.

Tests have been used not only for assessment of mathematics knowledge but also for the other subjects. There are articles[8] [9] describing design and implementation of tests for programming course. The tests have been adopted to particular demands and students' competency level thus making the tests to work in the most efficient way. Also P. Abreu, D. Silva and A. Gomes[10] (2019) discuss application of tests at programming course. The main goal is motivation of students for studies. Objective of the research is to determine if students' motivation to study increases if they are filling out the tests rather than writing a conventional test work and which test questions are more suitable for the assessment of the students knowledge. The authors of the paper conclude that assessment results depend on chosen question type and for the assessment of programming comprehension the multiple-choice question is the most suitable one and it also motivates students to study.

The research[11] explores application of the multiple-choice questions for the assessment of students' knowledge. The article describes use of the multiple-choice questions along with conventional exams. The results demonstrate that basic

[7] Tan, R.Z., Wang, P.C., Lim, W.H., Ong, S.H.C. & Avnit, K. (2018). Early Prediction of Students' Mathematics Performance. *Proceedings of 2018 IEEE International Conference on Teaching, Assessment, and Learning for Engineering*, 651-656.

[8] Chrysafiadi, K. & Virvou, M. (2018). Create dynamically adaptive test on the fly using fuzzy logic. *Proceedings of 9th International Conference on Information, Intelligence, Systems and Applications, IISA 2018*.

[9] Amelung, M., Krieger, K. & Rцsner, D. (2011). E-assessment as a service. *IEEE Transactions on Learning Technologies*, *Vol.4, 2*, 162-174.

[10] Abreu, P.H., Silva, D.C. & Gomes, A. (2019). Multiple-choice questions in programming courses: Can we use them and are students motivated by them? *ACM Transactions on Computing Education, Vol. 19, 1*.

[11] Bremner, D.J., Kernec, J.L., Fioranelli, F. & Dale, V.H.M. (2018). The Use of Multiple-Choice Questions in 3rd-Year Electronic Engineering Assessment: A Case Study. *Proceedings of 2018 IEEE International Conference on Teaching, Assessment, and Learning for Engineering*, 887-892.

skills of students can be tested with well drafted questions. The tests that provide instant results are appraised by both students and teaching stuff as they instantly indicate the students progress at the subject.

A research on variety of students' examination options has been done in three British universities[12]. There were explored 6 different knowledge assessment methods. The students favoured the multiple-choice questions whereas the oral exam and group work got the least students evaluation. The tests are used for assessment of students' knowledge also in US universities. The authors of the article[13] conclude that the tests enhance study process more than rereading of books or the lecture notes. Students, which had several tests during a Term, had better results of exams in comparison to those ones who took only few written test works. R. Bangert-Drowns, J. Kulik and C. Kulik[14] (1991) conclude alike: frequent tests enhance students' practical skills and improve results. Progress of the students increased along with number of tests, whereas the increase of the progress dropped when more tests had been added. This drives to conclusion that number of tests shall be moderate.

Scientists of the Washington University carried out interesting experiment[15]. They imitated classroom where three consecutive lectures were given that were followed by several days long study processes of various types – discussions on the lectures materials, the multiple-choice tests and the tests with short answers. Half of the questions had feedback (the right answers were given) – another half had not. After a month, the students had the overall test on a whole lecture material. The students that took the

[12] Furnham, A., Batey, M. & Martin, N. (2011). How would you like to be evaluated? The correlates of students' preferences for assessment methods. *Personality and Individual Differences, Vol. 50, 2*, 259-263.

[13] Nip, T., Gunter, E.L., Herman, G.L., Morphew, J.W. & West, M. (2018). Using a computer-based testing facility to improve student learning in a programming languages and compilers course. *Proceedings of the 49th ACM Technical Symposium on Computer Science*, 568-573.

[14] Bangert-Drowns, R.L., Kulik, J.A. & Kulik, C. (1991). Effects of Frequent Classroom Testing. *Journal of Educational Research, Vol. 85, 2*, 89-99.

[15] Butler, A.C. & Roediger III, H.L. (2007). Testing improves long-term retention in a simulated classroom setting. *European Journal of Cognitive Psychology, Vol.19, Issue 4*-5, 514-527.

tests with short answers had the best results. The feedback had no influence on achieved results. The research results clearly demonstrate that tests help to memorise the subject for longer period.

Research objective

The research aims to answer essential questions considering utilisation of the tests: Do the students treat tests seriously? Do the tests provide indeed unbiased results and what shall be done to obtain these results?

Basic summary

Assessment of students' knowledge by means of online tests is rather new knowledge evaluation and control method. New ideas, trends and methods provoke contradictory judgement as usual. Even now, quite opposite opinions about tests, their usefulness and assessment unambiguity can be heard. Sometimes the same arguments have been used on both sides, i.e., from those who are for and those who are against using the tests for knowledge assessment.

Main arguments „against" are:

- the tests reduce the lecturer's role at the assessment process;
- the tests burden comprehensive and profound assessment of teachable persons' knowledge.

Main arguments „for" are:

- the tests decrease labour-intensity of the tuition process;
- the tests enable objective assessment of the tested persons' knowledge.

It is important to observe the major components of the tests to insure assessment objectiveness:

1 Setting objectives of the test;
2 Selection of proper question type;
3 Exact wording of the question;
4 Evaluation and interpretation of the test results.

One must set objectives that should be achieved prior to commencing the test designing. If there are several objectives, they must be considered by their relevance and least important ones should be eliminated, and an equal number of questions should be compiled for each important objective.

When considering the type of question, the compiler should anticipate possible student errors in multiple choice questions (these are questions where students choose the answer from the provided options). Therefore, it is better choosing questions that students have to answer themselves. The question should be brief and clearly defined, with plain and easy perceivable wording. The question should be designed in a manner that does not cause big problems for the student and it could be answered with the aid of textbook, by reminiscing facts and algorithms. It is even better, if the questions could be compiled in a manner that allows to determine interim results and track errors.

Likewise, evaluation and interpretation of the results is an essential component of the test because, without comprehensive analysis of the results, one cannot conclude what aspects should be addressed further, what are the ways for improvement of the tuition methods and quality of the learning process. The mathematical and statistical methods are widely used for evaluation of the test results allowing effective utilization of computers at this phase that enables automation and optimization of the intended tasks thereby.

Department of Engineering Mathematics of Riga Technical University (RTU) assess knowledge of first year students by means of tests, thus replacing most of the home assignments of the Term 1. Authors of the article[16] compiled and implemented at the RTU ORTUS environment 16 tests with mathematical problems and 2 theoretical tests on Term 1 and 6 tests with mathematical problems and 2 theoretical tests on Term 2. As stated above, the tests replaced the largest part of the home assignments on Term 1 and partially

[16] Volodko, I. & Cernajeva, S. (2016). Application of the portal ORTUS in Studying Mathematics at the Riga Technical University. *Conceptual framework for improving the mathematical training of young people*. Budapest, Hungary: SCASPEE, 134-144.

on Term 2. Considering reluctance of the students fulfilling longer tasks, all tests are not labour consuming and contain 2 – 5 problems. Students must provide the right answer, not multiple choice, to all problems in all tests. Each test must be performed within 2 hours then the test is closed automatically. 3 attempts are allowed for each test and the best result is chosen for the final rating. The problems of consecutive attempts are similar but not identical. Considering large size of the database, the probability that the student will get the same problem is rather tiny.

The students had following tests on Term 1:

1.1 Determinants;
1.2 Operations with matrices;
1.3 Systems of linear equations;
2.1 Linear operations on vectors and dot product;
2.2 Cross product and mixed product of vectors;
3.1 Straight line in a plane;
3.2 Analytic geometry in three dimensional space;
4.1 Operations with complex numbers in algebraic form;
4.2 Operations with complex numbers in polar form and exponential form;
5.1 Limits1;
5.2 Limits2;
7.1 Extrema of functions, the largest and smallest values of a function in a closed interval;
7.2 Inflection points and asymptotes of a function;
8.1 First order partial derivatives of a function of several variables;
8.2 First order partial derivatives of a function of several variables;
8.3 Extrema of a function of two variables.

Unfortunately, our ORTUS environment offers limited functionality for a while – the all tests must be compiled so that the outcome that is filled in answer field must be nothing but a number. Currently, students cannot fill a function, matrix or equation in the answer field, fortunately such functionality is under development. Meanwhile, we compile all tests so that the outcome is a number. Here are some examples of the tests. For instance, test 1.2. Operations with matrices:

1 Given matrices $A = \begin{pmatrix} 5 & 4 & 1 & 2 \\ 3 & 1 & 0 & 7 \\ 1 & -1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 7 & 6 \\ 1 & 3 & 2 \\ 9 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix}$.

Calculate the element c_{23} of matrix $C = A \cdot B$.

2 Given matrices $\begin{pmatrix} 2 & 3 & 0 \\ 2 & 0 & 4 \\ 1 & -2 & 2 \end{pmatrix}$.

Calculate the sum of the elements in the first row of matrix $B = A^2 - 3E$.

3 Given matrices $\begin{pmatrix} 0 & 2 & 2 \\ 3 & 2 & 4 \\ 2 & -1 & 5 \end{pmatrix}$.

Find the element b_{21} of matrix $B = A^{-1}$.

4 Given matrix equation $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & -4 \\ 1 & 3 \end{pmatrix}$.

Find the sum of the elements of matrix X.

Another example – test 3.1. Straight line in a plane:

1 The straight line passes through point $A(3;-1)$ parallel to vector $\overrightarrow{BC}$, where $B(5;-2)$, $C(4,1)$. Find the slope of the line.

2 Find the cosine of the angle between the lines

$$3x - y + 5 = 0, \quad 2x - 6y - 9 = 0.$$

3 The straight line passes through point $A(-2;1)$ and form the angle 45^0 with the Ox axis. Find the second coordinate of the point of intersection of the line and the given line

$$x + 3y - 5 = 0.$$

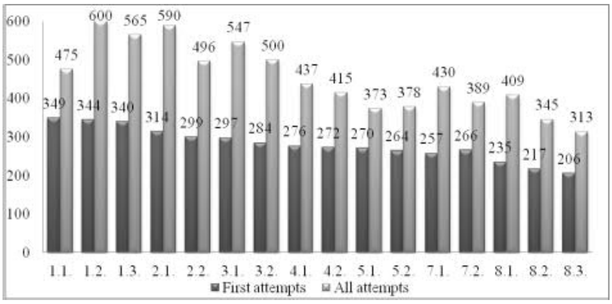

Figure 2.1.1 The number of students that performed the test.

The following statistics of the results is obtained from the tests of first year students of Faculty of Computer Science and Information Technology (FCSIT) on Term 1 from the current academic year (2018/19). 455 students are enrolled in the course. Figure 2.1.1 shows the number of students who have performed the tests: the first bar indicates number of initial attempts taken (which corresponds to the number of students who performed the tests at least once), the second bar – the total number of attempts.

It can be read from the diagram that the number of students who have performed the tests decreases: if the first test was performed by 349, then the last - merely by 206 students. There are two reasons: first, some students drop out of the university in the middle of a Term, second, students get tired and are performing assigned tasks less when the Term is running to an end.

The next two diagrams (Figure 2.1.2 and Figure 2.1.3) show the average rating of the tests (in percentage): the first bar – the average rating of the initial attempts, the second bar – the average rating of the all attempts, the third – the average rating of the last attempt, and the fourth – the average rating of the best attempt.

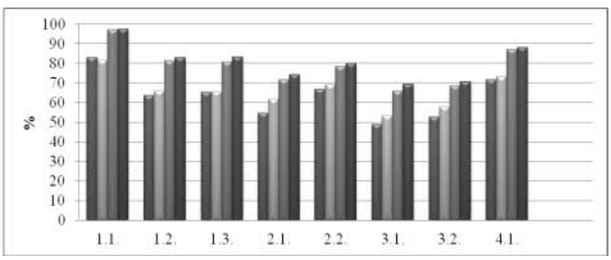

Figure 2.1.2 Average rating of the tests 1.1. – 4.2. in percentage.

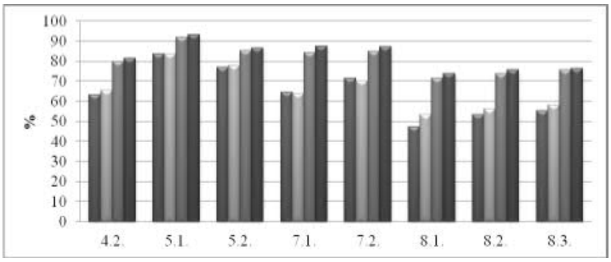

Figure 2.1.3 Average rating of the tests 5.1. – 8.3. in percentage.

Considering all ratings of the tests put together, the following results are obtained: the average rating of the initial attempt is 63.83%, 65.98% – of the all attempts, 79.84% – of the last attempt, and 81.69% – of the best attempt.

We can conclude from the diagrams (Figure 2.1.2 and Figure 2.1.3) that generally the last attempt is the best one whereas the first attempt often is not the most successful one. Generally the students do not repeat the test and do not exercise solving the problems further when receiving the highest rating.

The best results were obtained in the test 1.1. The Determinants and in the test 4.1. The Limits1 whereas the worst results were in both tests of Analytic Geometry – 3.1. Straight line in a plane un 3.2. Analytic geometry in three dimensional space.

Comparing the number of students who got the maximum rating to the number of students that performed the test (Figure 2.1.4), we can see that more than half of the students came up with absolutely correct answer to 12 tests, whereas less than half of students gave correct answers to 4 tests.

The test results determine 10% of a student's Term grade. Theoretical tests determine next 5% of the Term

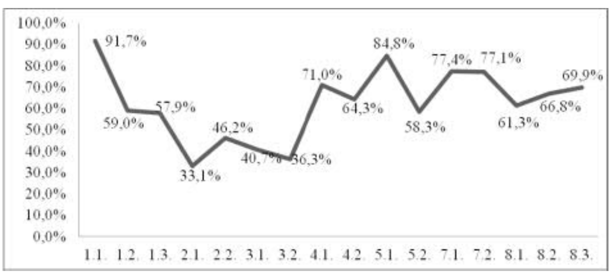

Figure 2.1.4 Number of students (%) who got the maximum rating.

grade. Students pass intermediate exams in the middle and in the last week of the Term. Students are exempted from the Term exams when passing intermediate exams successfully. Students have to pass theoretical tests prior to every intermediate exam. Both theoretical tests comprise 9 multiple-choice questions, i.e., students must opt one or more right answers from the 4-5 given alternatives. Answers to the all questions are available in the lecture notes. There are 3 attempts also enabled for these tests, yet the question of each consecutive attempt changes. Theoretical tests force students reviewing the theory prior to the intermediate exams,which do not comprise theoretical questions anymore, yet theoretical knowledge eases solution of the problems.

As statistics indicate, students are more successful at the tests comprising problems rather than theoretical tests. The first bar of Figure 2.1.5 shows the total number of students who have performed the tests, the second bar - total number of all attempts, the third bar – the number of students who scored the highest possible rate, i.e., answered all questions correctly. The Figure shows that only 10.5% of the students answered the first theoretical test (TT1) correctly and the second theoretical test (TT2) even less – merely 9.8%.

Even though, the number of students who gave correct answer is small, the average score is pretty fair: it overreaches 60% already at the first attempt, whereas highest score exceeds 70% (Figure 2.1.6).

The tests replace only two home assignments at Term 2: Definite and improper integrals and Double integral. For the present, the rest of the home assignments students shall calculate and hand-in for evaluation in paper form because

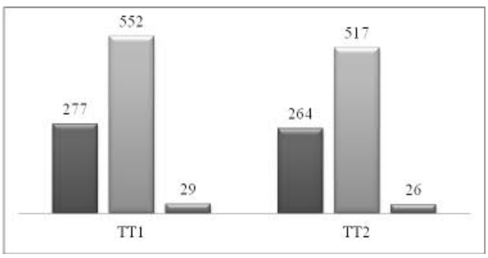

Figure 2.1.5 The number of students that performed the theoretical tests.

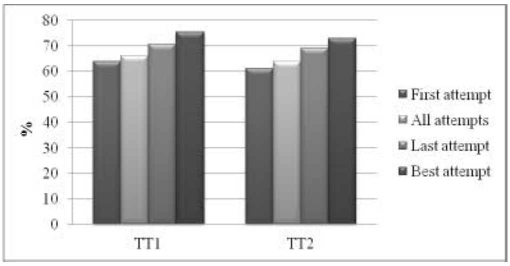

Figure 2.1.6 Average score of the theoretical tests in percentage.

currently RTU ORTUS e-studies environment does not allow writing the function in the answer field. This functionality is being developed and most probably after some years tests will replace all home assignments.

Likewise, on Term 2 students must have two theoretical tests and following tests of mathematical problems:

1.1 Calculation of a definite integral;
1.2 Applications of definite integrals (areas of plane figures given in Cartesian coordinates);
1.3 Applications of definite integrals (areas of plane figures given in polar coordinates and parametric form);
1.4 Improper integrals;
2.1 Double integral in Cartesian coordinates;
2.2 Double integral in polar coordinates.

In addition to the tests replacing the home assignments during both Terms, there are tests that follow each lecture for the purpose of exercising and self-control, results of which do not influence the Term grade. These are 22 tests on Term 1 and 15 tests on Term 2, which have no limitations in time and number of attempts. The students must choose one of four answers in these tests. As statistics indicate (Figure 2.1.7), students hardly ever are performing these tests. If the first test counts 357 attempts, then last tests get merely 2 or 3.

The results of Term 2 are even worse: if 77 students filled out the first test and 19 students the second test, then the rest of the tests counted a handful of attempts. When reading the results of the performed tests (Figure 2.1.8), one can get the impression that most of the students who open these tests are rather randomly choosing the answer,

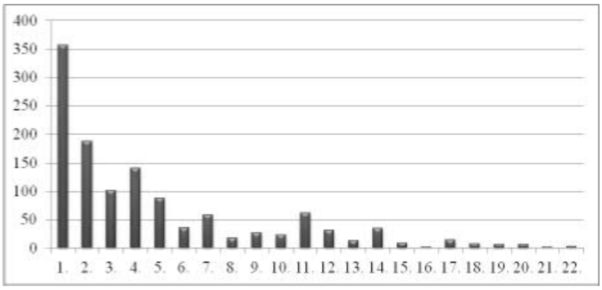

Figure 2.1.7 Number of attempts of voluntary tests of Term 1.

not solving the problem. Indeed, these tests require more efforts, i.e., 10 tasks shall be resolved.

Conclusions

Considering the data above, we come to the following conclusions:

1 If the students can choose, they prefer performing the tests rather than handing-in handwritten home assignments;
2 More than half of the students, who perform the tests repeatedly, are mastering problem solving and are gaining the highest grade as a result;
3 The students hardly ever are performing tests that do not affect the Term grade;
4 The students prefer performing the tests where the answer can be found by logic and they dislike performing the tests, which require labour-intensive solution of a problem;
5 Comparing the students’ test results with the results of the test works and exams, prevalence of congruence is

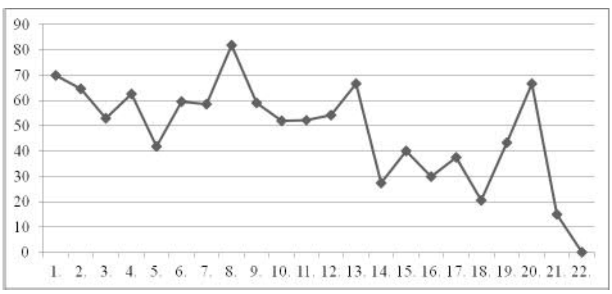

Figure 2.1.8 Scores of voluntary tests of Term 1 (in percentage).

observed – students who show good results in the tests are successful at other assessment tests;

6 System of the tests noticeably eases work of the faculty by releasing the teaching stuff from time consuming editing of home assignments.

Finally, we can conclude that assessment of students' knowledge by means of tests indeed cannot be the sole evaluation method, though properly compiled tests and relevant evaluation system of the results in combination with other assessment methods provide objective results.

2.2 Monitoring the Ukrainian University Students' Math Levels*

T. Krylova

*The article is published in the author's translation

The problem of developing intellectual abilities and creative mathematical skills in students is pressing and important for Ukrainian society.

The solution to this problem should be based on decent mathematical training of students. But, unfortunately, over the recent ten years, the mathematical training of secondary school graduates has considerably worsened, as evidenced by the results of EIT (external independent testing) in mathematics, as well as by results of college, technical schools and vocational schools graduates.

In order to properly organize the educational process in higher and applied mathematics, the teachers must know which student community he will work with, in particular, which students they can rely on during the classes. To do this, all institutions of higher education in Ukraine monitor the knowledge and skills acquired by students, in particular in mathematics.

«Monitoring (*eng*. "monitoring" from *monitor* – observational) – continuous observation of a process with the aim to find out its compliance with the desired result or previous assumption»[1] (Zagoruiko, 2010).

Educational monitoring performs the following functions: adaptive, diagnostic, integrative, comparative, pragmatic, formative, system-forming, informational, analytical-evaluative, stimulating and motivational, controlling, forecasting, corrective.

There exist the following types of pedagogical monitoring: didactic, managerial, educational, social and psychological, psychological and pedagogical, as well as medical.

Pedagogical monitoring is closely connected with pedagogical diagnostics.

[1] Zagoruiko, O. (2010). *Unabridged universal dictionary of the Ukrainian language*. Kharkiv, Ukraine: TORSING PLUS. [In Ukr.].

«Diagnostics (*gr*. "diagnostiko" – able to recognize) – the process of recognition, the doctrine of the principles and methods of diagnostics.

Educational diagnostics is a process of outlining the results of educational activities of students and teachers in order to identify, analyze, evaluate and adjust education»[2] (Kremin', 2008).

The concept «pedagogical diagnostics» was proposed by K. Ingenkamp[3] (1968). Pedagogical diagnostics is as old as pedagogical activity itself. It took a few millennia of teaching effort to define the results of educators' contribution with the help of pre-scientific methods. In the last two centuries scientific and supervisory methods have been used.

K. Ingenkamp (1968) highlighted the following aspects of pedagogical diagnostics:

- study (data collection, comparison, interpretation, analysis);
- forecasting;
- making pupils and students aware of the results of diagnostic activities.

Non-experimental methods are used to investigate the problems of study, which serve to detect or lack any psychological peculiarities, diagnostic methods (for quantitative measurement), experimental methods (for explaining mental phenomena) and forming methods (for identifying developmental opportunities).

To research the problems of study they apply non-experimental methods, with the aim of identifying or not any psychological peculiarities; diagnostic methods (for quantitative measurement), experimental methods (for explanation of mental phenomena), and forming methods (to identify developmental opportunities).

There exist the following aspects of diagnostic activity: comparing, analyzing, forecasting, interpreting, informing pupils and students about the results of this activity,

[2] Kremin', V. (Ed.). (2008). *Encyclopedia of Education*. Kyiv, Ukraine: Yurincom Inter. [In Ukr.].

[3] Ingenkamp, K. (1991). *Pedagogical diagnostics*: Transl. from German. Moscow, Russia: Pedagogika. [In Rus.].

monitoring the influence of various diagnostic methods on them.

Diagnostics aimed at improving the learning process should be focused on the following goals:

- internal and external correction in case of incorrect assessment of the results of training;
- identification of gaps in education, confirmation of successful learning outcomes;
- planning the next stages of educational process, motivation by encouraging success in learning and adjusting the complexity of the following steps;
- improving the learning environment.

Diagnostics interprets the learning outcomes in relation to the ways, methods of achieving these outcomes, reveals the trends, the dynamics of developing educational products.

The tasks of diagnostics are:

- to reveal the relative level of development of pupils, students, cadets under the influence of certain practices;
- identification of potential development opportunities.

Diagnostics (educational diagnostics) covers monitoring, verification, evaluation, accumulation of statistical data, their analysis, detection of dynamics, trends, forecasting of subsequent events.

They distinguish between the diagnostics of scope of knowledge and diagnostics of learning capacity.

Diagnosing of scope of knowledge is the assessment of consequences and outcomes achieved.

Scope of knowledge is also interpreted as the extent to which the intended goal was achieved. The most important principles of diagnostics and monitoring students' scope of knowledge are objectivity, systematicity, comprehensibility (clearness).

The system for diagnosing, controlling, verifying and assessing acquired knowledge and skills comprises the following steps:

1 Preliminary identification of the scope of knowledge of pupils and students;
2 Formative assessment in the process of studying each topic in question;
3 Repeated topic assessment;
4 Periodic assessment of acquired knowledge and skills after the full section or topic of the course;
5 Final assessment and accounting of knowledge, skills and abilities acquired by pupils and students at all stages of the didactic process; it is a diagnosis of the quality of actual learning and its relevance to the goal set at a specific stage;
6 Complex assessment, its main function being to diagnose how efficiently interdisciplinary connections were built. The practical criterion of complex assessment is the ability of pupils and students to explain phenomena, processes, events, based on the block of information that was acquired during the study of all subjects.

The basic structure of pedagogical monitoring comprises pedagogical control (initial (preliminary, diagnostic), on-going, periodic, repeated, thematic, intermediate, summative (outgoing), final).

"Pedagogical control is a system for assessing the results of learning, developing and educating students"[4] (Sliepkan', 2005).

Pedagogical control fulfils controlling, diagnostic, educational, developmental, stimulating, educational, organizational, measuring, evaluating, prognostic-methodical, or supervisory functions, as well as functions of control adjustment, planning. The following types of pedagogical control are distinguished: incoming (preliminary, diagnostic), ongoing, periodic, thematic, midterm, intermediate, repeated, attestation, summative (outgoing), complex, final, mutual control and self-control. There exist the following forms of pedagogical control: exams, tests, recitation, various types of control papers, tests, research

[4] Sliepkan', Z. (2005). *Scientific foundations of teaching process in higher school*. Kyiv, Ukraine: Vyshcha shkola. [In Ukr.].

abstracts, colloquiums, reports at seminars; laboratory, course, qualification, and diploma papers, etc.

The main ones among them are evaluation at lectures, at practical, laboratory and seminar sessions, at consultations, tests, exams and during extracurricular time.

Extracurricular evaluation includes the checking of homework, laboratory and control papers, lecture notes, research abstracts, corresponding to the part of lecture course offered for self-study; private conversations with students at consultations, conducting educational competitions and competitions for the best connoisseur of the subject, for the best student in the course, the best performance in laboratory and research work. The choice of forms of control depends on the purpose, content, methods, time and place.

The process of implementation of control measures is based on the requirements of organizational principles: systematicity, comprehensiveness, objectivity, differentiation, taking into account the individual characteristics of each student, humanity, transparency, unity of requirements, benevolence, educational nature of control.

All these principles of monitoring the acquired knowledge, skills and abilities of students are closely interconnected, complement each other and collectively define the requirements for the forms and methods of verification and evaluation of knowledge.

The focus on personality as a feature of learning outcomes evaluation presupposes the following: focus on students' acquisition and use of knowledge, on support of their individual development; monitoring compulsory results based on students' achievement, evaluating learning outcomes on a positive basis, ensuring unambiguous student performance evaluation. by any teacher, the adjustment of evaluation instruments on diagnostic base, the adjustment of verification tasks based on level differentiation, the openness of content, the evaluation periods and the requirements for learning outcomes, the purposeful involvement of students in supervisory activities (Dremova[5], 2004).

[5] Dremova, I. (2004). *Control of students' knowledge of algebra in high school*. Candidate's thesis for a candidate degree in pedagogy. Kyiv, Ukraine. [In Ukr.].

In pedagogical diagnosis it is important to determine the quality of the evaluation results. The most important criteria for assessing the quality of evaluation are objectivity, reliability and validity.

The most important principles for diagnosing and assessing the knowledge scope are objectivity, systematicity and visibility (openness).

Teachers of mathematical disciplines in Ukrainian higher educational institutions have a wealth of experience in determining the level of students' knowledge in mathematics.

During the first two semesters students of technical higher educational institutions must acquire the course of higher and applied mathematics which is of considerable volume. The complexity of mastering mathematics as a discipline is associated with its abstract nature. The main objective is to teach mathematics to the students, but for successful achievement of this goal it is not enough to deliver lectures skillfully and conduct practical classes, it is also necessary to control the level of acquisition of the proposed knowledge.

It is advisable to use the following forms of evaluation, broadly introducing the rating system of evaluation in the conditions of modular training:

- recitation from students at classes;
- various types of control papers (control paper (CP) to determine the remaining knowledge, «fast» CP with the verification of theoretical knowledge, testing, short-term CP, traditional CP, «rector's» CP, complex CP);
- conducting and checking laboratory works;
- independent control papers of evaluative and formative character;
- various types of homework;
- colloquium;
- attestation;
- pass/fail exams, examinations.

When conducting different types of CP there is always a need to rely on standards for student performance evaluation. It is advisable to report to students how many points they are given for correct performance of each task of the CP, each task of any other work or examination task, etc., how many points are removed for a slip of the pen, a mistake or a gross mistake.

In order to detect existing knowledge and acquired skills of freshmen from elementary mathematics, initial evaluation is carried out. At the very first class in higher and applied mathematics, a two-hour control paper is offered in order to reveal remaining knowledge in elementary mathematics ("zero" control paper). Professors of the Department of Higher Mathematics in Dniprovsky State Technical University (DSTU) conduct a so-called "zero" control paper, not testing, because the test results do not allow the teacher to make a conclusion on where, at what stage of reflection the student made a mistake, what kind of mistakes most students make and so on.

An evaluation scale is provided for each set of tasks.

The Department of Higher Mathematics of DSTU has drawn up variants of «zero» control papers for each course of study specifically, the tasks of control paper are updated annually.

The control paper contains the following tasks:

1 Perform arithmetic operations with ordinary and decimal fractions;
2 Make identical algebraic and trigonometric transformations;
3 Solve linear and square algebraic equations, index, logarithmic and trigonometric equations,
4 Present a formula for calculating the length of a circle, the area of a certain flat figure, the volume of a certain body.

For each set of tasks, a rating scale is provided.

Let us present one variant of a «zero» control paper:

1 To calculate:

1.1. $\left(\frac{7}{9}-\frac{47}{72}\right):\frac{5}{4}+\frac{7}{40}$ (5 points)

1.2. $(0{,}358-0{,}108)\cdot 1{,}6$ (5 points)

2 To present the following as a power expression:

$\sqrt[5]{\sqrt{32}}:\sqrt[3]{2}$ (10 points)

3 To reduce:

3.1 $\frac{\sqrt{x}+1}{x\sqrt{x}+x+\sqrt{x}}:\frac{1}{x^2-\sqrt{x}}$ (10 points)

3.2 $(\cos\alpha-\cos\beta)^2-(\sin\alpha-\sin\beta)^2$ (10 points)

4 To solve an equation:

4.1 $x-2{,}5=4{,}5-6x$ (8 points)

4.2 $x^2-3x+2=0$ (8 points)

4.3 $2^{-2x}=\frac{\sqrt{2}}{2}$ (10 points)

4.4 $\log_2^2(x+2)=4$ (10 points)

4.5 $\frac{4ctgx}{1+ctg^2x}+\sin^2 2x+1=0$ (10 points)

For each task, the points in the mark journal are written down separately in order to know not about the overall mathematical training of the first-year students but to identify the "gaps" in their knowledge.

For the seventh year in a row, the author conducts additional classes on the adaptation course in elementary mathematics, which is adjusted annually according to the analysis of the results of "zero" control paper.

Separately given points for each specific task allow the teacher to determine to which sections of elementary mathematics more attention should be paid at additional classes and consultations. At the end of an additional class, students are offered tasks for home independent study.

At the end of each topic, a short-term control paper (10-20 minutes), consisting of 1-2 tasks, is carried out. The student who has not received enough points should work out this section again, solve additional tasks, and write the control paper again.

Of course, it would be desirable to include additional lessons within the adaptive course of elementary mathematics in the timetable or in the syllabus on higher and applied mathematics.

In addition to the adaptive course of elementary mathematics, students are offered an electronic teaching aid on mathematics, developed by T. Krylova and O. Gulesha[6] (2009), where the first section is devoted to elementary mathematics.

Students who are focused on achieving their educational goals improve their knowledge of elementary mathematics, which leads to a better understanding of the course of higher and applied mathematics, they study and use their knowledge to solve applied, professionally directed tasks and problems of interdisciplinary nature.

"Zero" control papers are carried out at the beginning of each semester after the examination session, in order to identify and eliminate the shortcomings of the acquired knowledge, developed skills and abilities of students.

Let us provide one of the variants of such control paper:

1 Find the area $\Delta\, ABC$: A (3; 6; 0), B (−1; 3; 1), C (2; −1; 1).
2 Find the median equation AE $\Delta\, ABC$, A (−2; 0), B (2; 4), C (4; 0).
3 Compose the canonical equation of an ellipse and construct it, if $в = 4$, $\varepsilon = 0.3$.
4 Calculate:

4.1 $$\lim_{x\to\infty}\frac{3x^2-5x+1}{7x^2-15},$$

[6] Krylova, T., & Gulesha, E. (2009). Using Computer Testing in teaching Higher Mathematics. *Didactics of Math: Problems and Research: International collection of scientific works. Iss. 32.* – Donetsk, Ukraine: Ed. office of DonNU. [In Rus.].

4.2 $\lim\limits_{x \to 2} \dfrac{x^2 + 3x - 10}{3x^2 - 5x - 2}$,

4.3 $\lim\limits_{x \to 0} \dfrac{x\,tg\,5x}{\sin^2 x}$,

4.4 $\lim\limits_{x \to \infty} \left(\dfrac{x+7}{x+5} \right)^{2x+2}$.

5 Calculate the derivative of functions:

5.1 $y = \ln(3x^2 - x + 1)$,

5.2 $x^3 + y^3 - 3xy = 5$,

5.3 $\begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t. \end{cases}$

The correct solution for each example brings 10 points.

For a slip of the pen 2 points are removed, 4 points – for a mistake, 6 points for a gross mistake. At the current evaluation of student learning outcomes students can be offered testing. One of the variants of tests for differential equations can be presented as follows:

Differential equation	Define the type of equation
1 $tg\, x \sin^2 y\, dx + \cos^2 x\, ctg\, y\, dy$	(12 points)
2 $y' = (x + y)/x$	(12 points)
3 $xy' + y - e^x = 0$	(16 points)
4 $y' + x/y = -xy^2$	(16 points)
5 $y'' = -1/x^2$	(14 points)

6 $x^2y'' + xy' = 1$ (14 points)

7 Solve $y'' + 7y' + 2y = x - 1$,
$y(0) = 0, \quad y'(0) = 0$ (16 points)

After completing the study of the course in higher and applied mathematics, comprehensive control paper is carried out. Here is one of its variants:

1 Scalar product (20 points)

2 Defined integral (20 points)

3 Calculate $\lim\limits_{x\to\infty}\dfrac{2x^5 - 3x^2 + 15}{1 - 4x + 7x^5}$ (10 points)

4 Find the derivative of function: $y = \ln\cos 2x$ (15 points)

5 Present as Maclaurin series $y = e^{2x}$ (17 points)

6 Solve the equation $y'' - 2y' - 3y = 0$ (18 points)

By timely diagnosing the knowledge and skills acquired by students, changing the forms of assessment of current students' progress in mathematics, but always remaining a demanding professor, conducting additional classes and consultations in order to eliminate the gaps and shortcomings in students' knowledge, it is possible to promote students' deeper acquisition of mathematical knowledge and their high academic performance.

Diagnosing the earning capability is closely linked to the diagnostics of knowledge scope.

Learning capability is the ability of pupils or students to master the content of learning. The concept of learning capability comprises the potential of pupils and students, the thesaurus, generalization of thinking, the pace of progress in learning.

When properly applied, pedagogical diagnostics, in mathematics in particular, will considerably facilitate the elimination of gaps in the acquired knowledge and shortcomings in the acquired skills and abilities of students on the basis of a thorough analysis of their work, guided by the professor and of independent work as well, thus leading to overall improvement in the mathematical training of students from Ukrainian higher educational institutions, and will also improve the formation of their basic mathematical skills, laying the foundations for their general mathematical competence.

2.3 On the Use of Algorithms in Teaching Probability Theory*

O. Chernobai

*The article is published in the author's translation

Introduction

The theory of probabilities is a branch of mathematics that studies the patterns of random phenomena: random events, random variables, their functions, properties and operations on them. Mathematical models in the theory of probabilities describe with certain accuracy level the tests, results of which are complexly determined by testing circumstances (see e.g. Prokhorov[1], 1999).

The study of the theory of probabilities in modern conditions becomes especially relevant. This is caused by the fact that growing number of specialties require application of mathematical knowledge, practical skills and expertise of a sufficiently high level. The development of Ukrainian national high school includes the improvement of both mathematical education in general and its separate sections. The main areas of improvement are updating the content and technology of teaching higher mathematics and mathematical disciplines. In this article there will be reviewed specialties of algorithmic approach usage within teaching of the theory of probabilities, probabilistic process and mathematical statistics.

Analysis of latest publications related to the topic

In high school, depending on the specialization, the course of probability theory is either studied as an independent discipline, or is included in the course of higher and applied mathematics. The purpose, main tasks

[1] Prokhorov, Y. (1999). *Veroyatnost i matematicheskaia statistika: Entsyklopediia [Probability and mathematical statistics: Encyclopedia]*. Moscow, Russia: Bolshaia rossiiskaia entsyklopediia. [In Rus.].

and the motivation of the course were considered earlier (see e.g. abstracts papers[2] [3]).

In our report[4] it is stated that the process of learning the branch of probability theory by students has a number of features. Firstly, the scope of knowledge to be studied related to probability sections is quite large and time prescribed for these sections of higher and applied mathematics is limited. Secondly, there are a number of difficulties faced by students while studying the theory of probabilities. They are related to such aspects as presence of abstractively-logical analysis, probabilistic (ambiguous) assertions in the discipline, which are required to transform the content of the task into the language of probabilistic models for its further solution. The main difficulty is that events are less clear than figures, numbers, or expressions, and the probability and possibility are not as intuitive as length, area or volume. An event and its consequences are special types of mental objects, the mathematical formation of which is much more difficult than the formation of a figure (in geometry) or quantity (in arithmetic or algebra).

In addition, every year there is a decrease in level of mathematical preparedness of candidates for admittance to higher educational institutions. Modern students often come to an institution being unable to think logically and perform analysis. The traditional difficulty of mathematical disciplines – analysis of the tasks' text and, as a result,

[2] Chernobai, O. (2015). Pro deiaki osoblyvosti vykladannia kursu «Vyshcha ta prykladna matematyka» [About some specialties of teaching study course «Higher and Applied Mathematics»]. *Proceedings from Mathematics in modern technical university ґ14: III Mizhnarodna naukovo-praktychna konferentsiia (25-26 hrudnia 2014 roku) – 3rd International Scientific and Practical Conference.* (pp. 211–213). Kyiv, Ukraine: NTUU «KPI». [In Ukr.].

[3] Chernobai, O. (2017). Motyvatsiia pry vykladanni kursu vyshcha ta prykladna matematyka [Motivation within teaching of study course «Higher and Applied Mathematics»]. *Proceedings from Mathematics in modern technical university ґ16: Piata Mizhnarodna naukovo-praktychna konferentsiia (29-30 hrudnia 2016 roku) – 5th International Scientific and Practical Conference.* (pp. 183–184). Kyiv, Ukraine: NTUU «KPI». [In Ukr.].

[4] Chernobai, O. (2019). Alhorytmizatsiia v protsesi navchannia teorii ymovirnostei [Algorithmization in the process of teaching the theory of probabilities]. *Proceedings from Mathematics in modern technical university ґ18: Soma Mizhnarodna naukovo-praktychna konferentsiia (28-29 hrudnia 2018 roku) – 7th International Scientific and Practical Conference.* (pp. 197–200). Kyiv, Ukraine: NTUU «KPI». [In Ukr.].

the ability to solve the tasks in text format - is crucial in this subject as all tasks are in text format. Text tasks on the theory of probability, combinatorics, statistics, and probabilistic processes are much more diverse than algebraic ones. Apart from classical tasks (throwing up the dice, coins, randomly pulling the colored balls) there is a large number of similar contexts.

When solving a new task, to understand that this is the main task usually turns out to be quite difficult for the student. Not enough prepared students do not see an analogy, even in the tasks of pulling colored pencils or multicolored balloons out of the box.

In this regard, the teacher faces a rather difficult task of adapting students to studying his subject. One way to overcome these difficulties is to apply algorithmic approach to solving probabilistic tasks. Algorithms can be provided to students in the form of tables, sequence of actions and schemes.

The aim of the study

To develop algorithmic components of the methodical system of teaching probability theory's certain topics.

Presentation of main material

In the abstracts of papers mentioned above, it is said that one of the first topics in the course "Probability theory, probabilistic processes and mathematical statistics" is the classical definition of the probability of an event. Usually, after reading the task, the student faces chaos in his mind, everything is mixed: events, results, probabilities. The following algorithms help to structure analysis and to build a logical chain of consideration:

1 Define which experiment occurs in task being analyzed.
2 How many probable outcomes does the experiment have (n). It is good at this stage when students formulate questions aloud.
3 Introduce event A, the probability of which should be found in the task.

4 Determine how many outcomes contribute to the event (m). It is also important to formulate questions aloud here.

5 Apply the formula of classical probability $P(A) = \frac{m}{n}$.

At each stage, it is important to offer the student to formulate the question which should be answered at this stage. In doing so, one should obtain from the student a clear understanding of what is a test (experiment), what is an event, and what is the probability of the event.

Example. The group consists of 25 students, 5 of them are participants in the skills competition on Higher mathematics. What is the probability that a randomly selected student will be a participant of skills competition.

To solve the task, let's apply the algorithm.

1 Experiment: a student from the group is chosen.
2 Probable outcomes of the experiment ($n = 25$).
3 Event A – «randomly selected student is a participant of skills competition on higher mathematics».
4 Outcomes contributing to the event A ($m = 5$).
5 According to the known formula of the classical probability definition, we obtain a numerical value $P(A) = \frac{m}{n} = \frac{5}{25} = \frac{1}{5}$.

The classical definition of probability cannot be applied to an experiment with an infinite number of equally possible consequences. In this case, the geometric determination of probability is used. Within analysis a number of features, such as the elementary event in the experiment can be limited to the choice of the point, elementary events are equally possible, the number of elementary events is infinite, and their number forms a finite dimensional area, direct the student to the conclusion on possibility of applying a geometric probability – the probability of point falling within an area (section, part of the plane, etc.). The solution of the task, in comparison with the previous ones, is connected with the necessity of interpreting the experiment as a point choice in a certain range.

The algorithm for solving tasks on the geometric probability can be formulated as follows:

1 Define which experiment occurs in task being analyzed. As the number of outcomes of experiment defined in the task is infinite, geometric approach should be applied to probability calculation. For this purpose, experiment should be defined as point choosing in certain range.
2 Define *G* range of all possible outcomes and find its measure (length, area or volume) – *mes G*.
3 Introduce event *A*, the probability of which should be found in the task. Define the area ***Q***, which is a subset of the set *G*, and is the set of results contributing to the event *A*. Find the measure of the ***Q*** set – *mes* ***Q***.
4 Find the probability of the event *A* by the formula:

$$P(A) = \frac{mes\,Q}{mes\,G}.$$

Solving tasks on geometric probability causes a lot of difficulties. This is due to the difficulty of interpreting the task in text format in a way of putting a point into a certain area. In this case, it is inappropriate for a teacher to immediately give students the idea of this interpretation, but instead with the help of a series of questions he should stimulate the appearance of right idea within the students by themselves.

Example. Two students agreed to meet in a certain place in the time interval from t_1 to t_2 hours, as well as that the one who comes first will wait for the second within t_3 hours. Find the probability that the meeting will occur if each person can arrive at any time $t_3 \in [t_1; t_2]$.

Let's apply the algorithm to solve the task.

1 Experiment: two persons are located in a certain place within a certain time frame.
2 *G* range of all possible outcomes – set of points of a square with a side $t_2 - t_1 = T$.
3 Event *A* – meeting will occur.

4 The area Q corresponds to the shaded part of the square, if the moment of arrival of each person – A will take place under conditions $|x-y| \le t_3$, where $0 \le x \le T, 0 \le y \le T$.

5 These conditions are represented in the XOY coordinate system. The set of all results corresponds to the area of the square $ONCK$, and A event – area of the hexagon $OEDCBA$.

Using the geometric definition of probability, we will get:

$$P(A) = \frac{S_{OEDCBA}}{S_{ONCK}} = \frac{T^2 - (T - t_3)^2}{T^2} = \frac{T^2 - T^2 + 2Tt_3 - t_3^2}{T^2} = \frac{t_3(2T - t_3)}{T^2}.$$

The mentioned task was considered in the worksheet on probability theory of M. Semko[5] (2011).

Within study of axioms and theorems on probability theory, it is advisable to present them in the form of the following tables (see e.g. Chernobai [6], 2018):

To solve tasks using the theorems of addition and multiplication of probabilities, we propose the following algorithm:

1 Formulate an event, the probability of which must be found in the task.
2 Formulate an event through which you can express the sought event by adding, multiplying, and subtracting events.
3 Find the probability of an event formulated in p. 2.
4 Express the sought event through the event formulated in p. 2, by adding, multiplying and subtracting events.
5 Pass on to the probability of the sought event and apply the addition and multiplication probability theorems.

[5] Semko, M., Zadorozhnia, T., Kuchmenko, S., Mamonova, H., Rudenko,I., & Chernobai, O. et.al. (2011). *Teoriia ymovirnostei ta matematychna statystyka (robochyi zoshyt dlia indyvidualnykh zaniat) [Probability theory and mathematical Statistics (workbook for individual tasks)].* Kyiv, Ukraine: Instytut matematyky NAN Ukrayiny. [in Ukr.].

[6] Chernobai, O. (2018). Osoblyvosti vykladannia teorii ymovirnostei u suchasnykh umovakh. [Specialities of probability theory teaching in modern conditions]. *Proceedings from Accounting and taxation: realities and perspectives r18: III Vseukrainska naukovo-praktychna internet-konferentsiia (18-20 kvitnia 2018 roku) – 3rd All-Ukrainian Scientific and Practical Internet-conference.* (pp. 665–666). Irpin, Ukraine: UDFSU. [in Ukr.].

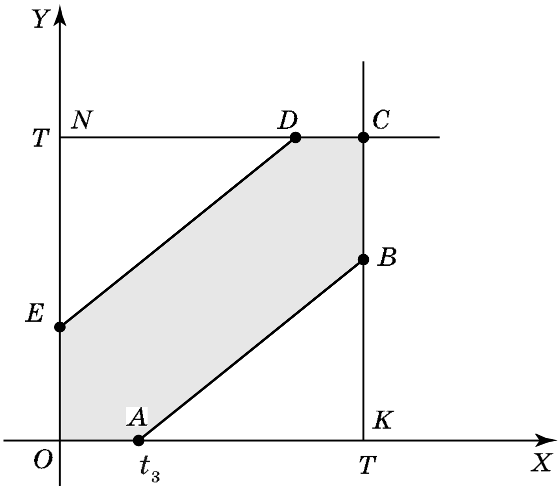

Figure 2.3.1 Geometrical presentation of task conditions.

Example. To complete the task, the manager addresses two independent performers. The probability that the first performer will execute the task is equal to 0.7 and the second – 0.8. Find probability that the task will be executed.

Let's solve the task using the above-mentioned algorithm:

1 Let's define event A – «the manager's task is completed».
2 Let event A_1 be «task is completed by first performer».
3 Event A_2 «task is completed by second performer».
4 We find the probability of the sought event, using the sum of the probabilities of compatible but independent events $P(A) = P(A_1) + P(A_2) - P(A_1A_2)$ (refer to Table 2.3.1).
5 Thus, the probability of event A is equal to:

$$P(A) = P(A_1) + P(A_2) - P(A_1A_2) = 0{,}7 + 0{,}8 - 0{,}56 = 1{,}5 - 0{,}56 = 0{,}94.$$

Example. Three students take the exam in higher mathematics. The probability to pass the exam with excellent

$P(A+B)$	
A and B are mutually exclusive	A and B are compatible
Probability that only one event will occur	Probability that at least one event will occur
$P(A+B) = P(A) + P(B)$	$P(A+B) = P(A) + P(B) - P(AB)$

Table 2.3.1 Probability of the sum of two events.

$P(A_1 + A_2 + \ldots A_n)$	
Mutually exclusive events	Compatible events
Probability that only one event will occur	Probability that at least one event will occur
$P(A_1 + A_2 + \ldots A_n) =$ $= P(A_1) + P(A_2) + \ldots P(A_n)$	$P(A_1 + A_2 + \ldots + A_n) =$ $= 1 - P(\overline{A_1} \cdot \overline{A_2} \cdot \ldots \overline{A_n})$ $A = A_1 + A_2 + \ldots + A_n$ $\overline{A} = \overline{A_1} \cdot \overline{A_2} \cdot \ldots \overline{A_n}$

Table 2.3.2 Probability of the sum of several events.

mark for the first student is 0.5, for the second one — 0.3, and for the third one — 0.2. Find the probability that three students will pass the exam with excellent mark.

Let's apply the algorithm for solving tasks based on the theorem of independent events' probability multiplication (refer to Table 2.3.3).

1 Let's formulate event A — «all three students will pass the exam with excellent mark».
2 Let event A_1 —« first student will pass the exam with excellent mark»,
event A_2 —« second student will pass the exam with excellent mark»,
event A_3 —« third student will pass the exam with excellent mark». Then sought event can be presented as product $A = A_1 \cdot A_2 \cdot A_3$.
3 The probability of event A can be found by the formula for multiplying the probabilities of independent events:

$$P(A) = P(A_1) \cdot P(A_2) \cdot P(A_3).$$

4 So the probability of event:

$$P(A) = P(A_1) \cdot P(A_2) \cdot P(A_3) = 0{,}5 \cdot 0{,}3 \cdot 0{,}2 = 0{,}03.$$

Some principles and examples of using the algorithms of the course have been earlier considered by the author. And now we suggest an algorithm for solving a task using formula of total probability and Bayes formula. The formula of total probability is a consequence of the addition and multiplication theorems. For these formulas, students can be offered the following algorithm:

$P(A_1 \cdot A_2 \cdot \ldots A_n)$	
Probability of occurrence of all events together	
Independent events	Dependent events
$P(A_1 \cdot A_2 \cdot \ldots A_n) =$ $= P(A_1) \cdot P(A_2) \cdot \ldots P(A_n)$	$P(A_1 \cdot A_2 \cdot \ldots \cdot A_n) =$ $P(A_1) \cdot P_{A_1}(A_2) \cdot P_{A_1 A_2}(A_3) \cdot$ $\cdot \ldots \cdot P_{A_1 A_2 \ldots A_{n-1}}(A_n)$

Table 2.3.3 Probability of the product of events.

1 Formulate an event A, the probability of which must be found in the task (or for Bayes formula – event, which occurred as a result of experiment).

2 Formulate hypothesis $H_1, H_2, \ldots H_n$.

3 Find probability of hypothesis $P(H_1), P(H_2), \ldots P(H_n)$.

4 Perform check using formula $P(H_1) + P(H_2) + \ldots P(H_n) = 1$.

5 Define total probability formula for defined task:

$$P(A) = \sum_{i=1}^{n} P(H_i) \cdot P_{H_{i_i}}(A).$$

6 Find conditional probabilities $P_{H_{i_i}}(A)$. At this stage it is preferable that students at least orally define the probability of which event and under which conditions they are searching for.

7 Substitute the values found in points 3 and 6 in the formula of total probability (point 5).

For the Beyies formula, one more point is added (point 8).

8 Calculate the probability of the sought-after hypothesis with the Bayes formula:

$$P(H_i) = \frac{P(H_i) P_{H_i}(A)}{P(A)}.$$

Example. The department of the SFS of Ukraine receives information from three independently operating centers. From the first center comes 50% of information, from the second — 30%, and from the third — 20%. The probability of error while processing information from the first center is equal to 0,1; from the second — 0,05, and from the third — 0,15. What is the probability of receiving information with an error? Information was received with error, calculate the probability that it was received from the third center.

Let's solve the task using the above-mentioned algorithm:

1 Formulate an event A – «information was received with error».
2 Formulate relevant hypothesis:

H_1 – information was received from the first center,

H_2 – information was received from the second center,

H_3 – information was received from the third center.

3 Find probability of hypothesis:

$$P(H_1) = 0{,}5,\ P(H_2) = 0{,}3,\ P(H_3) = 0{,}2.$$

4 Perform check using formula:

$$P(H_1) + P(H_2) + P(H_3) = 0{,}5 + 0{,}3 + 0{,}2 = 1.$$

5 Define total probability formula for defined task:

$$P(A) = \sum_{i=1}^{3} P(H_i) \cdot P_{H_{i_i}}(A).$$

Find conditional probabilities:

$$P_{H_1}(A) = 0{,}1;\ P_{H_2}(A) = 0{,}05;\ P_{H_3} = 0{,}15.$$

6 Substitute the values found in the formula of total probability. We have:

$$P(A) = \sum_{i=1}^{3} P(H_i) \cdot P_{H_{i_i}}(A) = 0{,}5 \cdot 0{,}1 + 0{,}3 \cdot 0{,}05 + 0{,}2 \cdot 0{,}15 = 0{,}095.$$

7 Calculate the probability of the sought-after hypothesis with the Bayes formula:

$$P(H_i) = \frac{P(H_i)P_{H_i}(A)}{P(A)} = \frac{0{,}2 \cdot 0{,}15}{0{,}095} = 0{,}316.$$

Consequently, the probability that information with error was received from the third center is **0.316.**

For tasks in which there is a series of tests under the Bernoulli scheme, the following algorithm can be set:

1 Formulate an event A, the probability of which must be found in the task.

2 Set Bernoulli scheme:

- define what should be treated as one test,
- define quantity of tests n,
- check if tests are independent,
- split solutions of one tests into two groups: «success» and «fail». «Success» = {solutions, which contribute to event A}, «fail» = {solutions opposite to «success»},
- find probability of «success» – p і «fail» – q. It should be checked that p and q do not change from test to test in this series of tests.

3 To express the probability of event A via the probability of m success in n tests performed under the Bernoulli scheme $P(A) = P_n(m)$.

4 Apply the Bernoulli formula to p. 3 $P_n(m) = C_n^m p^m q^{n-m}$ or in case quantity of tests is big, the approximate formulas: if n is big and p is very small **(np < 10)** – Poisson formula:

$$P_n(m) \approx \frac{\lambda^m e^{-\lambda}}{m!}, \quad \lambda = np,$$

if n is big and p is not very small **($np \geq 10$)** – Moivre–Laplace formula:

$$P(m) \approx \frac{1}{\sqrt{npq}} \varphi(x), \quad x = \frac{m - np}{\sqrt{npq}}.$$

Example. Out of 25 people, 8 are entitled to a tax benefit. What is the probability that two out of three randomly selected individuals will be entitled to tax benefits?

Let's solve the task using the corresponding algorithm.

1 Formulate an event A – «two out of three randomly selected individuals are entitled to tax benefits».

2 Set Bernoulli scheme:

- one test is to select one person out of 25,
- define quantity of tests $n = 3$,
- tests are independent,

- 8 cases contribute to «Success» of test, 17 correspond to «fail»,
- probability of «success» $p=\frac{8}{25}=0{,}32;$ and probability

  of «fail» is $q=1-p=1-0{,}32=0{,}68.$

3 Express the probability of event A as per Bernoulli

formula: $P(A)=P_3(2).$

4 Calculate the probability using the Bernoulli formula:

$$P_3(2)=C_3^2p^2q^1=\frac{3!}{2!(3-2)!}(0{,}32)^2(0{,}68)^1=0{,}208.$$

The task was considered in the article of T. Zadorozhnia[7] (2016).

Let's formulate a task which can be solved using the approximate formulas (see e.g. study guides of O. Bashchuk[8], 2019; I. Rudenko[9], 2017).

Example. The firm, which performs repair of apartments, puts the promo leaflets in mailboxes. Previous experience has shown that about in nine out of a thousand cases orders will be placed for repair of apartments. Find the probability that when 300 leaflets are placed, the number of orders will be equal to two.

[7] Zadorozhnia, T., Kharenko, S., Kuchmenko, S., Chernobai, O., Bashchuk, O., & Skaskiv, L. et al. (2016). Zadachi pro podatky [Tasks about taxation]. *Matematyka v ridnii shkoli. – Mathematics in home school, 10*. 16-21. [In Ukr.].

[8] Bashchuk, O., Kuchmenko, S., Skaskiv, L., & Chernobai, O. (2019). *Vyshcha ta prykladna matematyka: zbirnyk vprav ta zadach [Higher and applied mathematics: collection of exercises and tasks]*. Irpin, Ukraine: Universytet DFS Ukrainy. [In Ukr.].

[9] Rudenko, I., & Chernobai, O. (2017). *Vyshcha ta prykladna matematyka [Higher and applied mathematics]*. Irpin, Ukraine: Universytet DFS Ukrainy. [In Ukr.].

Let's apply the algorithm of Bernoulli scheme.

1 Formulate an event A – «when 300 leaflets are placed, the number of orders is equal to two».

2 Set Bernoulli scheme:

- one test is to place promo leaflets,
- define quantity of tests $n = 300$,
- tests are independent,
- 9 cases out of 1000 contribute to «Success» of test,
- probability of «success» $p = 0,009$; and probability of «fail» is $q = 1 - p = 1 - 0,009 = 0,991$.

3 Express the probability of event A as per Bernoulli formula $P(A) = P_{300}(2)$.

4 In our case $n = 300$ is quit big and $p = 0,009$; is very small and $\text{np} = 2,7 < 10$, therefore Poisson formula is applied:

$$P_n(m) \approx \frac{\lambda^m e^{-\lambda}}{m!}, \quad \lambda = np = 2,7,$$

$$P_{300}(2) \approx \frac{2,7 \cdot e^{-2,7}}{2!} \approx \frac{2,7 \cdot 0,067}{2} \approx 0,09045.$$

Let's review the task, being solved applying Moivre–Laplace theorem.

Example. According to statistics, 2% of residents of certain city who rent out apartments do not pay taxes. Find the probability that 300 residents do not pay taxes out of 5000 residents who rent out apartments.
Let's solve the task applying boundary theorems of Bernoulli scheme.

1 Formulate an event A – «300 residents do not pay taxes out of 5000 residents who rent out apartments».

2 Set Bernoulli scheme:

- one test is renting out apartments,
- define quantity of tests $n = 5000$,
- tests are independent,
- 2 cases out of 100 contribute to «Success» of test,
- probability of «success» $p = 0{,}02$; and probability of «fail» is $q = 1 - p = 1 - 0{,}02 = 0{,}98$.

3 Express the probability of event A as per Bernoulli formula $P(A) = P_{5000}(300)$.

In our case $n = 5000$ is quit big, and probability $p = 0{,}02$ and $np = 100 \geq 10$, therefore Moivre–Laplace formula is applied.

Firstly, we find:

$$x = \frac{m - np}{\sqrt{npq}} = \frac{300 - 10}{\sqrt{5000 \cdot 0{,}02 \cdot 0{,}98}} = \frac{290}{98} = 2{,}9591.$$

Corresponding probability:

$$P_{5000}(300) \approx \frac{1}{\sqrt{npq}} \varphi(x) = \frac{1}{98} \varphi(2{,}9591) = \frac{0{,}051}{98} = 0{,}0005.$$

Conclusions

Algorithmization of the task-solving process on probability theory helps students to clearly see a plan for solving the task, analyze the task conditions, teaches them to think analytically, logically and in a structured way. This approach contributes to a better acquisition of knowledge, a more clear and conscious application of the basic concepts and theorems of probability theory. Besides, the use of algorithms does not only enable to teach students to solve tasks, but also develops the ability to analyze tasks and solve them not only by example, to draw concrete conclusions and to summarize the results.

Considered algorithms with the use of classical probability definition, theorems of addition and multiplication of probabilities, Bernoulli scheme can be also applied by mathematics teachers of general education institutions while teaching the theory of probabilities.

CHAPTER 3 MATH TEACHER TRAINING IN GRADUATE AND POSTGRADUATE EDUCATION

3.1 The Enhancement of a Mathematics Teacher Training by Use of Methodological Projects*

I. Malova

*The article is published in the author's translation

Introduction

A modern teacher wants to know not only what he should do to make his methods effective, but why this or that solution of a methodological task gives the desired effect.

The foundation of teacher`s methods stays on the principles of learning as they reflect its regularities. Besides, the causes of negative phenomena existing or emerging in the pedagogic activity should be sought, in particular, in the learning principles realization as it is the learning principles that regulate its organization putting it into correlation with the students' mastering of an appropriate subject. The causes may bear a theoretical character (for instance, it is necessary to amend the approaches to the educational principles) as well as the practical character (for instance? The principles are applied incorrectly in concrete pedagogical processes).

There is no universal approach to the ways of singling out the learning principles, their content and their quantity in methodological literature.

We suggest the criterion offered by M. Skatkin as for what normative regulation can be treated as a learning principle. In accordance with the criterion there are three main learning principles that are postulated in Russia's pedagogic literature, namely, the principles of scientific approach, humanistic vector and reality-oriented education answering life challenges. The chain "learning principles – methodological foundations – a methodological situational solution" requires from the teacher some time for its mastering and individual approach on behalf of the student.

One of the ways of the chain mastering presumes the teacher trainees involvement into methodological projects aiming at methodological tasks situational solutions. Such project may be observed in the organization of students' work at a mathematical task that presents a difficulty. The text focuses on the requirements to the methodological projects, the requirements' foundations, the ways of students' methodological projecting activity organization.

Below we presented *the results of literature analysis* on the problem of learning principles discussion.

A. Gayfutdinov[1] (2011) presented the results of analysis of textbooks in teaching methods for a number of disciplines (physics, mathematics, biology, geography). The researcher made a number of conclusions:

1 There is no need for singling out a separate block of methodological principles, only general didactic principles should be taken into account, and on this basis the teaching methods for a subject should be worked out;
2 Works on methods of teaching biology and mathematics, published in early 2000s, are known for outlining a large quantity of principles, the formulation and content of which evoke a number of questions: how many didactic principles should be taken into account, what content is put into this or that principle, how are didactic principles realized in the methods of teaching this or that subject.

M. Dammer[2] (2012) studying the principle of scientificity in the formation of the content of study showed the existence of hierarchy in the principles of learning, the necessity of the principles quantity limitation (their quantity shouldn`t be big), the importance of functionality widening of the already existing principles instead of the increase in their quantity.

[1] Gajfutdinov, A. (2011). Metodicheskie principy v teorii obucheniya [Methodological principles in the theory of learning]. *Kazanskij pedagogicheskij zhurnal – Kazan pedagogical journal*, *1 (85)*, 165-171. [In Rus.].

[2] Dammer, M. (2012). Rol' principa nauchnosti v formirovanii soderzhaniya obucheniya [The role of the scientific principle in the formation of learning content]. *Vestnik yuzhno-ural'skogo gosudarstvennogo universiteta. Seriya: obrazovanie. Pedagogicheskie nauki – Bulletin of South Ural state University*, *4 (263)*. 30-33. [In Rus.].

N. Semyonova[3] (2012), modelling the system of learning principles in the conditions of informational and communicative technologies development, singled out four principles, called basic ones: thorough assimilation of the basic information; learning individualization; temporal effectiveness of learning; the stability of the learning process control. The researcher points out the existence of three developed ideologies of the overall learning principles system creation in the pedagogical research:

1 The ideology of interconditionality, complementarity and interconnection of didactic classical and new learning principles, in the framework of which the classical principles are specified and enriched, new options for their realization emerge, and new principles may enter the system of classical principles;
2 The ideology of opposition that lies in striving at cancellation of existing principles formed within a certain educational paradigm, and their replacement with new ones;
3 The ideology of independence, regarding the possibility of independent existence of different complexes of principles, the relations between which never being established.

The site http://www.yourarticlelibrary.com/teaching/ has published a number of articles devoted to learning principles. One of the papers[4] (Mondal, 2017) discusses the importance of principles in learning, stressing that learning principle is understood as a fundamental truth that makes teaching and learning conscious and productive. Paper[5] (Mondal, 2018) points out the leading role of principles in the choice

[3] Semenova, I. (2012). Modelirovanie sistemy principov obucheniya v usloviyah razvitiya informacionno-kommunikacionnyh tekhnologij [Modeling of the system of principles of education in the development of information and communication technologies]. *Pedagogicheskoe obrazovanie v Rossii – Pedagogical education in Russia*, 5, 106-110. [In Rus.].

[4] Mondal, P. The Importance of Principles in Teaching. Retrieved from http://www.yourarticlelibrary.com/teaching/the-importance-of-principles-in-teaching/5994.

[5] Mondal, P. Important Techniques Used in Principles of Teaching. Retrieved from http://www.yourarticlelibrary.com/teaching/important-techniques-used-in-principles-of-teaching/6031.

of methods, technologies and ways of acting in various situations. Paper[6] (Mondal, 2019) on the basis of analysis of the learning process various concepts presents general learning principles: Learning is considered as the acquisition of knowledge, habits, skills, abilities, and attitudes through the interaction of the whole individual and his total environment; Learning is meaningful if it is organized in such a way as to emphasize and call for understanding, insight, initiative, and cooperation; Learning is facilitated by motives or drives; Needs, interests, and goals are fundamental to the learning process; Learning is facilitated by the law of readiness or mindset; Learning does not occur unless the learner is ready to act or to learn, etc. Paper[7] (Mondal, 2019) gives three groups of principles: Starting Principles (those connected with a child's growth and development); Guiding Principles (those correlating with learning procedure and used methods); Ending Principles (those facilitating the persevering of the learning goals as results).

Thus, in Russia and abroad authors note, firstly, the importance of learning principles in theory as well as in teaching practice, secondly, the fact of their selection, foundation and ways of presentation problem.

Let us give our opinion on the notion of learning principle, on the normative statements that may refer to the concept.

In the formulations, giving the core of the notion of learning principle, the defining concept appears to be: main regulatory statement, the system of baseline didactic requirements to the process of learning, basic didactic statement; leading idea, normative requirement.

Most authors in appropriate formulations point out the *function* of this statement (idea, requirement): to enhance the necessary efficiency of the learning process; to accomplish the goals in the best possible way; to determine the course on personal development; to organize and control the didactic process.

[6] Mondal, P. (2019). 16 Most Important Principles of Learning. Retrieved from http://www.yourarticlelibrary.com/learning/16-most-important-principles-of-learning/6056.

[7] Mondal, P. (2019). Principles of Teaching: Starting, Guiding and Ending. Retrieved from http://www.yourarticlelibrary.com/teaching/principles-of-teaching-starting-guiding-and-ending/6038.

It means that the formulation of the learning principle must embrace the defining notion as well as the notion's function in learning theory and practice. For example, the formulation '*A learning principle is a basic normative statement enhancing the learning process necessary effectiveness*' meets the two mentioned requirements.

A question may arise: «How can you tell whether a certain regulatory requirement is or isn't a learning principle»? To answer this question school of M. Skatkin[8] (1982) suggested a criterion with two essential features: «Such regulation must spread its channeling and guiding influence onto the most important learning elements – its content, methods, organizational forms – and not be reduced to any other regulations, not be ousted by them» (same source, p. 51). The didactic means should be added to the learning elements, and organizational forms should be excluded from their number as in the modern notion of learning method there is such a characteristics as the self-reliance of students which, in its turn, presumes the use of organizational forms of learning. A criterion named after M. Skatkin can be formulated.

Skatkin's criterion: *normative regulation is to embrace by its channeling regulating influence the content, methods, didactic means of learning and shouldn't be reduced to any other regulations or be replaced with them.*

Unfortunately, productive for practical pedagogy M. Skatkin school's idea of the way of checking whether a normative regulation is or isn't a learning principle, remained unnoticed. Solving a topical task of revealed in pedagogy learning principles' systematization, school of Yu. Babansky (1983) suggested correlating the principles not with the group of main elements (constituents) of a pedagogical process, but with each element in particular. So, the scientificity principle correlates most often only with the learning subject content, not with learning means and methods, and consequentially students don't pay due attention to the necessity of using scientific learning methods and means.

[8] *Didaktika srednej shkoly: Nekotorye problemy sovremennoj didaktiki [Didactics of secondary school: Some problems of modern didactics]* (2d ed.) M. N. Skatkin (Ed.) (1982). – Moscow, Russia: Prosveshchenie. [In Rus.].

Taking Skatkin's criterion into consideration, we are able to give the following scientificity principle formulation: «*The scientificity learning principle demands facilitation of students' successes in mastering a learning subject, for which purpose it is necessary to provide motivation, learning material understanding and the feeling of pleasure the learning process could evoke in students*»[9] (Malova, 2007). The formulation should point out the destination of the principle (to enhance the academic success) and the signs of its realization (the motivation, understanding, and the feeling of pleasure the students get from the learning process).

The teacher realizes the scientificity principle if (s)he is guided with systemic activity approach to teaching, as the approach is "responsible" for the students' successful learning activity.

The second principle that regulates the pedagogical process organization in mastering a school subject is the *one correlating learning and life.*

Having correlated the learning principles not with a group of the pedagogical process main constituents, but with each separately taken constituent, the pedagogicians correlated the principle of life-tied learning only with the learning process content, and not with methods or means; they also extrapolated the principle's influence on extra curriculum activity through students' involvement into socially useful work.

Let's examine the essence of link between learning and life with regard to the notion of "learning content" exposed by I. Lerner[10] (1982). Learning content was associated by the scholar with four constituents of the social experience, reckoning under learning content:

1 The system of knowledge about nature, society, thinking, technical devices, ways of acting;

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[9] Malova, I. (2007). Nepreryvnaya metodicheskaya podgotovka uchitelya matematiki [Continuous methodical training of the teacher of mathematics]: *Doctor's thesis*. Yaroslavl, Russia. [In Rus.].

[10] *Didaktika srednej shkoly: Nekotorye problemy sovremennoj didaktiki [Didactics of secondary school: Some problems of modern didactics]* (2d ed.) M. N. Skatkin (Ed.) (1982). – Moscow, Russia: Prosveshchenie. [In Rus.].

2 The system of general intellectual and practical skills and abilities;
3 The experience of creative activity;
4 The experience of emotional and evaluative attitude to the environment and to each other.

The significance of such a social experience for students is revealed in the following words: «The assimilation of the first element of the social experience enhances the formation of the worldview and equips with methodological approaches to the cognitive and practical activities. In other words, knowledge serves the tool of any activity. The assimilation of the second element allows new generations to realize the culture reproduction and its conservation. The assimilation of the third element enables further development of culture which is impossible without creative activity. The assimilation of the third element provides the correspondence of a person's activity and needs, values system, motives, that is, all the manifestations of emotional attitude to activity and its products, to people (same source, P.102). Thus, there is a bilateral relation between the social experience, reflected in the learning content, and students' subjective experience: social experience can and must enrich the students' subjective experience, whereas the experience obtained in the process of the content's assimilation may become significant for everybody.

Now the following formulation can be given: «*The principle of pragmatic orientation (life-tied learning) requires the reliance in learning on the bilateral ties of the social experience reflected in the learning subject content and the subjective experience gained by students in the process of its acquisition.* We reckon this principle to be realized in definite pedagogical processes, if, firstly, the social experience constituents, laid down into a learning subject's concrete content and/or the mastering process of it, are brought up for discussion with pupils who are given the status of the pedagogical process subjects (it is this very task that is the core of educational humanization), secondly, the subjective experience is taken into consideration referring to a subject metadisciplinary links (it is the task that makes the essence of competence-based approach), and thirdly, every learning situation is regarded as a source of students' subjective experience enrichment.

Thus, a teacher realizes the principle of learning tied to life if (s)he actualizes the humanization of education, ensuring the realization of competence-centered approach since the latter facilitates the success of the students' subjective experience integrated use.

The third principle which regulates the pedagogical process organization in relation to mastering of a certain learning discipline by students is the principle of humanistic vector.

It may be formulated as follows: «*The principle of humanistic vector in learning requires teachers' and students' goals coordination*». The formulation matches Skatkin's criterion, for the goals determine the content as well as methods and didactic means; this regulation shouldn't be reduced to the principle of scientificity which doesn't take into account concrete students' demands.

A teacher realizes the principle of humanistic vector if (s)he provides personality-oriented teaching, as the latter facilitates students' success in the role of learning subjects and their development.

Thus, three principles of learning are singled out and their relevance to three educational approaches is found out. The question arises: «What is to be done with other regulations presented in pedagogical research literature as principles of learning»? The answer to this question may be the hierarchy of regulations where the top level is formed with the above mentioned principles and every lower level is subordinated to them.

The goal of the research: basing on the three learning principles and the corresponding approaches to learning, to find out the requirements to students methodological projects in solving methodological tasks relevant for the theory and practice of education.

A Power Point presentation is chosen as a means of realization for the methodological project as such presentation enables the teacher's methodological activity and students' learning activity. Research work[11] (Rizvanov, Abaturova, Malova, & Shakirova, 2018) highlights the role of the collective subjective experience method facilitating the success of methodological project realization.

The systemic activity approach implies the existence of:

1 The goal as a planned result;
2 Orienting foundations that enable the goal achievement;
3 The activity subject with the corresponding object.

Personality-oriented teaching presupposes:

1 Students' subjective experience enrichment (goal);
2 The use of dialogue technologies in the organization of their cognitive and reflexive activity (orienting foundations);
3 The enhancement of students' position as learning subjects and personal development (students' activity subject).

The competence building approach and education humanization presume:

1 The determination of general methods of problem situations solution (goal);
2 The analysis of concrete problem situations (learning and methodological) with the methodological tasks setting perspective (orienting foundations);
3 The solution variants of arising learning and methodological tasks (students' activity subject).

Thus, a set of requirements can be formulated to the methodological project goals, to the process of these goals accomplishment and to the students' activity subject in this process.

The requirements to the project's goal answer the questions:

1 What is the structure of the Power Point presentation;
2 How do the slides present the teacher's and the pupils' activity;
3 Who can use the prepared project. Let us formulate the ***requirements to the goal of a methodological project***:

[11] Rizvanov, Z. Z., Abaturova, V. S., Malova, I. E., Shakirova, L. R. (2018). Methodical Projects as A Method for Learning Future Teachers. *Amazonia Investiga. 7 (15).* 172-177.

1 The slides of a Power Point presentation should meet the basic methods requirements; for example, if the project refers to a mathematics task, it should include slides of: theoretical knowledge necessary for the solution of the task condition actualization; the task condition analysis; of the task solution variants search; the task solution arrangement; the summing up of the general work over the task solution;
2 All the questions of the teacher's dialogue should be presented in the slides, and the students' expected answers can be direct (text) or indirect (graphical means), alongside with it, all the methodological tasks should be solved in the presentation;
3 A Power Point presentation should be worked out as a ready and complete methodological product, which can be used by any teacher or student having access to Microsoft PowerPoint program.

The abovementioned requirements are pointed out in students' tasks of a methodological project creation.

The working process over the project presupposes the formulation and solution of a large number of methodological tasks. The requirements answer the questions:

1 If there are any already known ways of methodological tasks;
2 How new methodological tasks are solved;
3 Why it is necessary to solve new methodological tasks in a definite way. Let us formulate ***the requirements to the project creation process:***

1 To use the activity orienting foundations presented in basic methods of mathematics teaching, and also methods worked out in other methodological projects;
2 A methodologist who guides the project should point out the directions in new methodological tasks solutions, considering students' potential for their independent realization, or organizing the mutual search for new methodological decisions;
3 A methodological decision should be well-grounded.

Students' activity subject in their work over the methodological project can change depending on a stage of the work; the requirements answer the questions what should be given preference:

1 In the project's initial variant creation;
2 At the stage of the project's correction;
3 On finishing the work over the project. Let us formulate ***the requirements to subject of the project creation activity***:

1 To single out the ways of students' self-reliant success enhancement;
2 To highlight the causes of methodological difficulties and the ways of coping with them;
3 To summarize methodological solutions.

The ways of students' self-reliant success enhancement are recommended to presentation in the slides that sum up the results of the work with mathematical material; the ways of coping with methodological difficulties should be highlighted in methodological comments to slides; the generalization of methodological solutions – in students' publications.

The research results

On the basis of the requirements that were outlined above, 200 projects on teaching mathematics to pupils in secondary comprehensive school, secondary professional colleges, higher school (on teaching informational technologies) were worked out.

Methodological projects are connected with the following learning problem:

- the overcoming of mathematical difficulties in the students' work over a mathematical task (from the school course of informational technologies);
- the organization of students' work over a mathematical theorem;
- meaningful reading of mathematical texts organization.

Let's present some methodological tasks connected with students' learning methods concerning tasks that required new methodological solutions.

Methodological task 1. To work out a format of the task condition short record that should facilitate the solution search.

If the task is textual, the success of independent search for its solution depends on situations, values and relations among them that are highlighted as well as on the way they are represented in the short record.

Example 1. *A Family consists of husband, wife and their daughter who is a student. If the husband's salary were doubled, the family's total income would grow by 67%. If the daughter's scholarship were three times smaller, the total income would shrink by 4%. What percentage of the family's total income makes the wife's salary?*

The short record (Figure 3.1.1) enables the search of the solution variant via the questions: "What is the initial point to start the solution of the percentage task?" (it is necessary to find out which of the values is taken for 100%); "What can be found from the task's data?" (what percentage is made by the husband's income; what percentage is made by 2/3 of the daughter's income, and, consequentially, her income); "Will these data help answering the question the task poses?". It is recommended to reflect the answers to the questions in the short record of the task condition marking them with another color to tell new data from those given in the task.

If a task is geometrical, the success of the independent search for its solution depends on the way the figures are drawn (basing on the definition or given characteristics), on the consideration of all the data and their correct reflection in the drawing (it is convenient to mark equal values with auxiliary variables).

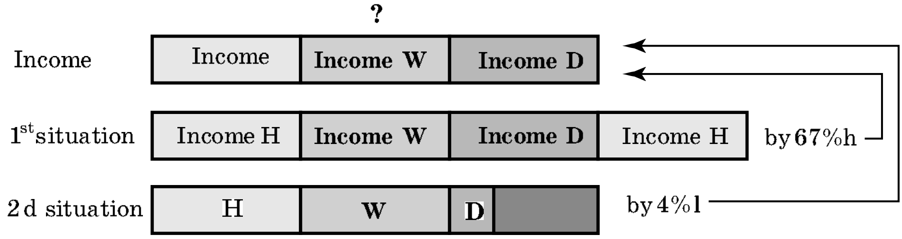

Figure 3.1.1 Short Record of the Task Condition as for the Family Income.

Methodological task 2. To motivate the search direction, the method or solution choice, additional construction, etc.

The success of an independent solution depends on the students' understanding of what can serve the grounds for finding a way of solution.

On presenting the first methodical task the motivating questions for the arithmetic method of a textual task solution were indicated. The generalized approach to reasoning from the task condition is actualized through the question "Is it possible to calculate anything basing on the task's data?" In case of a negative answer to this question the algebraic method (if it is possible to indicate the condition that comprises equation) or the method of auxiliary value introduction can be used. Particularly, in textual tasks the chosen value is taken for 1.

Example 2. *Some amount of 15% solution of a substance was mixed with the same amount of 19% solution of the substance. What is the percentage of the solution's concentration?*

The task condition's short record which indicates values can be convenient: the substance proportion, the whole solution mass, the substance mass in the solution (Figure 3.1.2). The motive for the solution mass designation as 1 is the question: "How do we act if the solution mass isn't given a name?" (in case of difficulties on behalf of the students, an additional question may be asked: "How do we act in the tasks on work if the work volume is not named?").

After introducing 1 into the short record the motive of using the arithmetic solution becomes evident: "Is it possible to calculate anything using the task's data?"

In solving planimetric tasks the questions like "What figures can be found in the drawing?"; "What do you know about them?"; "What data can be received from these

	$\alpha_{sb\text{-}nce}$	$M_{sl\text{-}n}$	$M_{sb\text{-}nce}$
I solution (15%)	0,15		
II solution (19%)	0,19		
I + II (?%)			

$M_{I+II} = M + M_{II}$ $\quad m_{I+II} = m_I + m_{II}$

Figure 3.1.2 The Short Record of the Task on Solutions.

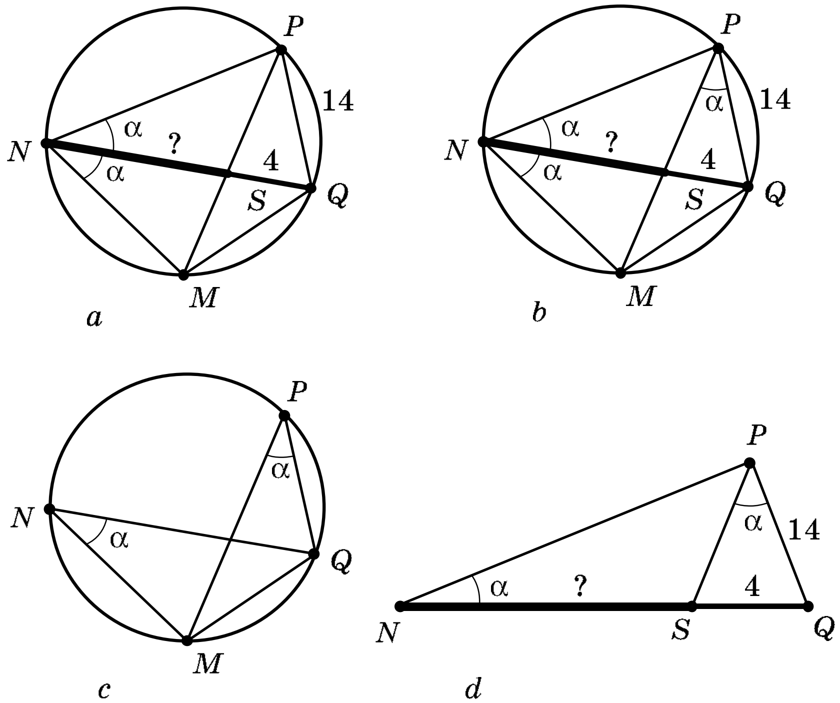

Figure 3.1.3 Drawings to the Task on the Inscribed Quadrilateral
Methodological Task 3.

figures?"; "Are the data sufficient to answer the question of the task?" can be asked. Alongside with the questions, separate drawings of the highlighted figures can be made.

Example 3. *In a convex quadrilateral NPQM the diagonal NQ is the bisector of the corner PNM and intersects with diagonal PM at point S. Find NS, if it is known, that round the quadrilateral NPQM a circle can be drawn, PQ=14, SQ=4.*

For students to obtain the experience of the answer to the question "What figures were received in the drawing?" (Figure 3.1.3 *a*), a slide in Power Point presentation shows the chord *MQ* and, after a pause, two angles inscribed on its basis (Figure 3.1.3 *c*); the linear segment *NQ* is built and after a pause two triangles ΔNPQ и ΔSPQ are built on its basis (Figure 3.1.3 *d*). The answer about equal angles to the question "What data can be obtained from the figures?" is designated with letter α in a separate drawing (Figure 3.1.3 *c*), as well as in the main one (Figure 3.1.3 *b*).

These steps motivate the use of triangle similarities:

To use animation effectively.

Animation has several functions:

- serves the illustration of a method of creating images (for this purpose the process of image creation should follow the one performed by students on paper);
- shows at what stage a student should perform this or that record in a copy-book (you should take into consideration the moment of animation's start);
- gives a chance to students to think over the answer (for the purpose the animation presupposes pauses.

Example 4. The base *AC of the isosceles triangle ABC is equal to 12. The radius circumference 8 with the center outside this triangle reaches the continuation of the sides of the triangle and touches the base AC in the middle. Find the radius of the circumference inscribed into the triangle ABC.*

In creating the drawing it is important to discuss: "What figures take part in the task?" (isosceles triangle and circumference); "From what figure is the drawing made?" (circumference); "How does the circumference interact with the triangle, what is the order of the triangle building?" Animation effects enable the illustration of the succession in the drawing: circumference; the touch point (the middle of the base); the straight perpendicular to the circumference radius drawn to the touch point (straight on which the base of the isosceles triangle lies); the height of the isosceles triangle; two tangents drawn from the top of the isosceles triangle; base tops.

Conclusion

The multi-year experimental work shows, that methodological projects enable the perfection of teaching methods competence of a teacher trainee, as they: motivate an independent creation of a methodological product; turn out to be effective mathematical help to students as well as methodological help to a student (teacher) in working out an up-to-date methodological solution; enrich the methods of teaching mathematics with new developments.

3.2 Project-Based Learning and Teaching Mathematics: Theoretical Framework **and Teachers' Beliefs***

I. Akulenko, O. Zydkov, A. Yakovenko

*The article is published in the author's translation

Introduction

Project-based learning is not new for Ukrainian schoolchildren and teachers, but it has not been used in the domestic education for many years. However, nowadays, educators and teachers focus on this method due to its powerful potential in the context of developing such important personality traits as independence, criticality, the ability to work in the team, to determine the goals of the activity and the ways to achieve them. According to the authors of the international study[1], it is these personal qualities that are the key ones to the successful life of a person in the 21st century. Pre-service Mathematics teachers should master design and project activities relating the organization of project-based learning and teaching while their methodical training.

Background

Numerous scientific studies in domestic and foreign pedagogy [2] [3] [4] [5] [6] focus on the organization, content and

[1] *21st Century Competencies: Foundation Document for Discussion. Phase 1: Towards Defining 21st Century Competencies for Ontario.* (2016). Edition Winter. Retrieved from http://www.edugains.ca/resources21CL/About21stCentury/21CL_21stCenturyCompetencies.pdf

[2] *Project Based Learning. Edutopia*. Retrieved from http://www.edutopia.org/project-based-learning

[3] *Project Based Learning for the 21st Century. Buck Institute for Education.* Retrieved from http://www.bie.org.

[4] Maida, C. A. (2011). Project-Based Learning: a critical pedagogy for the twenty-first century Policy Futures in Education. *Policy Futures in Education*, 9(6). Retrieved from https://www.researchgate.net/publication/275685844_Project-Based_Learning_A_Critical_Pedagogy_for_the_Twenty-First_Century. doi: 10.2304/pfie.2011.9.6.759.

[5] Markham, T. (2003). *Project-based Learning Handbook: a guide to standards-focused project-based learning for middle and high school teachers*, *2nd edn*. Novato, CA: Buck Institute for Education.

[6] Meltzoff, A., Kuhl, P., Movellan, J., & Sejnowsky, T. (2009). Foundations for a New Science of Learning. *Science, 17 July*, 284-288.

methods of implementing project-based learning. Genesis, historiography, the methodological and philosophical foundations of project method can be seen in the studies of J. Dewey[7] [8] (1900; 1916), V. Kilpatrick[9] (1925), A. Makarenko[10] (1983). J. Thomas[11] (2000) presented a literature review of project-based learning approaches in K-12 settings as well as of research on project-based learning's implementation and effectiveness. This review summarized a number of studies suggesting a positive relationship between project-based learning approaches and the quantity and quality of student learning. His review of the implementation literature also identified some common challenges that teachers face when implementing project-based learning. In the review of B. Condliffe, J. Quint, M. Visher, M. Bangser, S. Drohojowska, L. Saco, & E. Nelson[12] (2017) was described how project-based learning has been defined in the research literature and enacted in K-12 settings. This research paper assesses the project-based learning implementation and effectiveness, and recommends priorities for advancing the project-based learning further.

The studies of N. Alekseev[13] (1997), V. Guzeiev[14] (2000),

[7] Dewey, J. (1900). *The School and Society*. Retrieved from https://archive.org/details/schoolsociety00dewerich/page/164

[8] Dewey, J. (1916). *Democracy and Education*. Retrieved from https://en.wikisource.org/wiki/Democracy_and_Education

[9] Kilpatrick, W. (1925). *Foundations of Method: informal talks on teaching*. Retrieved from https://babel.hathitrust.org/cgi/pt?id=mdp.39015057278676;view=1up;seq=7

[10] Makarenko, A. (1983). *Oput metodiku rabotu detskoy trudovoy colonii* [Experience of working methods of a children's labor colony]. Moscow, Russia : Pedogogika. [In Rus.]

[11] Thomas, J. (2000). *A review of research on project-based learning*. San Rafael, CA: The Autodesk Foundation.

[12] Condliffe, B., Quint, J., Visher, G., Bangser, M., Drohojowska, S., Saco, L., & Nelson, E. (2017). *Project-Based Learning. A Literature Review*. New York, NY: MDRC.

[13] Alekseev, N. (1997). *Pedogagicheskie osnovu proektirovanie lichnostno orientirovanogo obucheniya* [Pedagogical foundations of the design of student-centered learning]. (*Doctor's thesis*). Tyumen State University, Tyumen, Russia. [In Rus.]

[14] Guzeev, V. (2000). *Planirovanie resul`tatov obrazovaniya i obrazovatel`naya tehnologiya* [Educational Results Planning and Educational Technology]. Moscow, Russia : Narodnoe obrazovanie. [In Rus.]

Yu. Gromyko[15] (1996), O. Zair-Bek[16] (1995), I. Malkova[17] (2008), E. Polat[18] [19] (2004), N. Pachomova[20] (2000), et al. consider the problems of project-based learning at the modern development stage of secondary school. A number of studies focuses on the problems of organizing project activity of schoolchildren in teaching Physics (Polikhun[21], 2007), Technology (Matyash[22], 2000), Chemistry (Momot[23], 2007), Biology (Talhina[24], 2009) etc. Some researchers study pedagogical potential of project-based learning in forming and developing: cognitive interests (Hrebennikova[25], 2005), creative potential (Polikhun[26], 2007), subject position in

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[15] Gromyko, Yu. (1996). *Proektirovanie i programirovanie razvitiya obrazovaniya* [*Design and programming of educational development*]. Moscow, Russia: Moskowskaya akademiya razvitiya obrazovaniya. [In Rus.]

[16] Zair-Bek, E. (1995). *Osnovu pedogagicheskogo proektirovaniya* [*Basics of pedagogical design*]. St. Petersburg, Russia. [In Rus.]

[17] Malkova, I. (2008). Konczepcziya i praktika organiziczii obrazovatel`nogo proektirovaniya v inovaczionoy shkole [The concept and practice of organizing educational design in an innovative school]. (*PhD thesis*). Tomsk State University, Tomsk, Russia. [In Rus.]

[18] Polat, E. (2009, 13 September). *Metod proektov [Method of projects]*. Retrieved from http://distant.ioso.ru/project/meth%20project/metod. [In Rus.]

[19] Polat, E. (2004). Chto takoe proekt? [What is a project?]. *Vidkrutuy urok*, *5-6*, 15. [In Ukr.]

[20] Pakhomova, N. (2000). Uchebnuy proekt: Motodologiya poiska [Training Project: Search Methodology]. *Uchitel`*, *1*. 41-45. [In Rus.]

[21] Polikhun, N. (2007). Rozvutok tvorchoi diyal`nosti starshoklasnukiv u proczesi navchanya fizuku z vukorustanuam proektnoi tehnologii [Development of creative activity of high school students in the process of teaching physics using design technology]. (*PhD thesis*). Kyiv, Ukraine [In Ukr.]

[22] Matyash, N. (2000). Proektnuy metod obucheniya v sisteme tehnologicheskogo obrazovaniya [The project method of teaching in the system of technological education]. *Pedogogika, 4,* 38–43. [In Rus.]

[23] Momot, Y. (2010). Proektna tehnologiya organizaczii pozaurochnoi robotu z himii uchniv zagal`noosvitnih navchal`nuh zakladiv [Design technologies for the organization of extracurricular work on the chemistry of students of secondary educational institutions]. (*PhD thesis*). Kyiv, Ukraine. [In Ukr.]

[24] Taglina, A. (2009). *Metod proektiv na urokah biologii* [*The method of projects at the lessons of biology*]. Kharkov, Ukraine: Ranok. [In Ukr.]

[25] Grebennikova, O. (2005). Proektnaya deyatel`nost` kak sredstvo razvitiya poznavatel`nuh interesov starsheklasnikov [Project activities as a means of developing the cognitive interests of high school students]. (PhD thesis). Velikiy Novgorod, Russia. [In Rus.]

[26] Polikhun, N. (2007). Rozvutok tvorchoi diyal`nosti starshoklasnukiv u proczesi navchanya fizuku z vukorustanyam proektnoi tehnologii (*PhD thesis*). Kyiv, Ukraine. [In Ukr.]

the education (Malkova[27], 2008). Following from John Dewey's understanding of the primacy of lived experience and reflection as the basis of learning and effective self-direction, the C. Maida[28] (2011) considers the practice of project-based learning – in the school, and outside, in the workplace and the community – as a critical pedagogy holding the potential for personal development, creativity, and social transformation.

Theoretical framework

Theoretical framework of the paper considers the analysis of scientists' views concerning the concepts of "design", "design in education", "pedagogical design", "project-based learning", "project activity" in the context of determining theoretical bases of training a pre-service Mathematics teacher to the organization of schoolchildren's project activity and project-based learning and teaching.

In philosophy (Sydorenko[29], 1984), project nature is considered as creative transformation ("re-creation") of existing reality on the basis of one's own original plan. The basis of our research is the position argued by I. Malkova[30] (2008) that design in education is a way of participation and influence of a person on his/her education that is characterized as a form of organizing educational practice and its quality, provided by form variety of a person's participation in educational practice, involvement in solving problems of his/her education, self-reflection of peculiarities of project practice organization and a person's subject position in

[27] Malkova, I. (2008). Konczepcziya i practika organizaczii obrazoval`nogo proektirovaniya v inovaczionoy shkole [The concept and practice of organizing educational design in an innovative school]. (*PhD thesis*). Tomsk State University, Tomsk, Russia. [In Rus.]

[28] Maida, C. A. (2011). Project-Based Learning: a critical pedagogy for the twenty-first century Policy Futures in Education. *Policy Futures in Education, 9(6).* Retrieved from https://www.researchgate.net/publication/275685844_Project-Based_Learning_A_Critical_Pedagogy_for_the_Twenty-First_Century . doi: 10.2304/pfie.2011.9.6.759.

[29] Sidorenko, V. (1984). Henezis proetnoy kul`turu [Genesis of project culture]. *Vоprosu filosofii*, *10.* 87-99. [In Rus.]

[30] Malkova, I. (2008). Konczepcziya i practika organizaczii obrazoval`nogo proektirovaniya v inovaczionoy shkole [The concept and practice of organizing educational design in an innovative school]. (*PhD thesis*). Tomsk, Russia. [In Rus.]

this practice. The organization result of education design practice is the determination and formation of a person's subject position in the education; herewith, the participants of education design get and implement the opportunities of influence on this practice and, thus, on his/her education. A separate type of designing in education is design activity in pedagogical process. Designing in education, in general, and in the teacher's pedagogical activity in particular, completely reformates the teachers' beliefs, their personal qualities, educational process, its purpose, content, process and result.

The concept of "pedagogical design" is widely used in the scientific discourse. O. Zair-Bek[31] (1995) considers "pedagogical design" as the applied scientific direction of pedagogics aimed at solving the tasks of development, transformation, improvement, contradiction resolution in the modern educational systems. We rely on the definition of pedagogical design proposed by N. Yakovleva[32] (2003) who interprets this concept as:

1 A purposeful activity of a teacher (or researcher in the field of pedagogy) to create a project that represents an innovative model of a mass-oriented pedagogical system;
2 A controlled process with a complex internal structure, the basis of which is the creativity of the designer.

To determine the theoretical foundations of our study, it is important to consider the specifics of pedagogical design, such as:

1 A result of pedagogical design (pedagogical project) can be implemented only partially; a significant part of the designed processes and phenomena can get out of control under the influence of random factors in the process of implementation;
2 Pedagogical designing is effective not for any object; e.g., different aspects of mental child's development such as

[31] Zair-Bek, E. (1995). *Osnovu pedogagicheskogo proektirovaniya* [*Basics of pedagogical design*]. St. Petersburg, Russia. [In Rus.]

[32] Yakovleva, N. (2003). Pedagogicheskoe proektirovanie inovaczionuh sistem [Pedagogical design of innovative systems]. (*PhD thesis*). Chelyabinsk, Russia. [In Rus.]

personal development, creativity, interpersonal relations, processes of socialization, etc. are difficult to design;

3 Pedagogical design has a poly-scientific character and involves the use of developments in a wide range of scientific fields;
4 While in the technical project, it is possible to describe the main technological processes, constructive knots and conditions that ensure its implementation, it is difficult to do it for results of pedagogical design;
5 Results of pedagogical design have flexibility, and sensitivity to adjusting.

The complex theoretical problem, which was covered in different ways in pedagogy, is the definition of the logic of pedagogical design, the selection of the main its stages and the content of activities in each of them.

We consider the stages of pedagogical design according to N. Alekseev[33] (1997) to be acceptable for our study:

- to determine the goals of the pedagogical project;
- to find out the system of pedagogical factors and conditions influencing the achievement of goals (orientation);
- to describe pedagogical reality to be designed (diagnostics of the initial stage);
- to fix (select) the level and operative units of pedagogical thinking to make a decision on creating a pedagogical project (self-reflection);
- to make a hypothesis about the variants of achieving goals and to assess the probability of their achievement in specific conditions (prediction);
- to build a specific model (pedagogical project) of pedagogical object (modelling);
- to construct a method for measuring parameters of a pedagogical object (extrapolating control);
- to realize a pedagogical project (implementation);
- to evaluate the results of project implementation and to compare them with the theoretically expected (assessment);

[33] Alekseev, N. (1997). Pedogogicheskie osnovu proektirovaniya lichnostno orietirovanogo obucheniya [Pedagogical bases of the design of person-centered learning]. (*PhD thesis*). Tyumen, Russia. [in Rus.]

- to build the optimized variant of a specific pedagogical object (correction).

Consequently, the concept of pedagogical design is specific in relation to the concept of design in education. We define it according to N. Yakovleva[34] (2003) as a focused, creative activity of the educational process subjects for the creation of a pedagogical project being an innovative model of the pedagogical system, in general, or its separate components, in particular.

One more concept included to the theoretical framework of the study is the concept of "project-based learning". It is used in scientific researches in a slightly different semantic perspective. T. Markham[35] (2003, p. 4) at Buck Institute for Education defines project-based learning as "a systematic teaching method that engages students in learning knowledge and skills through an extended inquiry process structured upon complex, authentic questions and carefully designed products and tasks". Project-based learning interpreted by K. Savina[36] (2010) is the way of interaction between the participants of educational process in the conditions of specially organized educational and cognitive and independent activity of students that consists in their motivated achievement of consciously set goals of creating real product in which personal development of learning subjects is modelled. Its essence according to M. Balkevičius, A. Mažeikienė & S. Švedienė[37] (2013) is to help students to look at learning as a self-regulating process,

[34] Yakovleva, N. (2003). Pedagogicheskoe proektirovanie inovaczionuh sistem [Pedagogical design of innovative systems]. (*PhD thesis*). Chelyabinsk, Russia. [In Rus.]

[35] Markham, T. (2003). *Project-based Learning Handbook: a guide to standards-focused project-based learning for middle and high school teachers, 2nd edn*. Novato, CA: Buck Institute for Education.

[36] Savina, E. (2010). Profesional`naya napravlenost` lichnosti studentov pedagogicheskogo koledgha v usloviyah proektnogo obucheniya [Professional orientation of the personality of students of the pedagogical college in the conditions of project training]. (*PhD thesis*). Moscow, Russia. [In Rus.]

[37] Balkevičius, M., Mažeikienė, Au. & Švedienė, S. (2013, July). The First Steps of Project-based Education in Lithuanian High Schools. *Procedia - Social and Behavioral Sciences, 83*. 483 – 492. Retrieved from: https://www.researchgate.net/publication/275538469_The_First_Steps_of_Project-based_Education_in_Lithuanian_High_Schools [accessed May 30 2019]. Doi: 10.1016/j.sbspro.2013.06.094

in which each student must form in his skill of planning, organizing and implementing activities.

We distinguish characteristic properties of the defined notion in determining the concept of “project-based learning”. This type of learning supposes the conscious statement of the goals of students’ activity. The purpose of the joint activity is the personal development of students. The purpose of the activity is decomposed in a number of goals, one of which is the creation of a real product in a specially organized educational and cognitive activity of students. In the project-based learning, more prerequisite conditions are created for the manifestation of the activity and independence of its participants, hence the external motivation is transformed into internal through the gradual attraction of new meanings, personal beliefs, new types of educational and cognitive activity of schoolchildren.

While fully sharing this view of scientists at project learning, we note that it is implemented in a specially organized, independent, educational, and cognitive activity of educational process subjects, which we describe, using the terms of “design activity” and “project activity”.

We consider design activity as a concept showing operation-activity aspect of designing. Therefore, we consider the concept of “design activity of educational process subjects” as a system of consequent interrelated procedures and operations performed with educational process objects or their models based on the prognosis and prediction of this activity results. We use the concept of “project activity” in somewhat different semantic perspective, focusing on the result of design activities.

Scientific researches concerning students’ project activity in the process of education are conducted in different directions. In the study of T. Miier[38] (2016, p. 163), the project activity of students is interpreted as a conscious and orderly activity of a student or students, carried out in a certain period of time in order to create a material or intellectual product

[38] Mier, T. (2016). *Organizacziya navchal`no-doslidnucz`koi diyal`nosti molodshuh shkolyariv [Organization of teaching and research activities of younger students]*. (Monograph). Kirovograd, Ukraine: FOP Aleksandrova M. [In Ukr.]

on the basis of an independent or collective implementation of pre-planned actions. The semantic conception of the definition proposed by Y. Polat, M. Bukharkina[39] (2007) is that the project activity is a joint educational and cognitive creative or gaming activity of students-partners, which has a common goal, coordinated methods, and means of action, and is aimed at achieving a common result in solving a certain problem being significant for the project participants. The intended purpose of the project activity is defined somewhat differently by S. Goncharenko[40] (1997), who points out that the project activity should be considered as reasonable and conscious activity, the purpose of which is the formation of a system of students' intellectual and practical skills. I. Malkova[41] (2008) determines its purpose in the context of forming students' subjective position in the educational process. In the scientific discourse there is a number of studies in which the procedure component of the project activity is involved: the process of forming the idea of the project activity (Hromyko[42], 1996), the analysis of the design results (Malkova[41], 2008), the development of creative activity of senior students by means of project-based learning (Polikhun[43], 2007).

Thus, we consider project activity of education process subjects in the broadest sense as their constructive and productive activity aimed at solving significant educational, learning or or real-life problem, at achieving the final result in the process of goal-setting, planning and implementing the project. This type of activity involves a conscious statement

[39] Polat, E. (2007). *Sovremenue pedagogicheskie i informaczionue tehnologii v sisteme obrazovaniya* [*Modern pedagogical and information technologies in the education system*]. Moscow, Russia: Hardariki. [In Rus.]

[40] Goncharenko, S. (1997). *Ukrains`kuy pedagogichnuy slovnuk* [Ukrainian Pedagogical Dictionary]. Kyiv, Ukraine: Lubid`. [in Ukr.]

[41] Malkova, I. (2008). Konczepcziya i practika organizaczii obrazoval`nogo proektirovaniya v inovaczionoy shkole [The concept and practice of organizing educational design in an innovative school]. (*PhD thesis*). Russia. [In Rus.]

[42] Gromyko, Yu. (1996). *Proektirovanie i programirovanie razvitiya obrazovaniya* [*Design and programming of educational development*]. Moskow, Russia: Moskowskaya akademiya razvitiya obrazovaniya. [In Rus.]

[43] Polikhun, N. (2007). Rozvutok tvorchoi diyal`nosti starshoklasnukiv u proczesi navchanya fizuku z vukorustanuam proektnoi tehnologii [Development of creative activity of high school students in the process of teaching physics using design technology]. (*PhD thesis*). Kyiv, Ukraine. [In Ukr.]

of its goals by the subjects of the activity; one of the goals is the creation of a real product (project). A project, by definition of S. Chard[44] (1998) is an in-depth investigation of a real-world topic worthy of a student's attention and effort. The study may be carried out with an entire class or with small groups of students. Projects typically do not constitute the whole educational program; instead, teachers use them alongside systematic instruction and as a means of achieving curricular goals. The project should have a significant personal significance for the subjects of learning (to be of subjective significance), and be socially important, related to real social processes, provide certain aspects of socialization, expand and enrich the experience of life and educational and cognitive activities of the participants of project-based learning.

In the field of scientific research, we focus on the project activity of pupils in learning Mathematics and a teacher in teaching Mathematics (in the narrow sense) and the design activity of a student, pre-service Mathematics teacher, in the process of his/her methodological training at higher school. Taking as a basis the above definition of "the project activity of the educational process subjects" concept, we consider the project-based learning activity of pupils in learning Mathematics in the narrow sense as the active, creative, constructive, and productive educational and cognitive activity of a pupil/pupils, carried out within a certain period of time with the aim of creating a material or intellectual product on the basis of independent/collective implementation of pre-planned methods of mathematical activity with mathematical objects or real world objects.

We attribute a teacher's project activity in teaching Mathematics (in the narrow sense) to a type of his/her methodical activity. Thus, we consider a teacher's project methodical activity in teaching Mathematics as active, creative, scientific, and productive activity carried out in a definite period of time with the aim of creating material or intellectual product (methodical project) on the basis of independent/collective performance of pre-planned ways

[44] Chard, S. (1998). *The Project Approach: Managing Successful Projects.* New York, NY : Scholastic Teaching Resources.

of methodical and mathematical activity with methodical, mathematical objects and the objects of surrounding reality or their models.

We attribute project activity of a future teacher in the process of his/her methodical training (activity of educational methodical design (Akulenko[45], 2013), to a specific type of student's learning, cognitive, scientific, practical, methodical activity being a system of consequent interrelated procedures and operations performed with methodical and mathematical objects and the real world objects or their models on the basis of forecast and prediction of this activity results in the form of educational methodical project.

The operational basis for conducting educational methodical design is the activity on modelling, forecasting and designing educational goals, content, forms, methods, tools, and results of teaching Mathematics, performed on the basis of previously implemented analytical and synthetic students' activity on these points.

The results of educational methodical design (educational methodical projects), should be considered in two aspects: cognitive and activity ones. The result of educational methodical design in cognitive aspect may be the transformation of methodical or mathematical object itself or its model, as: systems of didactic goals of learning a topic; certain technology of teaching and learning a topic; content or procedure model of the joint activity of a teacher and students in the process of implementation of the adapted method of teaching and learning, etc. in accordance with the specific conditions of its intended use. The result of educational methodical design in the activity aspect is the sequence of steps leading to the desired (predicted) result. This result forms a generalized way of educational methodical design activity.

The design activity of a pre-service Mathematics teacher in the context of educational methodical design has its additional

[45] Akulenko, I. (2013). *Kompetentnisno oriyentovana metoduchna pidgotovka maybutn`ogo vchutelya matematuku profil`noi shkolu (teoretuchnuy aspekt)* [*Competence-oriented methodical training of a future teacher of mathematics in a profile school (theoretical aspect)*]. (Monograph). Cherkasy, Ukraine: Vidavets Chabanenko Yu. [In Ukr.]

specific feature, since it is carried out not in the educational process at school, but during mastering methods and tools of math teaching while schooling at a higher school; therefore, a student performs educational methodical design not with methodical objects, but, mainly with their models, and he/she is limited to the possibility of verifying the results of educational methodical design. In addition, we note that it is precisely in the process of carrying out activities on educational methodical design a future Mathematics teacher masters the content and operation composition of the project activity of math teacher and pupils' project-based learning Mathematics, which he/she will organize in his subsequent work.

A prerequisite for the implementation of educational methodical design is the theoretical study and transformation of those objects that reflects and substitutes for components of real educational processes. It is an activity on educational methodical modelling. We consider educational methodical modelling as a process of constructing, studying and operating special objects (educational methodical models) being complex systems that reflect or reproduce important, for a researcher, certain characteristic properties, elements, connections in methodical objects. According to content, methodical models are divided into target (prognostic), content and procedure ones; the latter, in their turn, represented by organizational, managerial, instrumental, monitoring, and reflexive models[45]. Accordingly, a future teacher reproduces, integrates, constructs, transforms the elements of learning, organizing, managing, controlling, assessing activities of a working teacher in the process of educational methodical modelling.

The activity on educational methodical design is a natural extension of educational methodical modelling, because it involves further development of the built model and bringing it to the level of practical use. Educational methodical design involves the transformation of a methodical object or its educational model (for example, the system of didactic purposes of learning a topic; units of content or organizational forms or means of learning a topic, etc.) in accordance with the specific conditions of its intended use.

We consider educational methodical design in three aspects:

- as a systematic element of an integral system of pedagogical design;
- as a specific kind of scientific and practical activity on the development and substantiation of the target idea, the creation and transformation of a methodological object's educational model, which results in the creation of a methodological project as an objectively or subjectively new product (material or intellectual) reflecting elements of the methodical system of teaching Mathematics;
- as a system of consistent interconnected procedures and operations performed with methodological objects or their models, on the basis of forecast and prediction of this activity results as an educational methodical project.

Since a future teacher is largely confined to the possibility of verifying, clarifying and correcting his/her project in practice, the educational methodical design, in our opinion, is a derivative (secondary) activity compared with educational methodical modelling. The operational basis for conducting educational methodical design is the activity on educational methodical modelling, predicting and designing, performed on the basis of the pre-performed analytical and synthetic activity of students.

Activities on educational methodical design, performed by students in the process of their methodical training, are carried out at the following levels: pedagogical situation, didactic cycle or its sub-cycles, a separate lesson or a set of lessons (e.g. according to lecture-practice system of education).

The results of educational methodical design, carried out by a future Mathematics teacher, may be fixed in:

- curriculum for studying the topic;
- methodical recommendations for the implementation of a methodical scheme for introducing a new concept, fact, mode of activity, adapted to certain learning conditions (profile, level);

- description of the didactic training cycle or its separate sub-cycles, adapted to the specific learning conditions (profile, level): making a cognitive task or problem situation and creating students' positive motivation to solve it; updating students' basic knowledge necessary for mastering new material; presenting a new fragment of educational material and creating conditions for its initial learning; organizing further study of educational material to the necessary and possible level in this cycle; organizing feedback, control and application of students' knowledge; preparing students for extra-curricular work;
- scenarios of lessons, separate stages of lessons, extracurricular activities;
- exercise systems of various purposes, applied tasks;
- various products created using the tools of NIT (training programs and test control shells, text editors, table processors, multimedia presentation programs, websites, etc.);
- samples of students' project activity at Mathematics lessons and extra-curricular work;
- project of the individual educational trajectory of a student, etc.

Teachers' beliefs

These outcomes of students' educational methodical design however demonstrate their knowledge and understandings relies on factual propositions. But it's important to distinction between students' knowledge and their beliefs. In her study, P. Ertmer[46] (2005) clarified the significance of teachers' beliefs and why they are critical to consider the way of implementation project-based learning and teaching. The review[46] (Ertmer, 2005) suggested that teachers' pedagogical beliefs are strongly influenced by personal experiences, vicarious experience (observing models of other teachers implementing the innovation), and sociocultural

[46] Ertmer, P. (2005, December). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? *Educational Technology Research and Development, 53*(4). 25-39. Retrieved from https://link.springer.com/article/10.1007/BF02504683.

influences. The study underlines that many teachers' personal experiences do not support a belief in project-based learning and teaching.

In their review of the research on problem-based learning implementation in K-12 settings, P. Ertmer and K. Simons[47] (2006) argued that changing teachers' beliefs about their classroom role from that of director to facilitator is a key implementation hurdle for project-based learning.

Based on their observations of teachers' attempts to integrate the Learning by Design (LBD) approach into middle school science instruction, J. Kolodner, P. Camp, D. Crismond, B. Fasse, J. Gray, J. Holbrook, S. Puntambekar, and M. Ryan[48] (2003) found that one of the key implementation challenges involved teachers' willingness to change their role in the classroom and alter their conceptions of classroom control. J. Kolodner et al. found that this was too difficult for some of the teachers who had attempted to implement LBD; these teachers decided to end their participation in the research team's field test during the year or after one year.

M. Grant and J. Hill[49] (2006) noted that some teachers found implementation of project-based learning to be risky, because, in addition to modifying the teacher's role, they require teachers to tolerate changes to the traditional learning environment (e.g., noise level, student collaboration, and student movement) and feel comfortable with ambiguity and flexibility in classroom management. Given that teachers face accountability pressures, coping with the changes and levels of ambiguity can be difficult (Grant & Hill, 2006).

[47] Ertmer, P., & Simons, K. (2006). Jumping the PBL implementation hurdle: Supporting the efforts of K-12 teachers. *Interdisciplinary Journal of ProblemBased Learning, 1(*1), 40- 54. Doi: 10.7771/1541-5015.1005.

[48] Kolodner, J., Camp, P., Crismond, D., Fasse, B., Gray, J., Holbrook, J., Puntambekar, S., & Ryan, M. (2003). Problem-based learning meets case-based reasoning in the middle-school science classroom: Putting Learning by Design into practice. *Journal of the Learning Sciences, 12*(4), 495-547. Retrieved from https://stemedhub.org/resources/800/download/Kolodner_etal_2003_PBL_Meets_Cased-Based_Reasoning.pdf

[49] Grant, M., & Hill, J. (2006). *Weighing the risks with the rewards: Implementing student-centered pedagogy within high-stakes testing*. In R. Lambert and C. McCarthy (2006). *Understanding teacher stress in an age of accountability*. Greenwich, CT: Information Age Press.

Methodology, results and discussion

To study Ukrainian Mathematics in-service teachers' beliefs about the project-based learning and teaching, a survey of teachers was conducted.

Data collection tools: Standard closed ended questionnaire was developed by the researchers based on available literature to evaluate the teacher's concepts and performance of project-based learning Mathematics.

Data collection technique: The data were collected within one month; every participant filled in the questionnaire by himself / herself. Each questionnaire took from 10 to 15 minutes to be filled in; there was no missing questionnaire.

Ethical considerations: The purpose of the study was explained to every participant; the respondents were assured that the information will be confidential and used only for purpose of the study.

The questionnaire involved 125 respondents, 52% of whom work in city schools, 48% - in rural ones. The work experience as a Mathematics teacher varied: for 1-5 and 6-10 years (21%), 11-15 years (14%), 16-20 years (4%), 21-25 years (11), 26 years and longer (29%).

In general, 88% of respondents agreed that teaching Mathematics in modern school is impossible without project-based learning. The questionnaire showed that 80% of Mathematics teachers often or sometimes use project-based learning in their work; this fact bears witness to teachers' beliefs about the great value and importance of schoolchildren's project activity in learning Mathematics.

It should be noted that the percentage of those who uses the project-based learning and teaching in educational process of Mathematics is unevenly distributed among teachers according to their work experience. The teachers having 16-20-year and 21-25-year work experience often or sometimes use this method in their work (100%); the teachers having 1-5-year work experience do not use it at all (13%). This fact shows that the newly qualified teachers do not have sufficient preparation for the organization the schoolchildren's project-based learning of Mathematics and are not able to use it in teaching Mathematics. In support of this conclusion, 12% of them pointed out that they do not have sufficient knowledge about the essence of project methodology to use it in practice. The largest number of those who often use project-based teaching in their work (35%) is presented among the teachers having 11-15-year work experience. The teachers with work experience of more than 25 years always (19%) or sometimes (70%) use this method in Mathematics educational process.

It is an indicative fact that all teachers who work more than 10 years, believe that they have sufficient knowledge about the essence of project-based learning and teaching Mathematics. In our opinion, this situation is due to the fact that newly qualified teachers did not receive enough of the necessary knowledge while schooling in higher school, and those who have longer period of teaching experience were able to improve their qualifications, in particular, on the point of organizing project activities of schoolchildren.

Most respondents (82%) agreed that project-based learning helps schoolchildren to master the new ways of mathematical activity and to learn new mathematical facts. However, it is less effective for learning new mathematical concepts.

Concerning the expediency of using the project-based teaching Mathematics the students of different age groups, most respondents (66%) consider it expedient to use this method in senior school. At the same time, 52% of respondents consider it to be efficient method of teaching the students in K-7 – K-9 settings, 23% of respondent – the students in K-5 and K-6 settings. Thus, there is a steady tendency in the teachers' beliefs concerning the expediency of using project-based teaching of Mathematics in secondary school. According to the respondents, the use of projects in the educational activities of middle school students is less effective.

The importance of knowledge about the essence of project-based learning and project-based teaching Mathematics for pre-service and in-service math teachers was pointed out by 90% of respondents. 70% of respondents considered the knowledge about the essence of schoolchildren's project

№	Skills and teaching activities	Points
1	To formulate interesting and significant topic of educational project	4.73
2	To organize step-by-step independent work of students	4.66
3	To motivate students to discuss and create a project	4.64
4	To use simple examples to explain complex phenomena	4.61
5	To formulate a key issue of the educational project	4.60
6	To assess the results of students' project activity	4.58
7	To master a set of research methods	4.56
8	To organize the pupils' search of an optimal way to achieve the project goal	4.56
9	To assist students in analysing and synthesizing the results of educational project	4.54
10	To form a group of students to work at the project	4.51
11	To choose the age group of students to perform a project on specific subjects	4.45
12	To formulate the didactic goal of a project	4.36
13	To seek partners to work at the educational project	4.26
14	To present the educational project outside the school	4.17

Table 3.2.1

activity, its steps of performance, interdisciplinary links of educational themes in Mathematics, awareness of current educational Internet resources, the ways of motivating students for project activity to be of particular importance.

The respondents were asked to rank (in points from 0 to 5 the importance of mastering certain skills and teaching activities by prospective teachers to apply project-based teaching efficiently. The results are shown in Table 3.2.1.

The focus of the additional attention in the survey was to find out what organizational work experience should be gained by the students during their schooling in higher school for further successful application of project-based learning and teaching Mathematics. The results are shown in Table 3.2.2.

№	Organizational work experience	Points
1	To be the member of the project in higher school	4.43
2	To be the member of the project in school as an volunteer	3.98
3	To be the member of the project in school while having practice	4.29
4	To be attracted to managing students' research in Minor Academy of Sciences	3.92
5	To be attracted to seeking partners to perform project	3.89
6	To be attracted to various types of vocational guidance work in higher school	3.82

Table 3.2.2

Conclusion

A prospective Mathematics teachers need a special mastering and training in the organization of project-based learning and teaching Mathematics while studying in higher school (Akulenko & Zhydkov[50], 2018). Therefore, widespread and active use of project-based teaching Mathematics causes changes in mastering content, tools and methods of teaching Mathematics by pre-service teachers. They should be aware of the method genesis in the educational theory and practice, its theoretical, in particular, psychological and pedagogical bases, and the ways of using computers in its implementation.

The analysis of the questionnaire shows that to implement project-based teaching Mathematics successfully, pre-service teachers should be involved in the creation of educational projects while studying in higher schools and their subsequent implementation during the educational pedagogical practice

[50] Akulenko, I. & Zhydkov, O. (2018). Teoretychni osnovy pidhotovky maybutn′oho vchytelya matematyky do orhanizatsiyi proektnoyi diyal′nosti shkolyariv [Theoretical Bases of Training Pre-Service Mathematics Teacher for Organizing Project Activity of Pupils]. *Naukovi zapysky. Seriya: Pedahohichni nauky [Scientific Notes. Series: Pedagogical Sciences]. 16.* 9-13. [In Ukr.]

in secondary schools. In our opinion, it is promoted by the "New Information Technology in the Educational Process' elective course[51] (Zhydkov, 2017), which is provided for by the curriculum for students being pre-service Mathematics teachers.

The course curriculum provides the creation of a model for Mathematics educational project. Modelling the process of project-based learning and teaching Mathematics involves some stages: preparatory, training-designing and final. At the preparatory stage, students are introduced to the history of this method, its theoretical, in particular, psychological and pedagogical bases, the reasons for updating its implementation in the educational process in modern conditions. The training-designing stage involves preparing by the students a model of their own educational project in which they act as a teacher and a pupil at one time. Students learn to plan an educational project, learn the state educational standard and Mathematics curricula for different categories of secondary school pupils and predict its timing on their basis. Acting as a teacher, students give practically significant research task for pupils; acting as a pupil, they present the research results. At the final stage, the students carry out self-reflection on the implementation of the project.

[51] Zhydkov, O. (2017). Navchal'ne modelyuvannya studentamy zastosuvannya metodu proektiv u navchanni uchniv matematyky [Educational Modelling of Students' Use of Project Method in Teaching Mathematics]. *Visnyk Cherkas'koho universytetu: Seriya "Pedahohichni nauky" – Cherkasy University Bulletin: Series "Pedagogical Sciences", 13-14.* 48-54. [In Ukr.]

3.3 Flexible Model of Training Mathematics Teachers in the System of Postgraduate Pedagogical Education*

V. Kirman

*The article is published in the author's translation

The implementation of high-quality mathematical education is impossible without a modern and qualified teacher who has a professional pedagogical education. At the same time, there are several factors that determine the need for lifelong education throughout the profession. We can distinguish objective and subjective factors. Objective factors include dynamic changes in society that are the cause for a gradual transformation of educational content and innovative learning technologies. Subjective factors include, first of all, some gaps in teachers' subject, methodological, psychological and pedagogical competences, as well as the need to develop and improve their respective competences. Existing gaps can be caused by insufficient training in higher education institutions (which is currently associated with enrollees' low level of preparation for admission to pedagogical universities) and diffusion of competences that were not related to educators' previous pedagogical activity. Our observations and measurements[1] (Kirman, 2017) prove the existence of a large number of vulnerabilities, especially regarding mathematical and methodological competences of a math teacher. All of these reasons determine strategies for continuing education of math teachers.

The main institution responsible for teachers' continuing education is the system of postgraduate teacher education. In recent decades, a number of researchers have been developing the concept of postgraduate teacher education

[1] Kirman, V. (2017). Vektorna model' matematychnoyi kompetentnosti uchytelya matematyky ta pidkhody do yiyi identyfikatsiyi [Vector model of mathematical competence of mathematics teacher and approaches to its identification]. *Topical issues of natural and mathematical education. 2(10).* 94-101. [In Ukr.].

in Ukraine based on existing experience and today's needs[2] (Oliynyk & Danylenko, 2005) as well[3] (Baselyuk et al., 2012). Due to conducted researches the basic principles of postgraduate teacher education has been formulated and reflected in the new regulations. The basic principles are voluntary training and freedom to choose a place and form of teacher retraining.

Our work presents the model of postgraduate education of math teachers, its optimal forms for teacher training are substantiated and the principles of selecting educational material are developed as well. The model has been tested on the basis of the Dnipro Academy of Continuing Education and the Dnipropetrovsk Regional Institute of Postgraduate Teacher Education for three years, functioning of the system of postgraduate teacher education of math teachers was analyzed on the basis of the proposed model.

We assume that the system of mathematics teacher training is a subsystem of continuing teacher education. We have identified the main areas of continuing education of mathematics teachers, which primarily determine strategies for self-education and professional development. A methodological basis for defining these strategies is grounded on the competency-based approach; each teacher is supposed to be able to develop the capacity and readiness to undertake the appropriate type of activities. Based on this, we can identify the main areas of mathematics teachers' continuing education. They include: methodological education, general psycho-pedagogical education, special professional education. We will focus mainly on the last one. We distinguish three blocks in the module of mathematics teachers' professional education:

1 A workshop on elementary mathematics (mathematical workshop);

[2] Oliynyk, V., & Danylenko, L. (2005). *Pislyadyplomna pedahohichna osvita Ukrayiny: suchasnist' i perespektyvy rozvytku [Postgraduate Pedagogical Education of Ukraine: Modern Times and Prospects for Development]*. Kyiv, Ukraine: Milenium. [In Ukr.].

[3] Bazelyuk, V., Lushyn, P., Snisarenko, O., Snitsar, L., & Solodkov, V. (2012). *Pislyadyplomna osvita v umovakh yevrointehratsiyi: sutnist', zmist, tekhnolohiyi, hotovnist' do zmin [Postgraduate education in the context of European integration: essence, content, technologies, readiness for changes: study guide]*. Kyiv, Ukraine: Pedahohichna dumka. [In Ukr.].

2 Modern issues of theory and methodology of teaching mathematics;
3 Scientific and methodological (scientific-theoretical) foundations of the school course of mathematics.

It is obvious that during the retraining course, it is impossible to acquire new and improve previously learned skills. We can only say about the actual choice of self-education in the nearest (3-5 years) perspective. Thus, it is advisable to divide the retraining course into three stages: the stage of diagnostics (diagnostic), the stage of actualization of basic knowledge and skills (actualization), the stage of formation of self-education plan of (methodical and recommendation-based). Simple analysis and experience show that the most effective form of course organization is a distance-based form. There are several reasons for this. They become apparent after analyzing a comparative table of training course forms for teaching staff. Herein B is for blended learning, I is for intramural learning, D is for distance learning. The table provides expert assessments of the effectiveness of the respective activity scored on three-point scale (0 – no opportunity, 1 – there is some opportunity, 2 – the opportunity is effectively implemented).

Let's analyze the Table 3.3.1. The ball for the appropriate opportunity is calculated on the principle of voting[4] (Voloshin & Mashchenko, 2010) if this form becomes the most important among the expert points. It is easy to notice that the blended learning form has the corresponding advantages.

The problems and gaps in the professional skills of mathematics teachers of different categories have been identified via analysis and observation. We propose to use the indicator of the level of a teacher's mathematical competence in the paper[5] (Kirman & Schvydun, 2016) and

[4] Voloshyn, O., Mashchenko, S. (2010). *Modeli ta metody pryynyattya rishen' [Models and methods of decision-making technique]*. Kyiv, Ukraine: Publishing and Printing Center «Kyiv University". [In Ukr.].

[5] Kirman, V., Shvydun, L. (2016). Eksperymental'na aprobatsiya tekhnolohiy monitorynhu matematychnoyi hramotnosti [Experimental testing of mathematical literacy monitoring technologies]. *Scientific Notes of Kirovohrad State Pedagogical University, 10*. 52-64. [In Ukr.].

	I	D	B
Opportunity to follow in-service teachers training	0	2	1
Opportunity to freely choose time when performing tasks, planning own activities.	1	2	2
Opportunity to diagnose/be diagnosed objectively	2	0	2
Opportunity to get objective and effective control	2	0	2
Opportunity to have professional communication at a convenient time	1	2	2
Opportunity to have an informal communication with colleagues, exchange experience, demonstrate achievements	2	1	2
Opportunity to plan educational activities independently and manage it	0	2	2
Total voting points	**3**	**4**	**6**

Table 3.3.1 Comparative table of advantages of forms applied while organizing courses for pedagogical workers' professional development.

the article[6] (Kirman, 2018). It is the probability to solve a randomly selected task from a set that is relevant to the teacher's professional activity, and the activity profile also determines the probability for distribution to the sets. According to the results of monitoring studies, evaluations of the indicator of mathematical competence level have been obtained (mathematics teachers of the region are considered as the main entity). It is shown that the indicator mode does not exceed 0,6, the percentage of teachers with a high indicator is not more than 15%, at least 25% of teachers have an indicator that does not exceed 0,4. In general, we can distinguish the main groups of problems of mathematics teachers' professional level: solving non-reproductive tasks, proving the classical theorems of algebra, Introduction to Analysis, geometry, the analysis of complex mathematical concepts, theoretical and practical aspects of modern methods of teaching mathematics, mastering information and communication technologies of teaching. As we can see,

[6] Kirman, V. (2018). Pidkhody do otsinyuvannya parametriv modeley matematychnoyi kompetentnosti [Approaches to parameter estimation of models of mathematical competence]. *Science and Education a New Dimension, VI (65), 155.* 23-27. [In Ukr.].

the problem correlates with the offered blocks of the special-professional module for mathematics teachers.

At the same time, it is possible to talk about the considerable variability of qualitative and quantitative characteristics of specialists who undergo retraining courses. We suppose that the most significant ones are the following: work experience, qualification grade, types of students enrolled whom the teacher has worked with in recent years, motivation, ability to be aware of own problems, level of logical thinking, operational-mathematical level (ability to solve basic mathematical tasks from the school course), theoretical and mathematical level, theoretical and methodological level, level of instrumental competence (possession of training tools). It is also important to take into account the tasks that teachers face in continuing learning, the possibility of their implementation. All this allows us to formulate the principles for the construction and operation of the distance component, which is the core of blended learning. We distinguish such principles as the following ones.

1 *Relevance*
The content of teaching should meet teachers' real needs, predicting situations that any teacher may face in his / her professional activity. In recent years, preference for teacher training mainly has been given to methodological, technological, theoretical and pedagogical aspects. At the same time, the accumulated statistics and observations[7] (Kirman, 2017) show the real need to increase the level of teachers' mathematical competence.

2 *Availability and visibility*
The distance learning system should be easily implied; all the instructions should be clear. It is very important to deliver information plainly. In our opinion, it is effective to use small video lectures for mathematical topics that form thematic blocks, and lectures in the form of presentations are optimal for methodical topics.

[7] Kirman, V. (2017). Vektorna model' matematychnoyi kompetentnosti uchytelya matematyky ta pidkhody do yiyi identyfikatsiyi [Vector model of mathematical competence of mathematics teacher and approaches to its identification]. *Topical issues of natural and mathematical education. 2(10).* 94-101. [In Ukr.].

3 *Demonstrativeness*
Platform lectures and assignments cannot be exhaustive. This is primarily due to the course participants' individual needs. Therefore, a significant part of the materials on the platform should be in a form of review, help a course participant to independently find the literature and to work out the relevant material, which is both on the platform and in the external information environment. We consider the system of distance learning as a subsystem of specialized information space, including invalid. Therefore, the most important thing for the distance learning system is to navigate course participants in this environment. The elements of such environment contain media resources, textbooks, scientific and popular-science literature (with any data storage device).

4 *Scientific character*
The relevant principle reveals dual character. On the one hand, the content of teaching should be formed on the basis of the latest achievements in the theory and methodology of teaching mathematics. On the other hand, training tools, such as ICTs, should be used to their fullest potential.

5 *Consistency*
We regard the remote component as a teleological system, which is a subsystem of the general system of teachers' continuing professional education. The aim of the latter is to increase the level of professional competence.

6 *Comfort*
The mode of the system operation should be as comfortable as possible. This includes available access to the platform materials, ability to adjust the performance of tests, independence from the type of devices and system software that the course participant works with.

7 *Completeness*
Any professional activity issue should be reflected in the distance learning system. However, there are two modes. The first is direct. The relevant question is sufficiently reflected in the lectures or practical classes of the distance course. For the second mode it is possible to

get acquainted with the relevant question in the external information environment, the distance learning system itself can help to find the relevant information (external navigation function).

8 *Flexibility*

Distance learning comprises a very wide range of pedagogical staff with different needs and capacities, which is why we should take into account teachers' individual characteristics. Such mechanisms can be implemented through an excessive number of lectures, workshops, platform-based tests, and creating and adjusting individual educational trajectories based on test results and discussions with course participants. It is obvious that such mechanisms cannot be implemented without the active involvement of teachers and course curators.

9 *Adaptability*

The change dynamics require constant adjustment of the retraining program, so the distance education system needs to constantly adjust to such changes. This is achieved both through the redundancy of the materials/ information and the mechanisms for their constant data seed/replenishment and revision.

10 *Synergies*

The proposed structure and dimensional characters should not become dogma. Any system that develops in the external environment undergoes crises, but the system must not be destroyed. Therefore, a mechanism for "phase transitions" is needed. Such transitions will not be critical if the goals are transparent and the system's structure is simple.

Formulated principles allow us to construct and substantiate the structural and parametric model of mathematics teachers' blended learning. The dimensional characters of the model include the duration of training (intramural and distance components), the content of training in the intramural and distance components. The structural model characterizes the sequence of stages of the educational process, a set

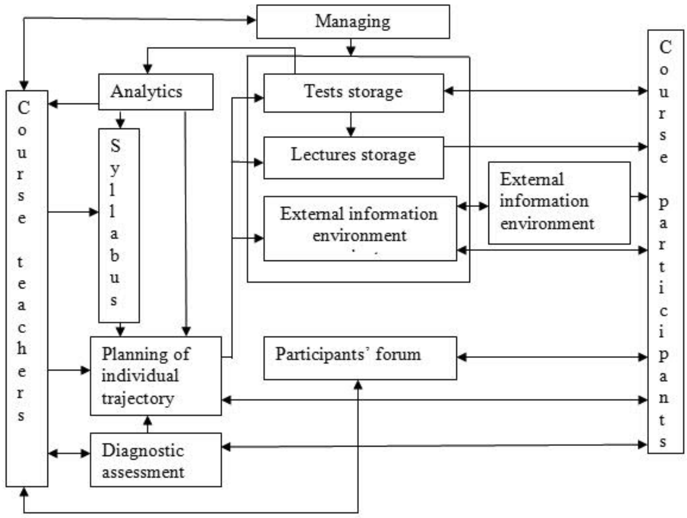

Figure 3.3.1 Structural model of the blended learning system of mathematics teachers' postgraduate education.

of procedures and technical means, the interaction of participants (Figure 3.3.1)

From the practical perspective, the organization of blended learning includes an introductory intramural component, a distance component, and a final intramural component. The first blended learning component involves the implementation of diagnostic and partially actualization stages (introductory lectures). The distance component implements mainly the actualization stage, as well as the methodical and recommendation-based one. This component includes acquaintance with video lectures, texts and presentations, performance of required and training tests, communication through forums and chat. Forums can be organized by lecturers and participants who may offer their own topics. The final face-to-face stage includes the final test and discussion with further methodological recommendations for teachers' self-education. The result should be an individual trajectory of self-education for the next 3-5 years.

Organizing blended learning introductory lecturers plays an essential role. The aim of introductory lectures is to prepare participants for independent work with texts of lectures and workshops, video lectures and presentations.

The introductory lecture should describe the entire structure of the corresponding block of mathematical competences as well as logical connections if the material relates to scientific and methodological basics of the school mathematics course.

Details of a special-professional module of the retraining course include three main blocks: scientific-theoretical foundations of the mathematics course, modern theory and methodology of teaching mathematics, a workshop on elementary mathematics.

The first block reveals the modern scientific approach to the main content lines of the school mathematics course, the second one includes the regulatory provision of teaching mathematics, modern methods and forms of teaching mathematics, digital technologies teaching, forms of assessment of students' educational achievements (with elements of testology and general theory of educational dimension), the third contains the main types of tasks based on external independent testing or standardized external testing, multifigured geometric tasks, tasks of mathematical modeling (special attention is paid to so-called K-tasks), tasks of basic functions study, equations and inequalities, basic problems of mathematical analysis.

The aim of the first block's lectures is to illustrate the mathematical concepts and facts taught at different levels of argument in a school course, from rigorous positions, using rigorous considerations and formal definitions. Here are some examples of combinatorial content lines.

The study of combinatorics begins with the basic principles which are the rules of multiplication and addition. They are viewed on an intuitive level, therefore, not proven. Let us show how to illustrate the relations between formal and informal approaches to basic combinatorial rules.

We formulate the basic rules of combinatorics[8] (Yadrenko, 2003). Let there be l of non-overlapping bounded sets $A_1, A_2, ..., A_l$. Then the number of ways to select one element from these sets is obviously equal to the value

[8] Yadrenko, M. (2003). *Dyskretna matematyka [Discrete Mathematics]*. Kyiv, Ukraine: Express. [In Ukr.].

$$Z = |A_1| + \ldots + |A_l|,$$

here $|A_j|$ is the number of elements in the set A_j. If we consider the choice of an element of the set A_j as an action with a number j then it is possible to formulate the so-called rule of addition in terms of action. *Namely, let's do one of the following l actions. Moreover, the first action can be done in t_1 ways, the second action can be done in t_2 ways,..., the action l can be done in t_l ways. Then, if each of these two actions does not contain the same execution mode, then you can perform one of the specified l actions using $t_1 + \ldots + t_l$ ways. This is the so-called an additive rule in combinatorics.*

Obviously, the addition rule is a differently formulated property about the number of elements in the union of non-overlapping bounded sets:

$$\left|\bigcup_{i=1}^{l} A_i\right| = \sum_{i=1}^{l} |A_i| .$$

The rule of adding is simply illustrated with the help of the so-called a decision tree (Figure 3.3.2).

Now we are going to formulate a multiplication rule in terms of action. *For example, it is supposed to do l actions sequentially. If the first action can be done in t_1 ways, the second one can be done in t_2 ways, the third one can be done in t_3 ways, etc. then all l actions together can be done in $t_1 \cdot t_2 \cdot t_3 \cdot \ldots \cdot t_l$ ways.* It is important to keep in mind that the number of ways to perform each subsequent action does not depend on how the previous ones were performed.

You can justify this principle with the help of a decision

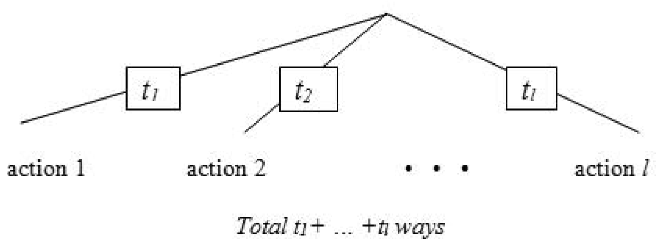

Figure 3.3.2 The rule of adding.

tree. Supposing there is a vertex P_0 (vertex of level 0). It is connected with vertices of level 1 - $P_1,\ldots, P_{t_1}$. Each vertex P_j of level 1 is connected to vertices t_2 of level 2 - $P_{j,1},\ldots, P_{j,t_2}$.

Then, similarly for each vertex of level 2 P_{j_1,j_2}, the edges are drawn to the vertices t_3 of level 3. Thus wise, it is easy to show by the method of mathematical induction that the number of vertices of level l $P_{j,j_2,\ldots,j_l}$ is equal to $t_1 \cdot t_2 \cdot \ldots \cdot t_l$. The number of trajectories of length l is equal to the same number (that is, those ones which contain l edges) leading from level 0 to the vertices of level l (Figure 3.3.3).

We should recall that if there is l of $A_1, A_2,\ldots, A_l$ set, then the set of ordered $(a_1, a_2,\ldots, a_l)$ sets, where $a_1 \in A_1,\ldots, a_l \in A_l$, is called the Cartesian product of sets $A_1, A_2,\ldots, A_l$ and denoted as $A_1 \times A_2 \times \ldots \times A_l$. The multiplication rule implies an important formula for the number of elements of Cartesian product of sets:

$$n(A_1 \times A_2 \times \ldots \times A_l) = n(A_1) \cdot \ldots \cdot n(A_l).$$

Let us pay attention to the tasks that we propose to participants for a joint discussion. They include not only school course tasks but also generalizing tasks that reveal the in-depth content of the mathematical concepts of the school course. Let us illustrate the following problems again with the example of a combinatorial content line.

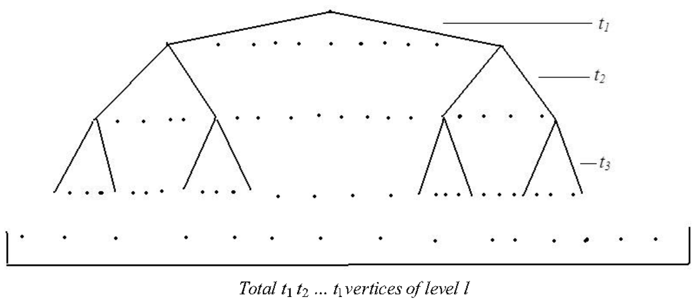

Figure 3.3.3 The rule of multiplication.

It is very easy to establish that, if $|X| = n$, $|Y| = m$ (here $|X|$ is the number of elements of the set X), then the number of all possible mappings of the set X in the set Y is equal to m^n. Indeed, let $X = \{x_1, x_2, ..., x_n\}$ $X = \{x_1, \dots, x_n\}$ $X = \{x_1, \dots, x_n\}$, $Y = \{y_1, y_2, ..., y_m\}$. Each of $x_i \in X$ can match one of m elements Y. Then by the multiplication rule we have that the number of mappings is equal to m^n.

Let us suppose that $X_0 \subset X$, $|X_0| = k$, $|X| = n$, $|Y| = m$, $m > n > k$. How many mappings $f : X \to Y$ exist that $|f(X_0)| = k$?

From the statement of the problem it follows that you can set bijection between X_0 and $f(X_0)$. Then you can build the necessary mapping after following these steps:

step 1 – selecting a subset $f(X_0) \subset Y$ (C_m^k ways);

step 2 – creating bijection between X_0 and $f(X_0)$ ($k!$ ways);

step 3 – creating constriction of mapping f in $X - X_0$ (m^{n-k} ways).

So we can build our function in $C_m^k k! m^{n-k}$ ways.

We should give some comments. The concept of “function” is not taken into consideration in the school mathematics course, but it is implicitly present in it. So functional dependencies are presented in algebra and Introduction to Analysis, transformations are discussed in geometry. It is important for a teacher to feel that functions are directly related to combinatorics[9] (Yamnenko, 2009), and the language of functions can be used to define the main combinatorial compounds (placement, permutations, combinations with repetitions and without repetitions). The corresponding

[9] Yamnenko, P. (2009). *Dyskretna matematyka [Discrete Mathematics: study guide]*. Kiev, Ukraine: Publishing and Printing Center «Kiev University». [In Ukr.].

Video lesson title	Total views (162 participants)
Propedeutical line of mathematical analysis	330
Line function	368
Exponential	197
Power function	186
Arrangement of movements	126
Dilatation or homothetic transformation	311
Distance between passing lines	155
Stochastic content line	369
Inclusion-exclusion principle	253
Basic combinatorial compounds	215
Bernoulli distribution	175

Table 3.3.2 Frequency of views of mathematics video lectures.

tasks, exemplified above, make it possible for a teacher to become aware of the scientific and theoretical foundations of both functional and combinatorial content lines of the mathematics school course.

We have tested the components of distance learning. Before using the platform, diagnostic testing and introductory lectures (in-person form) are conducted, participants of advanced training are familiarized with questions and types of tests. Being engaged in distance learning activities, participants complete the required tests; they have the opportunity to work with offered lectures on elementary mathematics and teaching methods, as well as to pass training tests. Mathematics lectures are hosted on a video platform. During the year of testing, the system was used by 162 participants. The frequency of views of some video collections is shown in Table 3.3.2.

The distance learning participants were offered training tests that they could perform at will. Twenty training tests are distributed along the main content lines of the school mathematics course. The activity of participants and the topics of the tests are shown in Table 3.3.3.

As we can see, unfortunately, the activity in performing optional tasks is not high enough; it does not correlate with

Test title	Number of Views (162 participants)
Algebraic expression	75
Algebraic function	48
Vector coordinate method in planimetry, plane transformation	*30*
Vector coordinate method in stereometry, space transformation	*31*
Integral	35
Combinatorics, probability calculus, mathematical statistics	52
Polyhedrons	21
Inequality	70
Stereometry basics	23
Exponential and logarithmic function	34
Sequences	30
Derivative	32
Equations	77
Solid of revolution	27
Goniometric function	25
Trigonometric equations and inequality	45
Triangles	30
Functions. Basic properties of functions	29
Congruous numbers	84
Tetragon	28

Table 3.3.3 Retraining course participants' performance of selective training tests.

the percentage of tasks that participants successfully coped with during diagnostic testing.

When performing the tests, the participants show high enough scores (note that there is an opportunity to repeatedly pass the appropriate test). Summary information on performing the remote component test scores is shown in Table 3.3.4.

The total score for the course participants for work on the platform was given on the basis of the results of 6

The subject of the credit test	Grade point average (maximum 10)	Total number of hits to the test
Text Problems	9,28	6789
Stochastic	9,41	908
Algebra and Analysis	9,03	1584
Geometry	9,45	832

Table 3.3.4 Summary information on trainees' mathematics tests passing on distance component (162 participants).

tests with weighting coefficients (one general training test and one technical). Figure 3.3.4 shows information about participants' performance of the credit tests. Here we observe characteristic bimodality of the distribution, which indicates the heterogeneity of the groups of participants and the possible clustering, which could be a base for distributions that is given by this one. At the same time, high test performance and high frequency of testimonials testify to participants' desire to achieve high results, but there are serious problems in mathematical preparation among some teachers.

As we can see, there is a significant difference between the work activity with selective training tests and test scores. It is conditioned by the desire of a large number of specialists to demonstrate high performance, but low motivation for a real increase in the level of their own mathematical competence.

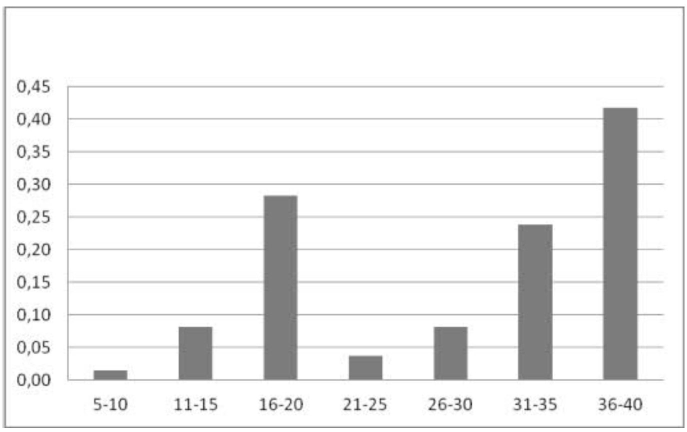

Figure 3.3.4 Bar chart of distribution of total performance points in the remote component (%, maximum score – 40).

Figure 3.3.5.a Performance of the test "Vector-coordinate method in planimetry" (maximum score – 10, 30 participants).

At the same time, there is a great potential for professionals, due to such training tests, to improve the relevant components of mathematical competence in a short time. As an example, let us consider the bar charts with the performance of tests "Vector-coordinate method in planimetry" and "Vector-coordinate method in stereometry" (Figure 3.3.5a, 5b).

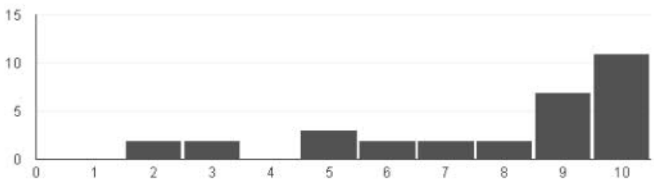

Figure 3.3.5.b Performance of the test "Vector-coordinate method in stereometry" (maximum score – 10, 31 participants).

Visual analysis of Figure 3.3.3 shows some increase in performance for the second test (done by the same participants except two of them). More detailed analysis using the sign test also shows an increase in performance. At the same time, it should be noted that the application of this statistical criterion can be considered correct due to the pseudorandomized sample of respondents. Note that the content of a significant number of test tasks a) and b) is identical.

Conclusions

In our research, based on systematic analysis, it is substantiated that the optimum form of postgraduate training courses for mathematics teachers is blended learning. The didactic goals made it possible to formulate the basic principles of construction of blended systems of teaching mathematics and to construct a structural model for such systems. Open principles, flexibility and adaptability of blended learning systems can be considered as the main principles. It has been researched that video lectures are preferred for mathematics lectures, and computer-assisted lectures are useful while delivering lectures in didactics. Testing of the blended learning system showed a sufficiently high activity among course participants while mastering theoretical material from the scientific-theoretical foundations of the school mathematics course, performing the required tests. A large number of teachers are not motivated enough to improve their ability to solve medium- and higher-level tasks, while at the same time, the distance education system can help to improve their skills.

A detailed and in-depth evaluation of the effectiveness of an open, adaptive, blended learning system requires longer observations in future studies.

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OUR AUTHORS

Akulenko Iryna – Doctor (habil.) of Science in Pedagogy, Full Professor, Department of Algebra and Calculus, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*. *e-mail*: akulenkoira@ukr.net

Bobyliev Dmytro – PhD in Pedagogy and Education, Associate Professor, Department of Mathematics and Mathematics Teaching Methods, Kryvyi Rih State Pedagogical University, *Ukraine*
e-mail: dmytrobobyliev@gmail.com

Bodnenko Tetyana – Doctor (habil.) of Science in Pedagogy, Associate Professor, Department of Automation and Computer-integrated Technologies, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: bod_t@ukr.net

Burda Mykhailo – Doctor (habil.) of Science in Pedagogy, Full Professor, Full Member of the NAES of Ukraine, Department of Mathematical and Computer Education, The Institute of Pedagogy of the National Academy of Pedagogical Sciences of Ukraine, *Ukraine*. *e-mail*: mibur5@ukr.net

Cernajeva Sarmite – Master of Education Science, Master of Mathematics, Lector Riga Technical University, *Latvia*
e-mail: sarmite.cernajeva@rtu.lv

Chernobai Olga – Ph. D. in Physical and Mathematical Sciences, Associate professor, Department of Higher Mathematics, University of State Fiscal Service of Ukraine, *Ukraine*. *e-mail*: chernobai.olga@gmail.com

Chumak Olena – PhD in Pedagogy and Education, Associate Professor, Donbas National Academy of Civil Engineering and Architecture, *Ukraine*
e-mail: chumakelena17@gmail.com

Kirman Vadym – PhD in Pedagogy and Education, Associate Professor, Dnipro Academy of Continuing Education, *Ukraine*
e-mail: vadym.kirman@gmail.com

Kondratyeva Oksana – PhD in Pedagogy and Education, Associate Professor, Department of Higher Mathematics, Cherkasy State Technological University, *Ukraine*
e-mail: kav@uch.net

Kravchenko Zoia – PhD in Pedagogy and Education, Associate Professor, Department of Natural and Mathematical Education, Kharkiv Academy of Continuous Education, *Ukraine*.
e-mail: zoyakrav@ukr.net

Krylova Tetyana – Doctor (habil.) of Science in Pedagogy, Full Professor, Professor of the Department of Higher Mathematics, Dniprovsk State Technical University, *Ukraine*.
e-mail: lebmax@ukr.net

Kulyk Lydmila – PhD in Pedagogy and Education, Associate Professor, Department of Physics, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: kulyk_l@mail.ru

Lovyanova Iryna — Doctor (habil.) of Science in Pedagogy, Associate Professor, Department of Mathematics and Methods of its Teaching, Kryvyi Rih State Pedagogical University, *Ukraine*
e-mail: lovira22@i.ua

Malova Iryna — Doctor (habil.) of Science in Pedagogy, Full Professor, Professor at the department of Mathematical Analysis, Algebra and Geometry Bryansk State Academician I.G. Petrovski University; Senior Researcher Southern Mathematical Institute of the Vladikavkaz Scientific Center of the Russian Academy of Sciences
e-mail: mira44@yandex.ru

Plysiuk Olena — Teacher-Methodist, Teacher of mathematics and economics, Municipal Institution "Ostroh Regional Lyceum-boarding School with an Intensive Military-physical Training", *Ukraine*
e-mail: alonarp5@gmail.com

Serdiuk Zoya — PhD in Pedagogy and Education, Associate Professor, Department of Mathematics and Methods of Teaching Mathematics, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: serdyuk_z@ukr.net

Sitak Iryna — PhD in Pedagogy and Education, Department of Mathematics and Computer Technologies, The Institute of Chemical Technologies (the town of Rubizhne) of the East Ukrainian Volodymyr Dahl National University, *Ukraine*
e-mail: sitaksrsna@gmail.com

Tarasenkova Nina — Doctor (habil.) of Science in Pedagogy, Full Professor, Department of Mathematics and Methods of Teaching Mathematics, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: ntaras7@ukr.net

Tinkova Darya — Postgraduate student, Department of Mathematics and Methods of Teaching Mathematics, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: tinkovads@gmail.com

Vlasenko Kateryna — Doctor (habil.) of Science in Pedagogy, Associate Professor, Professor of the Department of Higher Mathematics, The Donbas State Machine-Building Academy, *Ukraine*. *e-mail*: vlasenkokv@ukr.net

Volodko Inta — Doctor (habil.) of Mathematics, Full Professor, Riga Technical University, *Latvia*
e-mail: inta.volodko@rtu.lv

Yakovenko Anastasia — Postgraduate student, Department of Algebra and Calculus, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: borkovaa1@gmail.com

Zydkov Oleg — Postgraduate student, Department of Algebra and Calculus, Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*. *e-mail*: zhydkov63@gmail.com

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