COMPETENTIZATION AND MATHEMATICAL EDUCATION

MONOGRAPH

BUDAPEST

SCASPEE

COMPETENTIZATION

AND

MATHEMATICAL EDUCATION

Monograph

Edited by

prof. N. Tarasenkova

BUDAPEST

2021

Tarasenkova, N. (Eds). (2021). ***Competentization and mathematical education** : Monograph*. In L. Kyba (A. Ed.). Budapest, Hungary: SCASPEE.

The authors:
I. Akulenko, O. Bondar, O. Chernobai, N. Kornodudova, T. Krylova, I. Kulish, B. Lazarov, O. Matiash, H. Mikaelian, L. Mykhailenko, I. Nekoz, N. Orlova, S. Skvortsova, N. Tarasenkova, A. Varynska, I. Yakunina, & O. Zadorozhna.

Scholarly Editor: Dr. habil., Full Prof. N. Tarasenkova
Author's Editor: L. Kyba

ISBN 978-615-01-3972-2
SCASPEE, Budapest, 2021

DOI: http://doi.org/10.31174/SEND-MON004

This book will be of interest to all researchers in the field of didactics of mathematics.

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THE JOURNAL IS INDEXED IN:

INDEX COPERNICUS: ICV 2014: 70.95; ICV 2015: 80.87; ICV 2016: 73.35; ICV 2018: 90.25; ICV 2019: 89.50;

ICV 2020: 94.89

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DIRECTORY OF REEARCH JOURNAL INDEXING

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Cite this book:

APA Tarasenkova, N. (Eds). (2021). *Competentization and mathematical education: Monograph*. In L. Kyba (A. Ed.). Budapest, Hungary: SCASPEE.

SSU *Competentization and mathematical education*: monograph / I. Akulenko, O. Bondar, O. Chernobai, N. Kornodudova, T. Krylova, I. Kulish, B. Lazarov, O. Matiash, H. Mikaelian, L. Mykhailenko, I. Nekoz, N. Orlova, S. Skvortsova, N. Tarasenkova, A. Varynska, I. Yakunina, & O. Zadorozhna; Eds. prof. N. Tarasenkova, & L. Kyba. – Budapest : SCASPEE, 2021. – 126 p.

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PREFACE

The book documents and promotes the experiences of educators and is aimed at making headway on the problem of improving the quality of mathematical education in the context of competency-based approach. Tertiary and high school mathematical education should meet the expectations of the domestic stakeholders and answer the world standards. However, there is some controversy between the modern requirements of the society and the existing level of mathematical training. Besides, the current status of math training in Ukraine and other countries should be strengthened. And this determines the relevance of the research.

Meeting the fundamental challenge of improving the quality of mathematical education of pupils and students on the basis of studying and implementing the best European developments is possible under the conditions of solving a number of research tasks in such concenters:

- Learning Mathematics at Secondary & Vocational Schools (Chapter 1);
- Mathematical Training at the University (Chapter 2);
- Math Teacher Training in graduate and postgraduate education (Chapter 3).

Chapter 1 of the monograph focuses on : the master plan of a model mathematics curriculum for grades 5-6 of a New Ukrainian School; the methodology of incorporating didactical mapping into integrated approach; the content and learning specifications of finance and tax word problems in teaching mathematics at high school.

Chapter 2 of the monograph considers: the analysis of the pedagogical methods used in the educational process in technical universities; the realization of the applied orientation in teaching and learning higher mathematics in technical universities; integrated teaching of higher mathematics in technical universities.

Chapter 3 of the monograph brings to light: the problems of employing teacher moments as tools for fostering mathematic education students' teacher knowledge in geometry; the problems of incorporating the study of mathematics and a foreign language (MLIL) in math teacher's education and devising appropriate instructional materials.

It should be mentioned that the papers presented in the monograph do not exhaust all aspects of the problem. We hope to continue the discussion as its information field is extremely broad. We believe that every outcome of the studies (both – theoretical and experimental) allows of improving the training of specialists in math education and gradually raising the quality of math education to the society's today's demands.

Acknowledgements
The work was supported by the Ministry of Education and Science of Ukraine
(state registration number 0119U101735).

The monograph is published with the support of the Society for Cultural and Scientific Progress in Central and Eastern Europe, Budapest, 2021

CHAPTER 1

Learning Mathematics at Secondary & Vocational Schools

Master plan of a model mathematics curriculum (grades 5-6 of a new ukrainian school)

Svitlana Skvortsova, Nina Tarasenkova

Introduction

The reform of Ukraine's education system is conditioned by the state's European integration progress, efforts to adhere the world norms and rules of organization of the educational process and implement the best practices, including standardization of education in each of its links.

The current reform of general secondary education was launched in Ukraine in 2016. During this time, the Conceptual Principles for the Reform of Secondary Education “New Ukrainian School”[1] (2016), the Political Proposal for the Reform of General Secondary Education “New Ukrainian School”[2] (2016) were approved, the Laws of Ukraine “On Education”[3] (2017), “On Complete General Secondary Education”[4] (2020), Professional Standard for Teachers[5] (2020), the State Standard for Primary Education[6] (2018), the State Standard of Basic Secondary Education[7] (2020) were adopted.

In 2016, the implementation of the Concept of the New Ukrainian School in Primary Education began, and in 2021 the Concept is being implemented in basic secondary education. After the competitive selection and approval of model curricula, including mathematics curriculum for grades 5-6[8] (2021), the All-Ukrainian innovative educational project "Development and implementation of educational and methodological support for general secondary education in the context of the State Standard of Basic Secondary Education"[9] (2021) was launched. Within its framework, a new educational content of basic secondary education is being developed, particularly in mathematics.

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[1] The Concept of the New Ukrainian School. (2016) : Retrieved from: http://mon.gov.ua/activity/ education/zagalna-serednya/ ua-sch-2016/ konczepcziya.html [In Ukr.].

[2] Order of the Cabinet of Ministers of Ukraine of 14.12.2016 № 988-r "On approval of the Concept of Iimplementation of State Policy in the Field of Reforming General Secondary Education “New Ukrainian School” for the period up to 2029". (2016): Retrieved from: https://zakon.rada.gov.ua/laws/show/988-2016-%D1%80#Text [In Ukr.].

[3] Law of Ukraine “On Education”. (2017): Retrieved from: https://zakon.rada.gov.ua/laws/show/2145-19#Text [In Ukr.].

[4] Law of Ukraine “On Complete General Secondary Education”. (2020) : Retrieved from: https://zakon.rada.gov.ua/laws/show/463-20#Text [In Ukr.].

[5] Professional standards by professions "Primary school teacher of general secondary education", "Teacher of general secondary education", "Primary education teacher (with a diploma of junior specialist)".(2020) : Retrieved from: https://zakon.rada.gov.ua/rada/show/v2736915-20#Text [In Ukr.].

[6] State Standard for Primary Education. (2018) : Retrieved from: https://zakon.rada.gov.ua/laws/show/87-2018-%D0%BF#Text [In Ukr.].

[7] State Standard of Basic Secondary Education. (2020) : Retrieved from: https://zakon.rada.gov.ua/laws/show/898-2020-%D0%BF#Text [In Ukr.].

[8] Order of the Ministry of Education and Science of 12.07.2021 №795. (2021): Retrieved from: https://imzo.gov.ua/osvita/nush/vseukrains-kyy-eksperyment-nush-5-9-kl/ [In Ukr.].

[9] All-Ukrainian Experiment “New Ukrainian School”, grades 5 – 9 (2021) : Retrieved from: https://imzo.gov.ua/osvita/nush/vseukrains-kyy-eksperyment-nush-5-9-kl/ [In Ukr.].

This paper presents the author's vision of the conceptual foundations of the mathematics course for the adaptive cycle of basic secondary education (grades 5-6), its content, structure and features of deployment based on the author's model curriculum[10].

Argument

The mathematics course of the first cycle of basic secondary education is an important component of teaching, development and upbringing of students in the 5th-6th grades, which is a logical continuation and development of the mathematics course of primary education and the basis for teaching mathematics in the second cycle of basic secondary education.

Within the system of lifelong education this course is based on the content and compulsory / expected results of the mathematical education field of the State Standard of Primary Education and the Standard Educational Programs for Primary School.

The model curriculum is developed on the basis of the State Standard of Basic Secondary Education (further - SS) and the Standard educational program (further – SEP) general secondary education institutions(grades 5-9) and is aimed at achieving **the goal of mathematics education**, namely – at the development of the student's personality through the formation of mathematical competence in relation to other key competencies for successful educational and further professional activity, which implies:

- mastering the system of knowledge,
- improving the ability to solve mathematical and practical problems;
- developing logical thinking and mental properties;
- understanding the use of mathematics in private and public life.

Teaching mathematics in the first cycle of elementary school performs a number of important tasks of individual development, including:

- building the ability to explore challenging situations and identify problems that can be solved using mathematical methods;
- modeling processes and situations, developing strategies and action plans for solving problems;
- critical evaluation of problem-solving processes and results;
- developing mathematical thinking for cognition and transformation of reality and mastering the language of math.

Teaching mathematics in grades 5-6 ensures the formation and development of students' key competencies (Appendix 7 to the SS) and their common cross-cutting skills. It is obvious that this process occurs in the course of mastering the content and

[10] A model curriculum "Mathematics. Grades 5-6" for institutions of general secondary education (Authors – S. Skvortsova, N. Tarasenkova). (2021) : Retrieved from: https://drive.google.com/file/d/1ykOgcS2OiQbBxXAfxFoW-SxykuwZMlFm/view

achieving the expected learning outcomes, determined by the model curriculum, by means of training assignments offered in the textbooks created on the basis of this program.

The author's program is built on the scientific principles, principles of consistency, systematicity, accessibility, the connection between learning and life, integrativeness and continuity in teaching mathematics in primary school and in the first cycle of basic school, prospects — focus of the content and the results to be got at the first cycle of basic education, on mastering mathematics in the second cycle of basic school.

The principles of scientificity, systematicity and consistency are realized through the definition of the content of education — mathematical concepts, facts, methods of activity, which are consistent with conceptual framework and terminology commonly used in mathematical science. The logic of presenting mathematical content unfolds from simple to complex and provides at each subsequent stage of learning an increase in student competence by expanding the set of numbers (natural numbers with zero, common fractions with the same denominators, decimal fractions, and rational numbers) and transfer and reconstruction of known mathematical concepts, facts, properties, and methods of action in new environments. Equally important for the development of mathematical competence of the student is the gradual transition to a higher level of abstractness of content, forms of its fixation and methods of processing, which corresponds to natural changes in the intellectual sphere of students of this age, including the transition from visual to abstract thinking. The program provides for the generalization and systematization of what was learned at the previous stage of learning and reviewing of what has been learned at the end of the current school year, this ensures that learners' knowledge, abilities, and skills are integrated into the system. Connection with life and integration with other educational areas is carried out as a result of the inclusion of the following issues important for the life of a modern person in the content of education: analysis of data from schemes, tables, diagrams, the concept of arithmetic mean, percentage calculations, and mathematical modeling. The program pays sufficient attention to the formation of students' computing skills (which are classified by the European Community as key competencies of the XXI century).

The mathematics course of the first cycle of basic education is integrated in its structure and includes the arithmetic of non-negative integers / common fractions / decimal fractions / rational numbers, as well as algebraic and geometric propaedeutics, functional propaedeutics, and data analysis. In this regard, the following cross-lines are specified in the model program:

1. Numerical systems.
2. Propaedeutics of studying functions.
3. Expressions, equality and inequality.
4. Mathematical modeling.
5. Geometric shapes. Geometric quantities.
6. Data analysis.

Since the issues of numbering and arithmetic operations with numbers and quantities are a priority in grades 5-6, the content line "Numerical Systems" is system-forming. The formation of the concept of number is based on the experience (gained in primary school) of applying the knowledge of numbering within a million, computational skills of arithmetic operations of addition, subtraction, multiplication and division with non-negative integers (oral and written techniques); on the understanding of a common fraction as one or more equal parts of a whole. In primary school, the concept of number was introduced as a quantitative characteristic of equivalent sets and as a result of measuring the value, so the main quantities (length, mass, capacity, and time) were considered in close connection with the formation of the concept of number, the study of arithmetic operations with numbers. This created a propaedeutic basis for building models of the world around us and realized the connection of mathematics with other sciences.

The purpose of the content line "Numerical systems" is the formation and development of students' concept of number (natural number, number zero, and fraction (with the same and different denominators, decimal), integer (positive, negative), rational), the development of the concept of basic quantities, and the formation of computational skills, including computational skills for actions with named numbers. This line expands the possibilities for performing arithmetic operations not only in individual numerical sets, but also through the introduction of the action of exponentiation; it contains issues of numbering (oral and written), comparison of numbers; arithmetic operations with numbers; basic quantities - length, mass, time, and arithmetic operations with named numbers; percentages and proportions, which provides a block of basic knowledge "Numbers and expressions", which is put in Annex 7 to the SS.

Expansion of the set of numbers ranges from the study of non-negative integers (natural numbers and zero) in the 5th grade, common and decimal fractions in grades 5-6, to integers (positive and negative) and rational numbers in 6th grade. Within each numerical set, students perform arithmetic operations of addition, subtraction, multiplication and division, exponentiation (square and cube). In the 5th grade the arithmetic operations of addition, subtraction, multiplication and division with non-negative integers and decimal fractions, addition and subtraction of common fractions with the same denominators are studied. Grade 6 studies the addition and subtraction of common fractions with different denominators, the multiplication and division of common fractions, as well as the addition and subtraction, multiplication and division of integers, which is further generalized to rational numbers. It should be noted that for the 5th grade students a new action is the rounding of natural numbers and decimal fractions to a certain level, which is the basis for further study of approximate calculations.

In the 5th grade, having the primary school experience of understanding a common fraction as one or several equal parts of a whole, experience of comparison fractions with the same denominators, and the experience of finding a fraction of a number and a number after its fraction, students are brought to the highest level of generalization and gain an understanding of the common fraction as a fraction of two natural numbers. A new educational content for fifth-graders is the concept of decimal fraction, comparison of decimal fractions, and arithmetic operations with decimal fractions. At the same time, the rules for comparing common fractions with the same denominators, the rules for finding the fraction of a number and the number by its fraction remain in the form in which they were introduced in primary school. Only in the 6th grade, after learning multiplication and division by a fraction, the rules of finding a fraction from a number and a number by its fraction find their development.

In the 5th grade, percentages are introduced and students find the percentage of a number and the number by its percentage based on the rules of finding the fraction of the number and the number by its fraction. In the 6th grade, taking into account the generalized rules of finding a fraction of a number and a number by its fraction, finding the percentage of a number and a number by its percentage acquires a new meaning and percentage calculations are considered. The introduction of relationships and proportions in 6th grade not only expands the range of mathematical concepts, but also opens up new opportunities for students to learn new ways to solve problems that arise in the environment, through mathematics.

The content line "Numerical systems" is aimed at the formation of such students' skills as(Appendix 8 to the SS): distinguishing and differentiating among those problem situations that are solved by mathematical methods; analyzing data, describing the relationships, presenting data in various forms; determining what exactly may be the result of solving the problem situation; selecting, arranging, recording, converting audio, text, graphic information of mathematical content, in particular — in the digital environment; using mathematical concepts, facts and the suggested sequence of actions to solve problem situations; knowledge of mathematical terms and symbols and applying them properly; performing operations with mathematical objects and using various forms of information presentation; using the necessary accessories and ICT; formulating and reflecting in a user-friendly form the results of solving the problem situation, in particular with the use of ICT; presenting the results of solving the problem situation; evaluating different ways of solving a problem situation; connecting different elements of mathematical knowledge and skills, drawing conclusions, supporting the opinion with arguments; expressing ideas meaningfully, accurately, and concisely.

In the process of studying numbering the number is correlated with a pointt that is marked on a number ray (5th grade) or coordinate line (6th grade), then the location of the point on the plane is characterized by two coordinates, and it is depicted on the coordinate plane (6th grade). Knowledge of the arithmetic operations of multiplication and division, exponentiation is used in the study of processes and situations that take place around us, when creating their mathematical models. Students apply knowledge about groups of interrelated quantities and write these dependencies in the form of a formula, fill in the tables (grade 5). Then, among the dependencies between the quantities, direct and inverse proportional dependencies are singled out, and students learn to build a dependency graph (6th grade). Thus, the content line "Propaedeutics of studying functions" is actualized, the purpose of this line is to prepare students of 5-6 grades for studying the course of mathematics in the second cycle of basic school. This content line implements the basic knowledge block "Functions" of Annex 7 to the SS and is based on that experience of primary school graduates that embraces understanding the dependence of the result of arithmetic operation on the change of one of the components, to recognize situations that can be described by groups of interrelated quantities: mass of one object, quantity of objects, total mass; length of one, quantity, total length; capacity of one container, number of containers, total capacity; price, quantity, cost; labor productivity, working time, total output; speed, time of movement, distance traveled - and knowledge of the relationships between the quantities of one group and understanding the dependence of the change of one quantity on the change of another quantity with a constant third quantity.

The content line "Propaedeutics of the study of functions" contributes to the achievement of specific results defined in Annex 8 to the SS, namely: the differentiating among the problem situations of those that are solved by mathematical methods; data analysis, describing data relationships, presenting data in various forms; selection, ordering, recording, conversion of audio, text, and graphic information of mathematical content, in particular - in the digital environment; conversion, presentation and dissemination of information of mathematical content using various means, including digital; determining the components of the mathematical model of the problem situation and the relationships between them; building a mathematical model of the problem situation using a certain mathematical apparatus; constructing a mathematical model in a standard situation; performing operations with mathematical objects and using various forms of information presentation; linking different elements of mathematical knowledge and skills, formulating conclusions with the support of opinions with arguments; using mathematical concepts, facts and the proposed sequence of actions to solve problem situations; possession of mathematical terms and symbols and their appropriate use; ability to express ideas in a meaningful, accurate, and concise manner.

Finding the values of numerical expressions involves both compliance with the order of actions and the transformation of expressions on the basis of laws and rules of arithmetic operations, expanding parentheses, putting the common factor outside the parentheses, and reduction of similar terms. The logic of the development of the content of education involves the transfer of knowledge and methods of action on letter expressions. Thus, the following content line "Expressions, Equalities, and Inequalities" contains algebraic items: expressions (numerical and alphabetical), equalities (numerical, alphabetical, equations, formulas, and proportions), inequalities (numerical and alphabetical). The purpose of the content line "Expressions, Equalities, and Inequalities" is to prepare graduates of the first cycle of basic education to study a systematic course of algebra and provide a block of basic knowledge "Equations and Inequalities" (Annex 7 to the SS).

It should be noted that algebraic propaedeutics was started in primary school - students are already able to distinguish between numerical and alphabetical expressions; read and write both simple expressions and expressions for several actions, including parentheses; find the meaning of numerical and alphabetic expressions, following the order of actions, as well as performing simpler transformations – on the basis of laws and rules of arithmetic operations and the definition of multiplication. In grades 5-6, students continue to work with numerical and alphabetical expressions on the same principles that were learned in elementary school. The increase in competence is associated with the development of the line of transformation of expressions through the transition from the distributive law of multiplication with respect to the addition, and to factoring out the common factor, and then the reduction of similar terms.

In elementary school, equations were singled out among numerical equalities; students solved both simple equations and complex structure equations (those where the right part is represented by a numerical expression, one of the components is a numerical or alphabetical expression) mainly in a way based on the rules of finding the unknown component of the arithmetic operation. The students also got acquainted with the method of selection and, possibly, with the method that involves the application of the properties of equalities (additional questions in the SEP for primary school, produced under the guidance of O. Savchenko). The types of equations considered in primary school continue to be studied in grades 5-6, the increase in competence in grade 5 is provided by equations that involve actions with common and decimal fractions, conversion of numeric and alphabetical expressions, including finding the sum / difference of terms with the unknown, and in grade 6- equations in which you want to find the unknown term of the proportion, to reduce similar terms, and equations that contain the unknown in both parts. In the 5th grade, the methods of solving equations studied in primary school are used on broader contents, and in the 6th grade, the equations are being solved based on the basic properties of the equations.

Primary school graduates have sufficient experience in comparing numerical expressions and finding individual solutions of alphabetical inequalities. Thus, some solutions of alphabetical inequalities were found by the method of selection, the method of reduction to the equation and, possibly (additional questions of the program), on the basis of the dependence of the result of arithmetic operation on the change of one of the components at constant third. In grades 5–6, the line of inequalities continues, its development occurs due to the introduction of double inequalities and strict and non-strict inequalities in grade 5.

As a result of mastering the content line "Expressions, Equalities and Inequalities" it is planned to achieve specific results defined in Annex 8 to the SS: students connect various elements of mathematical knowledge and skills, draw conclusions, support their opinion with arguments; use mathematical concepts, facts and the suggested sequence of actions to solve problem situations; identify the components of the mathematical model of the problem situation and the relationship between them; choose a mathematical model to the standard situation; select ways and develop an action plan needed to solve the problem situation; perform operations with mathematical objects and employ various forms of information representation; seek alternative ways to solve the problem situation; evaluate different ways to solve a problem situation; know mathematical terms and symbols, and use them expediently; express their ideas meaningfully, accurately, and concisely.

This creates a mathematical basis for the study of situations and the identification of problems that can be solved using mathematical methods for modeling processes and situations, developing strategies, action plans for solving problem situations, critical evaluation of the process and the result of solving problem situations , which is the purpose of the content line "Mathematical modeling". This content line includes "symbolic models of problem situations" (L. Friedman) — word math problems: simple and composite, including typical; and involves the use of arithmetic and algebraic methods for solving problems.

The content line "Mathematical modeling" represents the development of the content line "Mathematical problems and research" according to the SEP for primary school, produced under the guidance of O. Savchenko. Traditionally, a certain part of the course of elementary school mathematics is formed by word math problems, which are an independent element of the content of education. Elementary school graduates are familiar with both simple problems that can be answered with a single arithmetic operation, and with complex problems that cannot be answered with a single arithmetic operation. Among the compiled problems, typical problems that contain the same value for two cases were identified: problems that set the task to find the fourth proportional, to perform a proportional division, to find the unknowns by two differences, to perform a double reduction to one; on processes: on joint work, and on movement. Elementary school students mostly solved problems by an arithmetic method, although they also got acquainted with the algebraic method of solving problems — making equations.

In primary school, the purpose of teaching how to solve story-based mathematical problems is to form the students' general ability to solve problems: to analyze the formulation of the problem, present its results in the form of an auxiliary model (short record and / or schematic drawing), find the ways solve the problem from the question to the numerical data (analysis) or from the numerical data to the question of the problem (synthesis), make a plan for solving the problem, write down the solution of the problem by actions with explanation and expression, and answer the questions asked in the problem. After the solution was found, the problem was still worked with (the solution of the problem was checked, the problem was researched into by changing the required when compiling and solving inverse problems, and it was also researched by transforming the problem itself). In grades 5-6, students continue to solve these types of problems; the development of competence is due to the complexity of the problems of proportional division, and in the 6th grade, these problems, as well as the problem of finding the fourth proportional, are solved by adding proportions. The increase in competence also takes place due to the introduction of new types of tasks: percent (the 5th grade) and percent calculations in the 6th grade. In grades 5-6, problems are solved by both arithmetic and algebraic methods.

The content line "Mathematical modeling" is aimed at the formation of students' abilities (Appendix 8 to the SS): to distinguish among problem situations those that are solved by mathematical methods; investigate the problem situation, obtain data, verify the accuracy of data; analyze data, describe the relationships between them, present data in various forms; select the data needed to solve the problem situation; determine what exactly may be the result of solving the problem situation; choose ways and develop an action plan needed to solve the problem situation; assess the need for and sufficiency of data and determine the insufficiency or redundancy of data to solve the problem situation; choose a mathematical model for a standard situation; identify the components of the mathematical model of the problem situation and the relationship between them; build a mathematical model of the problem situation, using a certain mathematical apparatus; look for alternative ways to solve the problem situation; evaluate different ways to solve the problem situation. The content line "Mathematical modeling" ensures the achievement of the overall result "Development of mathematical thinking for cognition and transformation of reality, knowledge of mathematical language", which is defined in Annex 8 to the SS. Word math problems are an effective means of developing students' skills to: identify and describe the relationships between math objects and real-world objects; use mathematical concepts, facts and the proposed sequence of actions to solve problem situations.

Geometric propaedeutics is provided by the content line "Geometric figures. Geometric quantities ". The purpose of this content line is to develop students' spatial representations, the ability to observe, compare, generalize and abstract; to nurture students' practical skills to build, draw, model and construct geometric figures by hand and with the help of simple drawing tools; to prepare the students to the study of a systematic course in geometry. This content line implements the block of basic knowledge "Geometry and measurement of geometric quantities", which is given in Annex 7 to the SS.

In the beginner math courses, students formed ideas and concepts about geometric shapes on a plane, their features and properties (point, line, ray, segment, curve, polyline, polygons, including rectangle and square, circle and circumference), and were offered only the definitions of rectangle and square, formulated through the nearest genus and species differences. Students have already learned to recognize geometric shapes in space (cube, rectangular parallelepiped, pyramid, cone, cylinder, sphere), to compare geometric shapes with the articles of the environment. The educational activity was connected with measuring and calculating geometrical quantities (length of a segment, perimeter of a polygon) (formula of perimeter of a rectangle and a square), the area of a rectangle and a square), solving of problems of geometrical maintenance (on calculation of length) and inverted to them. The development of this line in the first cycle of basic education takes place by considering the degree of angle, measuring angles with a protractor, properties of measuring segments and angles, the concept of complementary rays, classifications of triangles by sides and angles, consideration of triangle inequality, properties of rectangular parallelepiped and cube. a rectangular parallelepiped and a cube in the 5th grade; circumference and its elements, the area of a circle, a circular sector, the concept of parallel and perpendicular lines in the 6th grade.

The content line "Geometric shapes. Geometric quantities” ensures the achievement of concrete results (Annex 8 to the SS): distinguishes among the problem situations those that are solved by mathematical methods; singles out similar situations; investigates the problem situation, receives data, checks their reliability; analyzes data, describes the relationships between them, presents data in various forms; selects the data needed to solve the problem situation; determines what could be the result of its solution; selects, organizes, records, converts audio, text, graphic information of mathematical content, in particular in the digital environment; selects ways and develops an action plan needed to solve the problem situation; builds a mathematical model of the problem situation, using a certain mathematical apparatus; evaluates different ways of solving a problem situation; identifies and describes the relationships between mathematical objects and real-world objects; connects various elements of mathematical knowledge and skills, draws conclusions, supports his opinion with arguments; uses the necessary accessories and information and communication technologies; owns mathematical terms and symbols, expediently uses them; is expressed meaningfully, accurately, and concisely.

The connection of the mathematics course of the first cycle of basic education with life and the integration of its content with other educational branches is realized through the content lines "Propaedeutics of studying functions", "Mathematical modeling", "Geometric figures". Geometric quantities ", as well as through the content line" Data Analysis ". The purpose of this content line is to acquire the simplest skills of working with ways of presenting information, analysis of information contained in tables, diagrams, charts, graphs, the formation of skills to use data to solve practical problems. The content line "Data Analysis" implements a similar block of basic knowledge "Data, Statistics and Probability" from Annex 7 to the SS. By means of this content line the semantic line "Work with data", which is a component of mathematical educational branch according to the State standard of primary general education, isdeveloped . In elementary school, students learned about different ways of presenting data — in the form of diagrams, tables, bar charts, they learned to read information from them. In the 5th grade the development of competence is done by introducing the concepts of "data table", "scale" in the context of analyzing data tables, diagrams, by finding the mean. In the 6th grade pie charts are introduced, the semantic basis of students' analytical activity is expanded by involving rational numbers.

The content line "Data Analysis" ensures the achievement of specific results defined in Annex 8 to the SS: it distinguishes among the problem situations that are solved by mathematical methods; investigates the problem situation, receives data, checks the accuracy of data; analyzes data, describes the relationships between them, presents data in various forms; selects the data needed to solve the problem situation; selects, organizes, records, converts audio, text, graphic information of mathematical content, in particular in the digital environment; converts, presents and disseminates information of mathematical content using various means, including digital; selects ways and develops an action plan needed to solve the problem situation; builds a mathematical model of the problem situation, using a certain mathematical apparatus; formulates and reflects in a user-friendly form the results of solving the problem situation, in particular with the use of information and communication technologies; presents the results of solving the problem situation, explains their application; evaluates different ways to solve a problem situation; possesses the knowledge of mathematical terms and symbols and uses them expediently; expressed ideas meaningfully, accurately, and concisely.

Thus, the content lines "Numerical systems", "Propaedeutics of studying functions", "Expressions, equalitiess and inequalities", "Mathematical modeling", "Geometric figures". Geometric quantities", "Data analysis" fully implement the blocks of basic knowledge, which are presented in Annex 7 and ensure the achievement of specific results in accordance with Annex 8 to the State standard of basic general education. These content lines are implemented in the sections according to which the program for grades 5-6 is structured.

The 5th grade program contains the following sections:

Section I. Generalization and systematization of what was studied in primary school.

Section II. Natural numbers.

Section III. Ordinary fractions.

Section IV. Decimal fractions.

Section V. Percentages. Arithmetic mean.

Section VI. General Reviewing.

The 6th grade program is structured in the following sections:

Section I. Generalization and systematization of what was studied in 5th grade.

Section II. Divisibility of natural numbers.

Section III. Ordinary fractions and actions with them.

Section IV. Ratios and proportions.

Section V. Rational numbers and actions with them.

Section VI. Reviewing.

Within each section, the sequence of content development and the logic of achieving the expected results is determined in the topics of textbooks.

The program specifies the content of educational material, types of educational activities for each grade and presents the relevant expected learning outcomes. The scope of educational material defined in the program is necessary and sufficient for the formation of students' subject mathematical competence and key competencies, as well as readiness to study mathematics at the next level of education – in the second cycle of basic general education.

On incorporating didactical mapping into integrated approach*

Borislav Lazarov

Introduction

Pursuing the goal of enhancing the effectiveness of teaching-learning process, the new Bulgarian educational legislation entrusts the innovative schools with the task of developing the curriculum which would introduce new didactical approaches (MES, 2015)[1]. Among them are project-oriented initiatives for upgrading the subject-oriented curricula through topic-oriented didactical scenarios in which the integrated approach is crucial (Lazarov, 2018)[2]. The integrated approach supposes that any included subject in a topic-oriented didactic unit (TODU) comes with its own methodology and the subject teachers prepare the separate parts of the TODU scenario at their own discretion, having in mind some specific educational goals. Development of a concept map materially facilitates the analytical constituents of the ingredients of the topic under consideration (Giouvanakis et al., 2016)[3]. Routine procedures are usually applied at this stage, though rethought in a new context. However, at the synthesis level, in most cases, there are certain problems with the multifaceted presentation of the topic. Then it is necessary to concert the concepts and to merge analytical conceptual maps. Such a technical procedure has its ‘didactic price’, which will be discussed below.

Theoretical base

A concept map matrix is shown in Figure 1. It is based on (Kitano et al., 2008)[4] and represents a modification of the Model of a Concept Map introduced by Novak and Gowin (1984)[5]. Here the concepts are accommodated in several hierarchic levels and a concept of higher level correlates with some concepts of lower levels (higher means a smaller level number).

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[1] Ministry of Education and Science. (2015). Regulation No 5 of 30.11.2015 for the general education. Darzhaven Vestnik, I. 95 of 08.12.2015. [in Bulgarian]

[2] Lazarov B. (2018). Topic-oriented Upgrade of Subject-oriented Educational System. In Tarasenkova, N. (Eds). Current Status and Prospects of Mathematical Education: Monograph. SCASPEE, Budapest, Hungary. pp 37-53.

[3] Giouvanakis, T., Evaggelos, K., Mpakavos, A. & Samaras, H. (2016). Creative thinking through concept mapping. International Journal for Cross-Disciplinary Subjects in Education, Volume 7, Issue 1. pp 2705-2711.

[4] Kitano, M., Montgomery, D., Vantassel-Baska, J. & Johnsen, S. (2008). Using the National Gifted Standards for PreK-12 Professional Development. NAGC&CEC-TAG. Corwin Press, Thousand Oaks, California, p 124.

5 Novak, J. & Gowin, D. (1984). Learning how to learn. Cambridge University Press, Cambridge, UK, pp 15-54.

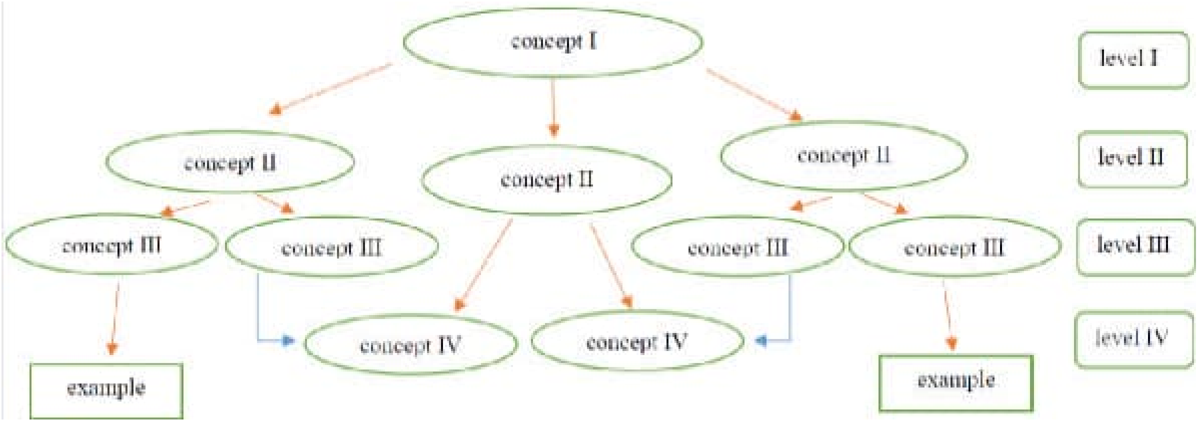

Figure 1 Concept map matrix.

The relationships between the concepts can be of two types: links (straight arrows in the figure) or crosslinks (broken arrows). The score of this particular map is calculated as

$$11 \cdot 1 + 4 \cdot 5 + 2 \cdot 10 + 2 \cdot 1 = 53.$$

This score corresponds to 11 links $(11 \cdot 1)$, 4 levels $(4 \cdot 5)$, 2 crosslinks $(2 \cdot 10)$, and 2 examples $(2 \cdot 1)$ (Kitano et al., 2008)[6]. The values of the entities are commensurate with that of one link. In our opinion, the scoring formula is incomplete. It does not take into account the role of the concepts themselves. Simply expanding the formula with the addend (number-times-value) does not read the contribution of concepts from given level to the complexity of the map. The value 10 for a crosslink is also overestimated.

The relationships in Figure 1 constitute an ontology and the classical Aristotle's' genus-species hierarchy is easily recognizable in it (Aristotle, IV Century BC)[7]. A *similarity index* could be appropriated to any two concepts in such ontology. Giouvanakis and colleagues (2016) give the definition via an example:

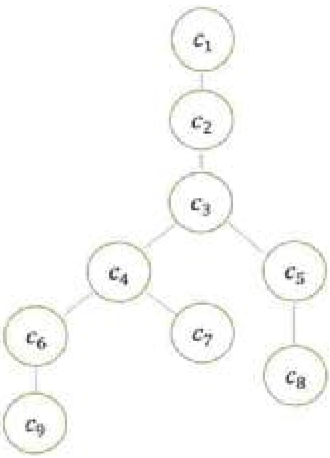

Figure 2 Ontology graph-tree.

[6] Kitano, M., Montgomery, D., Vantassel-Baska, J. & Johnsen, S. (2008). Using the National Gifted Standards for PreK-12 Professional Development. NAGC&CEC-TAG. Corwin Press, Thousand Oaks, California, p 124.

[7] Aristotle (IV Century BC). (1978). Topica, Chapter 5. Russian edition: Aristotle, Works, vol.2, chpt. 5, Myslj, Moskva, p 354.

Following the notations in Figure 2, let:

- d_1 be the number of the links from the concepts $c_{6,7}$ to the common concept c_4;
- d_2 be the number of the links from c_7 to c_4;
- d_3 be the number of the links from c_4 to the genus concept c_2.

The similarity index for the concepts c_6 and c_7 is defined by the formula

$$\mathrm{sim}(c_6, c_7) = \frac{2d_3}{2d_3 + d_1 + d_2} .$$

On the integrated approach in Bulgarian secondary school

Our standing point on the implementation of integrated approaches in secondary school is described more comprehensive in (Lazarov, 2018)[8]. In this section, we highlight the most important moments. The fundamental idea is to amalgamate two (or more) methodical units from different subjects in one common methodical unit keeping the relative independence of any subject teaching methodology. We speak for relatively independent inclusion of the methodology because the analytical knowledge developed by any methodology is expected to be synthesized in the frame of the common unit. The synthesis supposes transferability and multifunctionality of the complex *knowledge-skills-attitude* (KSA) into another context that differs from the ones, in which the analytical KSAs were developed. Therefore, a kind of synchronization of the subject methodic should be orchestrated and we have designed a special instrument for this purpose calling it *processing matrix*.

The processing matrix is a didactic tool that makes the implementation of the integrated approach via TODU more technological (ibid.). It includes:

(1) definitions of the objectives, indicators of progress, and benchmarks;
(2) selection of educational content to be examined;
(3) schedule and methodic for didactical support of the students;
(4) methodic for data collecting and processing the data collected;
(5) template for presenting information and frame for presentation design;
(6) assessment system;
(7) resources provided to the students and organizing a school conference;
(8) analysis of the outcomes of the initiative and drawing conclusions.

The processing matrix facilitates reconsidering the KSA developed in the traditional curriculum topics through the perspective of their potential application in a new context. Its proper design allows obtaining synergetic effect from the TODU, i.e. the resulting effect of the integrated teaching-learning process to exceed the effect of the expected one of the integrated subjects if they act separately (ibid.). However, amalgamating methodic of different curriculum subjects is tricky business and some

[8] Lazarov B. (2018). Topic-oriented Upgrade of Subject-oriented Educational System. In Tarasenkova, N. (Eds). Current Status and Prospects of Mathematical Education: Monograph. SCASPEE, Budapest, Hungary. pp 37-53.

antinomies arise especially in points (2), (3) and (6) of the processing matrix. Resolving them could help a proper concept mapping.

Tuning the concept map matrix

Basically, the structure of the concept map matrix in Figure 1 could be applied to a TODU for simple synchronization of concepts. However, a teaching methodology operates with more complex entities and relations than subject concepts and links between them. This is why we are going to conform the general concept map matrix with the purposes of the integrated approach. We call didactic map (DM) an analogue of the concept map for TODU with the modifications as follows.

First, we count the levels down up, i.e. the DM is supposed to be designed from the bottom to the top (a bigger level number means a higher level as in Figure 3). Next, let us define the nodes of the DM as *atoms*, which stand for 'atomic didactic components of the subject methodic'. An atom could be a theorem, an ethic category, a historical fact etc. We redefine the edges of the concept map as *beams*. This is to emphasize the multicomponent type of the relations between the atoms. The reformulation is not just a technical act but also a meaningful reconsideration of the didactical process of concept mapping. E.g. (see Section 7), a beam that links the atoms (Hristo Botev) (FP, 2019)[9] and (Bulgarian National Revival) in a DM could contain the knowledge in different levels about: the role of the hero in several stages of the revolutionary movement, some momentous of the historical period, his education etc. Any beam component reflects some specifics of the educational context in which the particular TODU takes place. In a DM, a beam can be either *direct* (d-beam), or *crossing* (c- beam).

We speak about *didactical mapping* when designing a DM related to integrated approach implementation into a TODU. Let us list several areas from a topic-oriented educational process, in which the didactical mapping is potentially useful. It:

- illustrates the hierarchy and relationships between different elements of a topic;
- facilitates the perception and memorization of key elements (bases and beams) as fewer characters achieve greater information saturation;
- allows easy modification and restructuring in the already constructed map fragments;
- allows for ideas to be shared, formalized quickly and accurately, and correctly displayed when preparing documentation;
- helps to clarify the logical consistency of the topic structure in the separate elements;
- allows of evaluating more precisely the complexity of a topic, defining its complex structure, decomposing separate elements in it.

Developing the theory

We are going to technologize the advantages listed in section 4 by introducing two quantitative attributes related to the new atom-beam structure of the concept map

[9] https://www.thefamouspeople.com/profiles/hristo-botev-5649.php (Access Date: 26 March, 2021).

matrix: distance and weight. We consider the educational objectives of a TODU in relation with some components of a *synthetic competence* that is developed via integrated approach (Lazarov, 2013)[10]. Thus, the attributes of a beam should reflect the observed indicators for the progress of these components of the synthetic competence. E.g., let the qualitative indicators Ik are the attributes of a d-beam, $k = 1,2,\dots,5$. Let the benchmark i_k is assigned to Ik for the transition from the origin atom to the beam-connected one. We take the value $J = \sum_{k=1}^{5} i_k$ as quantitative measure of the transition calling it *beam-value*. We take the beam-value instead of the corresponding link from the definition of the similarity index sim. When stating the indicators, it is important to consider the direction of the educational process. This means that any indicator reflects some elements of the progress in the transition from an atom to the succeeding one, thus the beams should be thought as vectors.

1. Distance in DM

We are going to invert the *ontological similarity* taking into account the structure of the concept map matrix in Figure 1 but with down-up counting of the levels. Let C' and C'' be two atoms in the first (bottom) level both connected with an atom C^* (it could be also C' or C'') and having a genus root C (it could be C^*); let δ be the level number of C and let d' and d'' equal the sums of the beam-values J for the beams connecting C^* with C' and C'' respectively.

1.1. Suppose the DM is an ontology graph-tree, i.e. there are no c-beams. In this case, we define *ontological distance* as:

$$\boldsymbol{D_C(C', C'') = \frac{d' + d''}{\delta}} \;\; \& \; \forall C: (\boldsymbol{D_C(C', C'') = 0 \;\Leftrightarrow\; C' \equiv C''}) \,.$$

Indeed, the values of sim are fractions (decimals) between 0 and 1, which makes difficult to feel the similarity. On the contrary, the ontological distance shows quite clear the relation between atoms. Here it is a kind of motivation to introduce the concept of ontological distance:

- the root C (the topic under consideration) is in the area of knowledge for which the concept map is intended;
- a hierarchical chain of atoms, which are common for C' and C'', is formed up from C^* to C;
- down the atom C^* there is a split into two different subject branches, hierarchically arranged to any of the atoms C' and C'';
- the farther away C' and C'' are from C^*, the larger the distance;
- the higher the root C in the map, the shorter the distance.

The consideration is that a long distance reflects some differences in fundamental details of the atoms, and vice versa – small distance means conceptual proximity.

[10] Lazarov, B. (2013). Developing Synthetic Competence along Individual Educational Trajectory. Proceedings of the International Conf. on Math. Edu. on Creativity & Giftedness. The Korean Society of Mathematical Education, (August 9–10, 2013), pp 251-262.

Another feature of D is its *root sensitivity*. This means that $D_C(C', C'')$ changes when taking the genus root C in different levels of the map. Figuratively, the distance decreases when upgrading the ontological tree "up". Let us clarify this by an example. Consider C' stands for 'pekinese', C'' stands for 'bulldog'. Now taking $C \equiv C^*$ to be 'dogs' $D_C(C', C'')$ seems to be large, but taking C to be 'pets' (keeping C^* for 'dogs') the distance becomes smaller. Mathematically, larger level of the genus root increases the denominator of D, hence diminishes the fraction. Didactically, it is crucial for the effectiveness of teaching-learning the atoms of the educational process to be immersed in a deeper origin.

1.2. Suppose, there are c-beams in the DM. The ontological distance in the case "there exists a c-beam between C' and C'' with the beam-value J" we define as

$$D_C(C', C'') = \frac{J}{\delta}.$$

1.3. In both cases 5.1.1 and 5.1.2, if C' and C'' are in different levels, we consider δ as the relative level with respect to the lowest one among the levels of C' and C''.

The ontological distance could be extended on more sophisticated cases. For instance, when "C' and C'' are indirectly connected with c-beams" or "C' and C'' are indirectly connected with c-beams and d-beams". In these cases, one should combine the ideas in stating $D_C(C', C'')$ in the two basic cases of DM presented in this section.

2. Weight of DM

We follow the Novak and Govin's (1984) ideas for the score of a concept map outlined above by modifying the algebraic sum. We will form the characteristic weight of the DM by considering levels, bases and beams with specific coefficients that depend on the structure of the respective component.

Let us evaluate the level structure of a graph-tree. In binary branching, each level doubles the number of atoms in the above level. Thus, level 1 will contribute one point to the map weight, level 3 gives four points, etc. The weight of a DM with m levels receives $L = 2^m - 1$ points from the level-map-characteristic. We adopt this simplest structure as a medium model. Indeed, graph-trees that are more sophisticated, could have different type branching in any particular knot, e.g., single or triple, etc., which in average make binary branching.

An atom participates with an individual coefficient c_k for including already studied material and a coefficient c_n for a new material. The material of the atom-structure could be of a rather different type: experimental work, learning a poem by heart, drawing a figure etc. Motivation: in an atom, the usage of concepts requires an appropriate time resource and complicates the synthesis in the same time. Moreover, introduction of a new concept requires additional technological time for operationalization. Therefore, the points that a particular atom contributes to the map weight are assumed to be $B = c_k + 2c_n$ in total.

A beam will contribute J points to the map weight. Here we make no formal difference between d-beams and c-beams. However, the complexity of the beam could be taken into account by stating the benchmarks i_k.

Thus, we define weight W of a DM having p atoms and q beams as

$$W = L + \sum_{t=1}^{p} B_t + \sum_{t=1}^{q} J_t.$$

Weight is a measure of the permissibility of a particular didactic scheme described by the DM. It is clear that when considering the values L, B, J, some interdependence and overlapping are allowed. For example, concepts at one atom affect the indicators of both the incoming beam and those of the outgoing one. We do not consider this as a drawback, but rather as a collateral feature of the integrated approach.

3. Editing a DM

A need for editing could appear when two independently designed DMs are formally combined into one. E.g., if the total weight is too big or there appear too long distances. The following operations are possible:

- adding an atom;
- removing an atom;
- replacement of an atom;
- adding a beam;
- removing a beam;
- adding an attribute to a beam;
- removing an attribute from a beam.

Any of these operations has a didactic price. It can directly reflect the corresponding action, but it can also take into account factors determined by the educational context. The price can be in the direction of gain (usually associated with weight or distance reduction) or loss (increase in performance values).

Sample technology of didactical mapping for TODU

DM structurizes the didactic process: the hierarchy of atoms is clarified, and hence the order of their introduction; beams are stated, cross-domain links are rethought. The technology of DM design could be as follows:

- a team of teachers (usually two subject teachers and a coordinator, preferably an ICT teacher) traces the topic for integrated consideration;
- the subject teachers independently highlight the topic from their methodology, each teacher on their own clarifies the educational goal and indicators of progress;
- the subject teachers prepare a preliminary list of atoms, taking into account the concepts learned, the already developed students' knowledge and skills, as well as the need of introducing new concepts, building additional skills, forming the positive attitude;
- atoms are ranked in levels, only d-beams are defined;
- the subject teachers design auxiliary DMs independently of each other, stating the topic under consideration at the top of DM (with the highest level-number);

- the team jointly draws up a DM for the TODU by uniting and editing the auxiliary DMs into one;
- the weight of the DM is calculated and, if necessary, ways of reducing it are sought;
- when drawing up the DM, the distance between the atoms is taken into account and, if appropriate, the DM is edited, possibly by adding c-beams.

Example

In this section, we present the didactical mapping in a TODU that could be titled 'Hristo Botev – translator and propagandist' (it was dedicated to the 140th anniversary of the Bulgarian hero Hristo Botev) in the experimental classes on mathematics at a secondary school in Sofia, Bulgaria. Botev cannot be fully studied in neither history nor literature, since Hristo Botev's multifaceted appearances does not fall within the narrow scope of any particular school subject.

1. Botev's hypostases

Figure 3 gives us an idea of a formal split of Botev's hypostases inside the school curriculum. The left branch of Botev's subtree is the subject of history lessons, while the right branch is studied in the school course of literature. (The examples in the rectangles on the bottom of this DM could be different and they do not form a separate level in the DM.)

The activities of the translator and the propagandist do not easily fit together, insofar as the translators are tasked with rendering some source, and the mission of the propagandists is to persuade through (their own) ideas. Thus, there is no place in the presented DM for a little known side in Botev's activity: the translation of the textbook *Lessons for the first four arithmetic rules* by Mikhailov (Penkov and Chobanov (1958)[11] have done a detailed review of this translation). Here, Botev's genius manages to synthesize a brilliant translation with propaganda, which gave us a reason to organize a corresponding TODU devoted to the celebration of his anniversary.

Figure 3 Ontology graph-tree of Botev's hypostases.

[11] Penkov, B. & Chobanov, I. (1958). Botevite „Urotsi za parvite chetiri aritmeticheski pravila" (Botev's "Lessons about the first four arithmetic rules"). Fiziko-matematichesko spisanie. №1. pp 61-67. (in Bulgarian)

2. Quantification of the DM

Let us calculate the ontological distance between the atoms C' (publicist) and C'' (poet) taking $J = 5$ for all beams (a motivation for this value is given in the next paragraph). We have C^*(Botev), C (Bulgarian National Liberation Movement), $d' = d'' = 10$, $\delta = 4$. Thus, $D_C(C', C'') = \frac{10+10}{4} = 5$.

3. Role of the deepness of the DM

Let us note that extracting the branch 'Botev' from the Bulgarian-National-Liberation-Movement didactic map as an autonomous DM for which $C \equiv C^*$ (hence $\delta = 3$), the distance increases: $D_{C^*}(C', C'') = \frac{20}{3} \approx 6.7$. In our TODU the deepness works as it is discussed in Appendix 1.

4. Adding branch in the DM

Another (third) hypostasis, namely 'Hristo Botev – translator and propagandist', cannot be found under the atom C^* (Botev) in Figure 3. It needs two new atoms to be added: (translator) in level III and (mathematician) in level IV as shown in Figure 4. The transition (translator) → (mathematician) requires studying a fragment of Botev's biography by Zahariy Stoyanov (1888)[12]. The bottom row of this branch will be supplemented by the example [Lessons for the first four arithmetic rules]-rectangle.

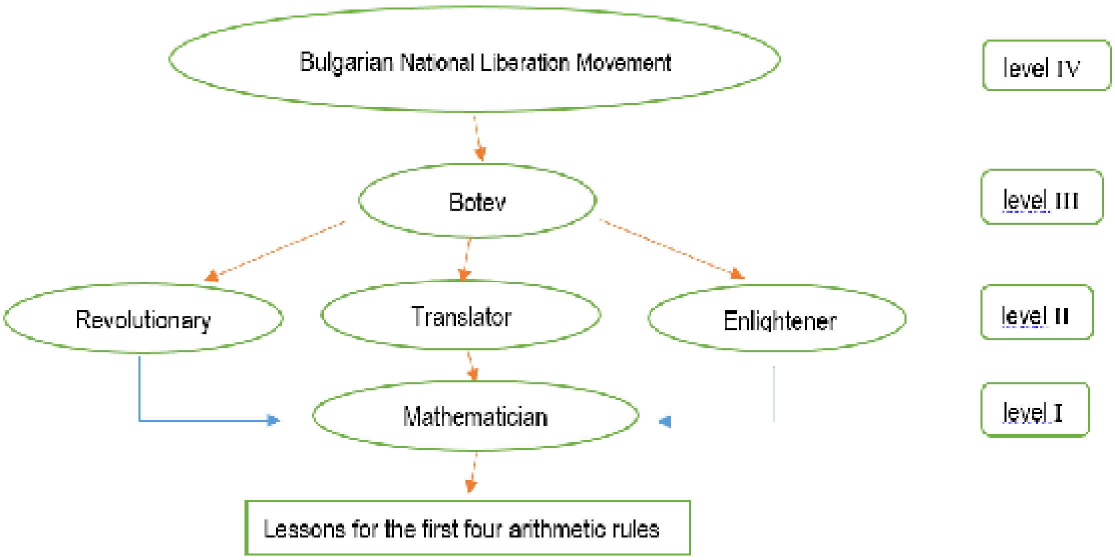

Figure 4 DM-branch for the third Botev 's hypostases.

Let us consider the beam from the atom (translator) to the atom (mathematician) with attributes (indicators) Ik, $k = 1,2,\dots,5$, as follows: I1 – knowledge, I2 – skills, I3 – attitude, I4 – transferability of KSA, I5 – multifunctionality of KSA. Let the benchmark of any indicator take value from the set $\{0; 1; 2\}$. We state: $i_1 = 2$

[12] Stoyanov, Z. *Hristo Botyov. Opit za biografia*. BZNS Pblsh, Sofia, (10th Edition 1960) pp 391-392. Initially published in 1888. (in Bulgarian)

(fundamental knowledge in Bulgarian history for that period); $i_2 = 1$ (basic math skills); $i_3 = 1$ (moderate positive attitude); $i_4 = 0$ (no requirements); $i_5 = 1$ (basic). Thus $J = 5$.

The introduction of the new bases allows the bases (revolutionary) and (enlightener) to be connected in a natural way through intersecting beams with beam-values $J_{1,2}$. Taking the genus root (Bulgarian National Liberation Movement), the distance in the chain

{(publicist) $\overset{J_1}{\leftrightarrow}$ (mathematician) $\overset{J_2}{\leftrightarrow}$ (poet)}

Is calculated via c-beams equals

$$\frac{J_1}{\delta} + \frac{J_2}{\delta} = \frac{5}{4} + \frac{5}{4} = 2.5.$$

In comparison with the distance $D_C(C', C'') = 5$, which we obtained above, this is twice less. Such a reduction of the distance could be done at a minimal didactic cost: preparatory independent work for the students to study the text (Stoyanov, 1888), one additional lesson, and monitoring the transferability and multi-functionality of the knowledge. In the Appendix 2, we give the test-item from the term test that checks these transferability and multifunctionality.

Conclusions

The didactical mapping presented here is so far a theoretical model in an experimental phase. We tried to recruit some teachers for implementing the didactic map in their pedagogical practice but we met their tacit resistance. Such attitude is understandable on this stage of our research. However, our plans include elaborating a map-shell that automatically calculates the weight and the ontological distance when stating the benchmark values. In our opinion, the technology and the model for didactical mapping could help in deeper reconsideration of the didactic process of integrated approach in topic-oriented training units.

The positive results reported in (Giouvanakis et al., 2016) encourage us to develop more TODUs that would integrate different school subjects, providing a priory calculated quantitative characteristics of a particular TODU. Further clarifications of the parameters are expected to be done considering the educational context in which the teachers are going to implement the integrated approach.

Note. This chapter is an improved and supplemented version of the author's proposal to the ITHEA 2020 Conference, which is included in (Lazarov, 2020)[13].

[13] Lazarov, B. (2020). Developing Synthetic Competence via Socratic Style Teaching in Secondary School Extracurricular Activities. Institute of Mathematics and Informatics, Sofia, pp 147-162.

Appendix 1

Botev's exercises from Lessons for the first four arithmetic rules (Stoyanov, 1888)

(1) The Sultan has 800 women, whose number increases by 75 each year. How much has His Majesty's harem grown in 10 years?

(2) Assen and Petar liberated Bulgaria from the yoke of the Greeks in 1190. In 206 years, the Bulgarians fell under the rule of the Turks. In what year did our homeland lose its freedom?

(3) A peasant was carrying 16 kilos of wheat to sell it in the town, on his way there he fell asleep, and the Circassians stole 5 kilos of wheat from him. How many kilos of wheat was left for the peasant?

(4) The Turks conquered Serbia in 1459 and ruled it for 371 years. In what year did Serbia gain its freedom?

(5) Muhammad II conquered Constantinople in 1453. Today's Sultan Abdul Aziz ascended the throne in 1862. How many years passed from the conquest of Constantinople to the reign of Abdul Aziz?

(6) Bulgaria fell under the rule of the Turks in 1396. For how many years did it prosper under the rule of the sultans? (How many years would you like it to last for?)

(7) Boris, Tsar of Bulgaria, accepted the Christian faith in 862, and Vladimir, Prince of Russia (the Kievan Rus), did it in 988. How many years before the Russians (the Kievan Rus) did Bulgaria convert to Christianity?

(8) As of 1873, the Turkish state had an income of 20,637,210 pounds, and an expenditure of 21,404,450 pounds. In the Civil list of HM Sultan the expenses for public education made up 82,025 pounds, when the costs of all items in the list were 1,126,840 pounds more than the cost of enlightenment. Find: how many pounds did not end up in the state treasury and what was the sum of the Civil list of HM (in pounds)?

(9) The pig is farrowed twice a year, with an average of nine piglets. If you have 12 pigs, how much will the pig population increase in two years, if half of those farrowed in the first year are farrowed twice in the second?

Comment

The problems were made for the Bulgarian primary school students living in Ottoman Empire in the last quarter of the XIX century. Including such problems in extracurricular activities for modern 9^{th} graders makes sense only in a TODU, which integrates subjects as history, literature, and mathematics. Any of these subjects does not provide enough space for understanding the full-scale propaganda underlying in the system of problems. These exercises work as an entire educational scenario only in the framework of studying the Bulgarian National Liberation Movement.

Appendix 2

Item 10 from the term test. Hristo Botev is the author of the following problem:

‘Assen and Petar liberated Bulgaria from the yoke of the Greeks in 1190. After 206 years, Bulgaria fell under the Turkish rule. In what year did our country lose its freedom?’

Write down the years in which the Bulgarian kingdoms fell under the Turkish slavery, and the names of the respective Bulgarian rulers under whose rule this occurred.

Tarnovo Kingdom: Year … Tsar …

Kingdom of Vidin: Year … Tsar …

Which kingdom does Botev have in mind? …

Answers

Tarnovo Kingdom: 1393 Tsar Ivan Shishman. Kingdom of Vidin: year 1396, Tsar Ivan Sratsimir. Botev refers to the kingdom of Vidin.

Cultivating imagination and developing aesthetics of math education

Hamlet Mikaelian

Introduction

Revealing the aesthetic potential of mathematics and its involvement in the educational process significantly increases the interest of students and the effectiveness of teaching[1 2]. It also greatly contributes to the effective solution of the problem of cultivating mental qualities of students. We addressed this issue in some of our papers. In[1] we showed the role of mathematics aesthetics in cultivating memory and volitional qualities[3] of students, their needs[3], attention span[4], and emotions[5]. We delineated the problem in the system of training mathematics education students[6]. In this paper the cognitive process of imagination constitutes the main stay of investigation. We consider the problem depending on the properties and types of imagination, as well as the aesthetics of mathematics influence on the development of imagination during math training.

Imagination, its role in teaching mathematics

The imagination (in Greek fantasia – φαντασία, φαντάζομαι – I imagine) is a mental activity, the creation of such mental situations and conceptions that in reality did not happen in the past, were not perceived by the man[7]. There are other definitions of imagination, too. Thus, the imagination is the capability of human consciousness to create images that has no alike in the past[8]. The imagination is the ability to present/imagine images of the objects that exist in reality or in a person's imagination when the given object is missing[9]. The imagination is a person's plunging into their inner world and creating there images, pictures and concepts[10].

While the perception creates the images of objects and phenomena that have a direct influence on our sensations, the imagination creates images of the missing objects and

[1]Mikaelian, H. S. (2015). Beauty and educational potential of mathematics. Yerevan: Edith Print. [in Arm.].
[2] Mikaelian H. S. (2019). Aesthetic foundations of mathematical education. Yerevan-Cherkasy. Edith Print. p.220. [In Rus.],
[3] Mikaelian H. S. (2019). Aesthetic needs and mathematics education. *Bulletin of Cherkasy University series pedagogical sciences.* Issue № 1. Cherkasy, Ukraine. p. 263-269. [In Rus.].
[4] Mikaelian H. S. (2020). Aesthetics of Mathematical Education and the Mental Phenomenon of Attention. *Bulletin of Cherkasy University series pedagogical sciences*. Issue № 1. Cherkasy, Ukraine. p. 67-74. [In Rus.].
[5] Mikaelian H. S. (2020). Aesthetic emotions in the process of teaching mathematics. *Bulletin of Cherkasy Universit, series pedagogical sciences*. Issue № 3. Cherkasy, Ukraine, p. 176-187. [In Rus.].
[6]Mikaelian H. S. (2017). Values as significant component in the National Educational Curriculum of Armenia (with presentation in mathematics education). *Ethische Bildung für eine nachhaltige, dialogische Zukunftp*. Gratz, Austria, p. 171-182.
[7] Gozli Davood G. https://www.researchgate.net/publication/342502764_Imagination_in_Gene ral_Psychology_Thin king_wi th_Lu ca_Tateo's_A_Theory_of_Imagining_Knowing_and_Understanding http://philosophy.niv.ru/doc/dictionary/encyclopedic/.
[8] Philosophy Encyclopedic Dictionary. (2004). Edited by A.A Ivin. Moscow, Russia. [In Rus.].
[9] The New Encyclopedia of Philosophy. (2001). In 4 vols, edited by V.S. Stepin. Moscow, Russia. [In Rus.].
[10] Koslov, N. I. nkozlov.ru/library/samorazvit/d4147/#.VaEFBvamqqko. [In Rus.].

phenomena based on the already existing images and concepts. Through imagination a person can virtually travel to other worlds and to the future, and return to their past. The role of imagination in any person's life cannot be overestimated; imagination is one of the prerequisites to the success of any human activities. Any more or less complicated psychological process cannot be imagined without imagination.

The role of imagination in mathematical activity, particularly in teaching mathematics is crucial, too. The mathematical objects, as items that do not exist in the nature, are completely the result of human imagination. Consequently, imagination has vital significance for mathematics and for its development. Any mathematical invention consists of discovering features and connections of mathematical objects that follow various connections and features in a mathematician's imagination, which should be "guessed", i.e. they should be presented as hypothesis, rejected in most of the cases, and those that can be proved are proved and then become truths. Before being uncovered, these truths are of imaginary nature, as they have not existed yet and appear for the first time in a mathematician's imagination.

It is clear that the process of making mathematical discoveries means close cooperation of imagination and thinking. And in the process of this intellectual search these two mental phenomena play completely different roles. The process of search itself is connected with imagination, it proposes the option or hypothesis and the mind checks the truthfulness of that hypothesis. The final act of finding or discovering is naturally performed in the result of joint and unified functioning of both those mental phenomena. Thus, the aesthetic essence of intellectual search is connected with imagination; the aesthetic subjective features of finding and discovering[1] depend on the result of joint and unified functioning of imagination and thinking.

It is worth mentioning that teaching performed only through thinking, for instance — the teaching of the theorem and its proving, or teaching problem solving becomes a mechanical learning. The teaching is deprived of the possibility to demonstrate the aesthetic features of intellectual search, finding, and discovering. And thus, for the learning process to be accompanied by the demonstration of the mentioned aesthetic features and to attract the student, and bring satisfaction, it is necessary for the student themselves to find not only the solution to the problem, but the proof of the theorem, as well. An experienced teacher can accompany the process of introducing the most important objects of mathematics, the concepts, with the demonstration of the mentioned aesthetic features. In all those cases, the appearance of the aesthetic features is related to the manifestations of imagination.

Types of imagination

Imagination is closely related to the will and therefore it can be volitional, non-volitional, and post-volitional. Non-volitional imagination shows off irrespective of a person's will and without using volitional efforts[2]. When we ask a child to describe their getting home from school, they imagine the situation and generally do not use

[1] Mikaelian H. S. (2019). Aesthetic foundations of mathematical education. Yerevan-Cherkasy. Edith Print. p.220. [In Rus.]
[2] Gozli Davood G. https://www.researchgate.net/publication/342502764_Imagination_in_Gene ral_Psychology_Thin king_wi th_Lu ca_Tateo's_A_Theory_of_Imagining_Knowing_and_Understanding http://philosophy.niv.ru/doc/dictionary/encyclopedic/.

great volitional efforts; in this case, imagination is non-volitional. When a student reads a novel, they have to use volitional efforts to imagine the characters and their correlation. Imagination works in parallel to the development of the actions, every time a new shade is added to the personages' portraits and their relationships. It is the example of volitional imagination.

The volitional and non-volitional types of imagination are closely connected with each other; if the image that occurs in our imagination for the first time results from a certain volitional activity, later on it appears more easily, i.e. becomes non-volitional. Such imagination is also called post-volitional[1]. Thus, in the initial stage of teaching any mathematical concept, e.g. — rhomb, it will be difficult for a student to imagine the corresponding example based only on its definition, and for this, the student will need to strain the will. In this case, the imagination is volitional. However, after having drawn the rhomb several times, a student will not have to strain the will to imagine any rhomb. That is to say, in the initial stage of imagining the rhomb the imagination was volitional, later on it became non-volitional.

The existence of volitional imagination is mandatory in the efforts used for the intellectual search, finding and discovering scientific beauty, knowing the non-evident truth, understanding the essence of the subject, aimed at overcoming[2] the complicated and difficult obstacle when demonstrating subjective features[2]. But in case with the subjective features of unpredictability and unexpectedness, the imagination seems to play a small role.

Revealing the objective features of scientific beauty[2] requires the participation of imagination, as well. The availability of order, symmetry, harmony, rhythm, and other features is not revealed immediately. Usually they are concealed behind separate parts of the object, they require demonstrations that are more scrupulous, they are seen only partially, etc., their disclosure calls upon active interference of imagination for completing the parts, unveiling the concealed features and details through presuming, and for performing other similar actions.

In the process of mathematical activity and teaching mathematics mainly the volitional imagination is demonstrated. Getting to know each mathematical concept due to its abstract nature requires an imaginary approach accompanied by certain volitional efforts. The same refers to the theorems, their proving, math problems, and especially to their solutions. Usually, while solving geometrical problems and proving theorems, drawings come handy. The very construction of such drawings requires a certain work of imagination, as well. However, offering the drawings prepared ahead, replaces the required volitional type of imagination with non-volitional type, which naturally requires a minimum tension of forces on the student's part. In some cases the constructions required for the solution of peculiar stereometry problems, especially the combined ones or the problems related to sections, cannot be solved even by the imagination of the students who are strong in mathematics and the teacher is compelled to offer the required drawing.

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[1] Gozli Davood G. https://www.researchgate.net/publication/342502764_Imagination_in_Gene ral_Psychology_Thin king_wi th_Lu ca_Tateo's_A_Theory_of_Imagining,_Knowing_and_Understanding http://philosophy.niv.ru/doc/dictionary/encyclopedic/.

[2] Mikaelian H. S. (2019). *Aesthetic foundations of mathematical education*. Monograph. Yerevan-Cherkasy. p.220. [In Rus.]

Imagination as a part of mental activity, the can be active-reproductive, creative, wish or passive[1].

Reproductive imagination is demonstrated when an item or phenomenon which the person has never dealt with before is imagined based on its description. A student imagines the events of distant past, the characters of a fiction book, the far away countries, etc. based on the descriptions from their textbooks. The process of teaching mathematics is greatly conditioned by the reproductive imagination of the student; it is based on that type of imagination. The newly studied geometric images and bodies are impossible to understand without imagining them, and imagining them is actualized through their description. The solutions of geometric tasks frequently require building new images for which the data given in the tasks are used. It is like the modelling of math word problems in secondary schools: building corresponding algebraic models based on their data is nothing but building reproductive images of imaginary situations described in the problems through those data.

The creative type of imagination is distinguished and given importance, as well, when the images are generated or built during creative activity. Here the main attention is concentrated on the creative works done in the fields of art, science and technology. However, as A. Einstein said, “a tasty soup is a greater creation than a mediocre painting”. Thus, both cooking a soup and the “tasty” result in any area of human activity is a good creation, which can be reached only via creative imagination. A lesson in general (and the lesson of mathematics in particular) is a creative process, both for the teacher and the student. The teacher’s creative approach includes the creation of conditions for their students to apply their creativity. We should add that creative activity in this case might embrace the whole process of the lesson: its plan, the main concept being introduced, observation of its examples and features, connections with other concepts, their proving, and applying. How will that main concept be introduced? Will the student feel the necessity of its introduction? Will they come to the corresponding idea? Will they formulate it, or will they mechanically learn the ready-made concept-model that does not leave any space for imagination? Those are the teacher’s creative or modelled approaches and the atmosphere created in the result of those approaches will be creative, the students will be engaged in pleasant cognitive activities, whereas in the other case it will be an imposed hard work. The same refers to the rest of the lesson stages.

Imagination properties

Imagination depends on a person’s ability to imagine, which is distinguished for its simplicity, clearness, stability, and vividness. The simpler the conception of the object in our imagination is, the better it serves its end. For instance, in mathematics the simplicity of imagining the geometric drawing greatly contributes to the solution of the problem, whereas the complicated and intricate drawings make the process of solution complicated or impossible. We would like to add that simplicity, as the feature of scientific beauty, has aesthetic attractiveness and makes the imagination process pleasant, pushes the mind towards it.

[1] *The New Encyclopedia of Philosophy*. (2001). In 4 vols, edited by V.S. Stepin. Moscow, Russia. [In Rus.].

Clearness is an important feature of imagination, too: incomplete, insufficient, unaccomplished, unclear conceptions are not sufficient enough for the final accomplishment of goals. When observing the objective features of scientific beauty we discussed the mutual connection of simplicity and clearness features in the process of cognition and creation of the beautiful, as well as their mutual contradiction and mutual addition[1]. That connection is preserved and is given importance in the process of building imaginary objects, too. Particularly, during the process of teaching mathematics when introducing concepts, formulating the theorems, proposing the proofs, problems and solutions, it is impossible to have a meaningful success without the clear work of imagination.

Stability is an important feature of imagination. Usually for the problem to be solved it is necessary to keep the concept created in our imagination for a comparatively long, continuous time which requires strain of mind and the latter can be attained if a student possesses volitional positive qualities. The things mentioned above, are especially important during mathematical activity where the problems are usually solved with difficulty and require a continuous work of mind and consequently of imagination. It should be noted that such work becomes easy due to the fact that stability is an objective feature of scientific beauty.

Another feature of imagination is its vividness. The more vivid our concepts about the imagination object are, the easier the completion of our plans and actions, foreseen for that object, is. We would like to mention that the vivid depiction of both the perceived and the imaginary object and its preservation in our memory is related to the interest shown towards it at the very moment of perception or imagination. Moreover, the vividness is directly proportional to the interest shown: the more genuine the interest is, the more vivid, colourful, and beautiful the object seems.

The development of imagination

A person's imagination develops in parallel to their age. At the beginning when a child has little knowledge about the surrounding world the correspondence of the imagined objects with the reality is not big: a child is ready to imagine a combination of several lines as a house, car, etc. With age, the imaginary concepts of a child start to correspond to reality more and more. While at the beginning the simplicity and vividness features of imagination dominate, later on the clearness and stability gradually become more important.

The aforementioned is taken into consideration during the process of teaching mathematics. It is applied in teaching concepts. The notion of a number and the actions with the numbers, for example, are very difficult for the first grade pupil to imagine. That is why such processes are accompanied by describing the pictures of various objects, which makes the process of imagination easier and simpler. And as the result of numerous depictions, a child forms the notion of the numbers one, two, and three and, in general, of natural number.

It is important to note that the development of imagination means the development of its features and the latter are directly related to aesthetic features of clearness,

[1] Mikaelian H. S. (2019). *Aesthetic foundations of mathematical education*. Monograph. Yerevan-Cherkasy. p.220. [In Rus.],

simplicity and stability. It means that imagination development means the enhancement of aesthetic attractiveness of the object depicted in imagination, which contributes to the increase of the educational process efficiency.

It is clear that in this and other cases there are general approaches to the development of the student's imagination. Let us mention the important ones of such approaches[1]:

- separation of an element or feature from the image of the whole and its mental imaginary depiction;
- building from separate members or through joining their features such an image in imagination that did not exist previously or acts in a new way;
- building through the essential features of the object the imaginary picture of such a new item that is vested with similar features;
- creating a new, complete image through mixing several images;
- building a new image through the generalization of the common features of objects that are similar to each other.

There are also definite tricks that enhance the imagination development. Here are some of them[11]:

a. The imaginary picture is usually built from various combinations of the images existing in our memory. That is why the bigger the quantity of such images is, the easier the building of the imaginary picture, imagination or conception is. While in the process of building the imaginary picture thinking, attention, will, and other mental qualities are employed together with imagination and memory. The aforementioned is vividly seen in mathematics and its teaching. For the building of imaginary pictures to become possible or easier it is necessary to have certain backup knowledge about the material, as well as experience in demonstrating corresponding examples. For example, let's suppose that we want to imagine, depict or build the section of a triangular pyramid that passes through one of its base's sides and of the centre of the height. For that we first should have the clear image of a triangular pyramid in our memory where, additionally, the height is given, too (Figure 1).

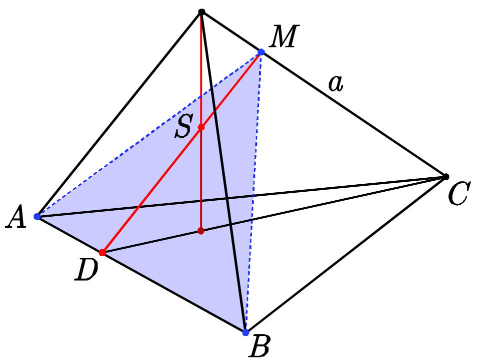

Figure 1

In that height our imagination easily distinguishes the central point S, as the section will pass through that point. What is next? We know that the section also passes through the AB side of the base. In our memory we have the knowledge that we can make only a single plane through the straight line AB and point S and it is exactly the given section, but how can we built that section? In order to build the required section we should find the cross-section of the cutting plane with the facets that do not pass through the straight line AB. (This conclusion is the result of thinking activity). However, we do not see any way for getting the intersection of those planes. Our memory and thinking come to help us: the intersection of two planes is a straight line and we notice that the

[1] http://psyznaika.net/voobragenie.html, Imagination. [In Rus.].

intersecting plane has one common point with each of the mentioned facets; those are points A and B. Thus, we still need to find another common point. Here comes the turn of our imagination to work. Where can we find such a point? The imaginary inspection of the facets and the small thinking effort enable us to conclude that the wanted point can be only on the straight line *a*. Here our imagination enables to observe on that straight line the imaginary point M through which the intersecting plane will pass. Let us turn to imagination again and build the straight line that passes through M and S that will cut the part AB in some D point. Isn't it possible to get that D point beforehand and by connecting it with S get the imaginary point M? Such guidance of our thinking makes our imagination work towards finding point D. The joint and not complicated work of our imagination and thinking enables us to notice that point D lies in the plane that passes through the segments *h*, *a*, and the plane of the pyramid base, consequently it is on the line of their intersection. But the mentioned intersection line is segment CD to which OC is a part. It seems that everything becomes clear now; it is necessary to draw in the plane of the pyramid base the straight line CO which will intersect with AB exactly in point D. After all these it will not be difficult for our imagination and thinking to "build" the required section AMB.

b. Development of the ability to concentrate on an imaginary object mentally. For this, the concept of the object in its details and integrity is required. Such approach is typical for teaching mathematics since any element of mathematical object can be of crucial significance in the solution of the problem and looking for its ways mainly depends on the integral idea about the object. For example, let us suppose that we want to solve a problem about the polynomial multiplication $(x^3y^5z)\cdot(y^2x^7z^3)$ or another similar exercise. Here we are obliged to associate the same variables with each other in our imagination, while we use the knowledge of commutative and associative laws of the product and start to imagine the same variables next to each other, one after another, and make their multiplication. Having solving the first problem, we take some steps in the next similar exercise applying our imagination and omit their recording. With the increase of a student's abilities, the number of the omitted steps increases. After gaining certain skills one can completely move to an imaginary field and solve the further problems in our heads. It is clear that the teacher, by all means, should employ the didactic principles of individualization and availability.

c. The availability of the goal is an important condition for the development of imagination. Really designating the tasks required for accomplishing the goal, building the plan, developing the ways, searching for the methods is firstly actualized through our imagination and in our heads. Here the process of teaching mathematics has an important role, as well. Any mathematical theorem or problem foresees a certain goal and the corresponding proving, solution, as well as the search for a new proving, new solutions, etc. It is an endless process of developing clear plans where the volitional imagination has inexhaustible possibilities of manifestation. In this imaginary world of mathematics, a human mind and imagination usually make unthinkable flights.

d. To live means to create, i.e. to search, to find new ways since often the traditional performance is not interesting and boring. Thus, the life is built in a way that similar and likewise actions are perceived, lived as a single action, i.e. they shorten human life. New places, new people, new books, new knowledge, and new tasks require searching, finding, and discovering. It is creation where human imagination is in the foreground. The process of teaching mathematics with its wonderful, endless, and inexhaustible architectural construction and beautiful and various applications is an inexhaustible field of mental phenomena's active cooperation, endless process of creative searches and the right to be its pioneer vests with imagination. One of the "gems" of geometry, Pythagorean Theorem, gave wonderful possibilities for searching a new proof. Its different proofs have been found by the ancient Greeks, Chinese and Indians, Leonardo da Vinci, the 20th president of USA James Garfield, other outstanding persons, and common mortals for whom making life interesting and pleasant is related to the most important psychological processes characteristic of a man: manifestations of thinking and imagination which is performed in its best way in the searches for the proofs of wonderful mathematical theorems. The number of such proofs for Pythagorean Theorem arrives at a few hundreds.

Finance and tax word problems in high school math

Olga Chernobai

The current curriculum in mathematics, approved by the Ministry of Education and Science of Ukraine (Matematyka, 2020)[1] states that the purpose of basic general secondary education is the development and socialization of students, the formation of their national identity, general culture, worldviews, ecological style of thinking and behavior, creative abilities, research skills and life support skills, ability of self-development and self-learning in the context of global changes and challenges. At the same time, the competency approach is considered as the basis for building the content and organization of the process of teaching mathematics. According to this approach, the final outcomes of teaching the subject are certain formed competencies, such as the students' ability to apply their knowledge in educational and real-life situations, being able to take responsibility for their actions. One of such competencies is social and civic competence, it requires students to gain the ability to analyze their own economic situation, family, local and state budget using mathematical concepts and methods.

It is worth noting that paying taxes is an integral part of the development of the city, a successful state and the world economy. The formation of civic consciousness of the state's population is extremely important for the formation of a perfect tax culture, a conscious attitude to taxation and the reduction of cases of tax evasion. The complete and timely flow of funds in the budget to certain extent depends on how perfect the tax system is and the existing level of the citizens' tax literacy, which should be integrated in the education process starting from childhood.

Civic consciousness is the readiness and ability of a citizen to actively participate in the affairs of society and state on the basis of a deep awareness of their rights and responsibilities. The foundations of civic consciousness are laid during the school years during various educational and tutorial classes. It is possible to demonstrate clearly enough the need for conscious taxpaying when teaching mathematics in the scope of all mathematical disciplines being taught in secondary schools and institutions of higher education.

At the same time, the traditional section of elementary mathematics, consisting of tasks with equations or word algebraic tasks, tasks for finding extreme values, which promotes the development of logical thinking, ingenuity, observation, the ability to conduct small studies can be effectively filled with finance and tax word problems.

In the current algebra curriculum for the 8th grade it is indicated that the student solves narrative tasks on: the use of relationships of economic phenomena; types and taxes calculations, payments; movement; productivity; cost of goods; joint work; mixtures and alloys. In the 9th grade program, a student is expected to solve the problems on: calculation and analysis of the family's financial capacity; calculation of the amount of taxes paid; decision-making on personal and collective financial issues, etc.

In order to spread basic tax knowledge for the development of civic consciousness of students, the staff of the Higher Mathematics Department of the University of the State

[1] *Matematyka*. (2020). Retrieved from [Mathematics] Ministerstvo osvity i nauky Ukraiiny [Ministry of education and science of Ukraine]: https://mon.gov.ua/ua/osvita/zagalna-serednya-osvita/navchalni-programi/navchalni-programi-dlya-pochatkovoyi-shkoli [in Ukrainian].

Fiscal Service of Ukraine compiled a collection of tax word math problems for the 5th-11th graders and a collection of finance and tax word math problems in higher and applied mathematics, which have been published in (Zadorozhnia, et al., 2016)[2] and (Bashchuk, Kuchmenko, Skaskiv, & Chernobai, 2019)[3] and (Rudenko & Chernobai, 2017)[4].

However, it is worth noting that students can be introduced to the concept of taxes earlier. For example, when studying multi-digit numbers, we offer a math dictation having tax content: “The tax on real estate other than land in the Dnipro city was paid in 2020 by five thousand four hundred people, and in 2021 by more than seventeen and a half thousand people. The city budget received fourteen million and three hundred thousand UAH in 2019, in 2020 – ten million and seven hundred thousand UAH more”. Similar tasks have been offered for analysis, see e.g. (Chernobai O.B., 2019)[5]

It is worth while noting that the concept of percentage is the basic mathematical concept when studying various fiscal, financial, social, and economic situations in the family, city, region, state or around the world. It should be highlighted that acquaintance with the concept of percentage takes place when studying the topic "Ratios and proportions".

The main purpose of the study of percentage is to introduce the concept of percentage as a form of writing numbers; to teach students to solve three main tasks on percentage (including those related to labor productivity, market economy, tasks with tax and financial content, socio-economic tasks).

It is worth recalling that the main requirements for the acquired competencies within study of the topic "Percentage" can be defined in the form of knowledge and understanding. Therefore, when studying this topic, students should:

- understand percentage as a form of writing integers and fractions;
- be able to write percentage in the form of ordinary and decimal fractions and give any number in the form of percentage;
- be able to find the percentage of a number, the number by its percentage, the proportion of numbers percentage;
- be able to apply knowledge of percentage to solve the simplest tasks of interdisciplinary, industrial, economical, tax and social content.

It is worth starting the teaching of the percentage topic with a brief historical reference, which may be similar to the following: "Percentage calculations originated in antiquity. Three main tasks on percentages arose in connection with the needs of commercial operations (determination of interest capital or time for which capital gives a certain profit). Later percentage started to be used when calculating other changing variables (population change, soil fertility, etc.). In the XIX century percentage has been already

[2] Zadorozhnia, T., Kharenko, S., Kuchmenko, S., Chernobai, O., Bashchuk, O., & Skaskiv, L. et al. (2016). Zadachi pro podatky [Tasks about taxation]. *Matematyka v ridnii shkoli. – Mathematics in home school, 10,* cc. 16-21. [in Ukrainian].

[3] Bashchuk, O. Yu., S. M. Kuchmenko, L. V. Skaskiv, та O. B. Chernobai. *Vyshcha ta prykladna matematyka: zbirnyk vprav ta zadach [Higher and applied mathematics: collection of exercises and tasks].* Irpin: Universytet DFS Ukrainy. [in Ukrainian]., 2019.

[4] Rudenko, I. B., & Chernobai, O. B. (2017). *Vyshcha ta prykladna matematyka [Higher and applied mathematics]*. Irpin: Universytet DFS Ukrainy. [in Ukrainian].

[5] Chernobai, O. (2019). On the use of Algorithms in Teaching Probability Theory. In N. Tarasenkova, *Curent Issues in Ensuring the Quality of Mathematical Education* (pp. 138-154). Budapest: SCASPEE.

widely used in statistics and other sciences, engineering, economics, chemistry, meteorology, agriculture and manufacturing. For example, the percentage indicates the various tolerances in the manufacture of products, efficiency, energy loss, operating costs, depreciation, the part of the task completion, the percentage of chemical compounds, mixtures, alloys, the composition of society in different categories of the population, humidity, seed germination etc.

As percentage was originally used for the commercial purposes, the definition of percentage was tied to it. For example, in Ferber's textbook "Arithmetics" the following definition was given: "Percentage (interest) means the fee that is charged for the use of a sum of money, a loan, which is taken for a certain period".

Teaching mathematics in modern conditions of distant, full-time or mixed study forms makes it necessary for the teacher to demonstrate a modern educational approach. Therefore, presentations are the main and effective means of presenting, consolidating, and teaching the educational material. Some of them were considered in the study (Chernobai O. , 2021)[6].

Given the fact that during the introduction of percentage concept there are used different methodological approaches, it is worth considering one of them. The most successful, according to the author's opinion, is the introduction of percentage by analogy to ordinary fractions. Several slides of acquaintance with the concept of percentage can be presented in Figures 1-3, which is of interest to the students.

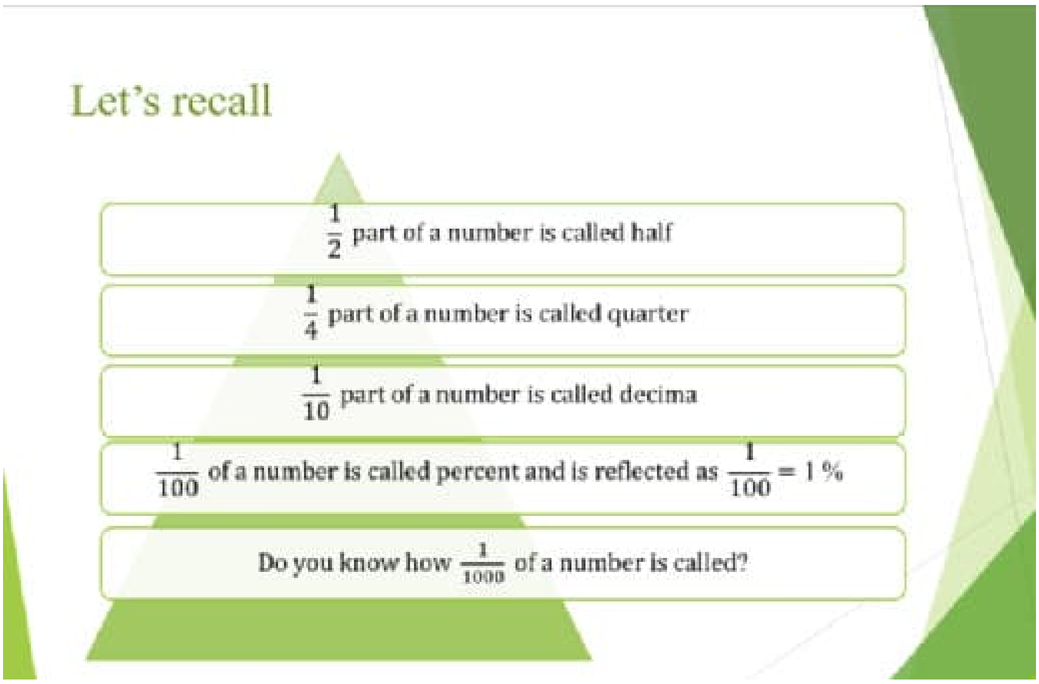

Figure 1 Percentage definition.

[6] Chernobai, O. (2021). Praktychne napovnennya matematyky zadachamy z finansovym ta podatkovym zmistom [Practical filling of mathematics with tasks having financial and tax content]. *Materialy IX mizhnarodnoji naukovo-praktychnoji konferentsiji "Problemy matematychnoji osvity"(PMO-2021) [Proceedings of the IX International Scientific and Practical Conference "Problems of Mathematical Education" (PMO-2021)* (pp. 176-177). Cherkasy: FOP Hordiienko I.I. [in Ukrainian].

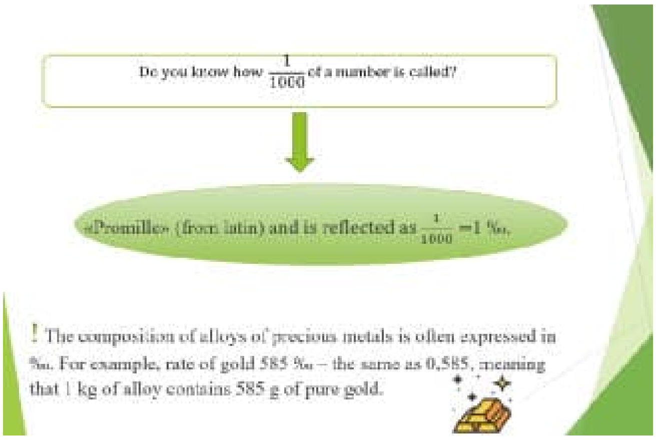

Figure 2 Promille definition.

Examples of the percentage usage are shown in Figure 3. At the same time, students are offered to give their examples, to describe cases where they have already faced the concept of percetage, which gives the opportunity to actively perceive the study material.

Figure 3 Examples of percentages application.

At the first stage of the study of percentage, we suggest formulating the rules in the form of algorithms.

The algorithm for converting a common fraction to a percentage can look like this:

1) convert a common fraction to a decimal;
2) convert a decimal fraction into a percentage;
3) if the common fraction does not turn into a finite decimal, you need to perform rounding with the required accuracy.

The algorithm for converting interest to decimal fraction can be formulated as follows:

1) the number of percentages must be divided by 100.

The above-mentioned algorithms should be demonstrated through examples. Either working online or offline with students, it should be noted that teaching mathematics should trigger positive emotions among students, so we offer to cover the methodological aspect of acquaintance with the percentage concept in the form of slides shown in Fig.4-5.

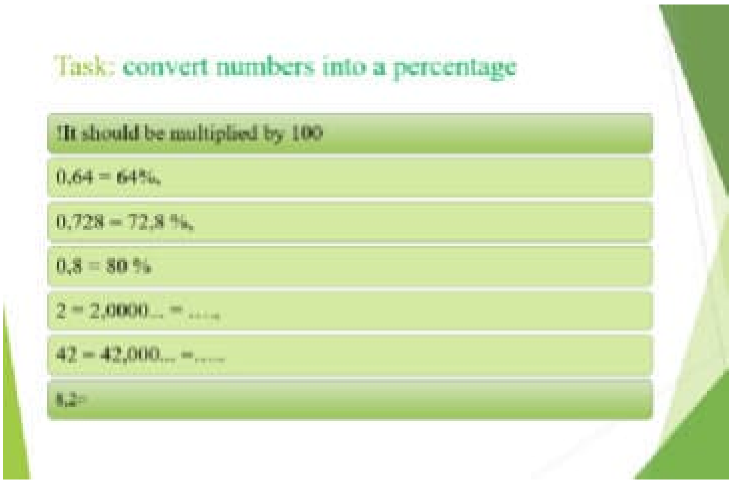

Figure 4 Convert numbers into a percentage.

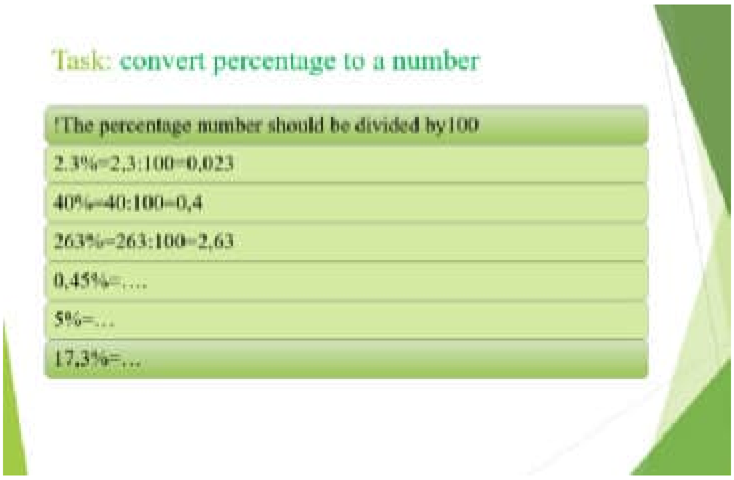

Figure 5 Convert percentage to a number.

The three main types of percentage tasks (finding a percentage of a number, finding a number by its percentage, finding the percentage proportion of two numbers) having financial content can be presented in the form of slides shown in Fig. 6-8.

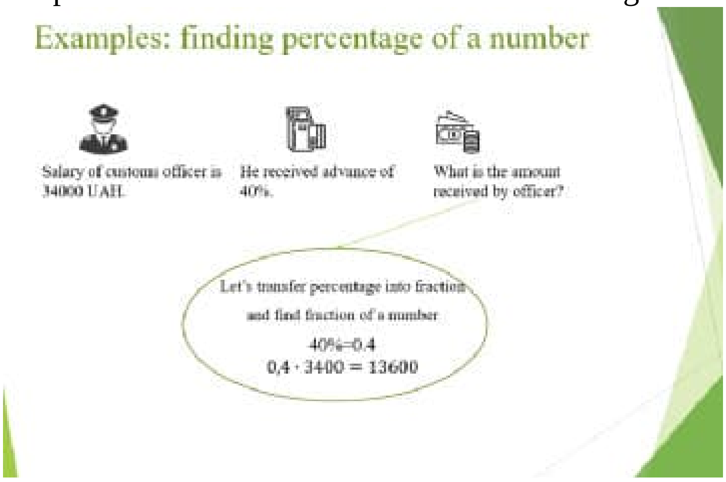

Figure 6 Finding a number by its percentage.

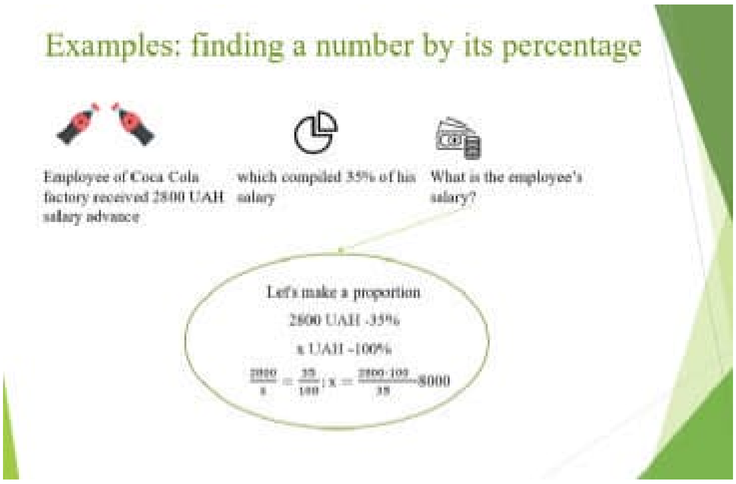

Figure 7 Finding a percentage of a number.

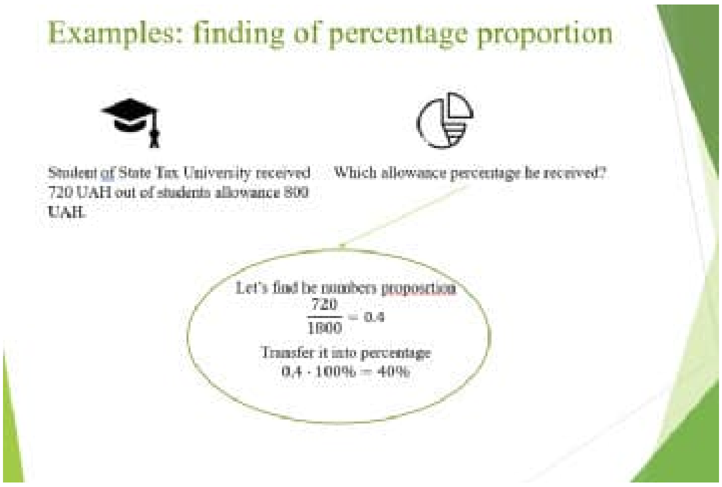

Figure 8 Finding the percentage proportion.

Word problems are of importance in the study of the topic "Percentage", the main functions of which are the development of students' logical thinking and demonstration of the practical application of mathematical knowledge in real life situations.

Whether a student acquires the knowledge necessary for further usage, depends on the skills of the teacher, their ability to combine traditional and innovative teaching methods. The tasks with social, tax content are the educational resources for the formation of social and civic competence.

Finance and tax word problems can be treated as examples of percentage-related tasks linked to everyday life. Such tasks were partially considered in studies (Chernobai, 2019)[7] and (Chernobai O.B./, 2020)[8]. Illustration of some of them can be reproduced in the form of slides shown in Fig. 9-10.

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[7] Chernobai, O. "*On the use of Algorithms in Teaching Probability Theory*." In Curent Issues in Ensuring the Quality of Mathematical Education, by N Tarasenkova, 138-154. Budapest: SCASPEE, 2019.

[8] Chernobai O.B./. (2020). Pro deyaki typy poshukovykh zadach [About certain types of search tasks]. In M. I. Burda, *Matematychna osvita: mynule, sohodennya majbutnie do 100-richchya vid dnya narodzhennya O.F. Semenovycha*: monohrafiya [Mathematical education: past, present, future to the 100th anniversary of the birth of O.F. Semenovich: monograph] (pp. 102-107). Kharkiv: SHNTM "Novyi kurs", 2020.-203c. [in Ukrainian].

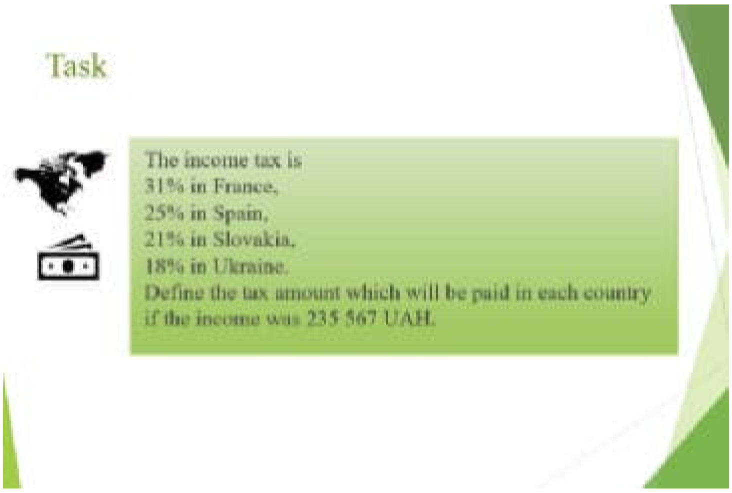

Figure 9. Task on taxes

Figure 10 Solving task on taxes.

During the study of the topic "Percentage" there is a gradual improvement of practical skills related to actions with decimal and ordinary fractions, logical thinking, the connection of mathematics with everyday life. This prepares students for the wider use of mastered methods in the next stage of studying mathematics in general, and financial literacy in particular.

While solving word problems students are also learning to use mathematical models related to everyday life and their social position. Solving word problems is integrated in the study of all topics covered by the program.

To ensure these skills, we highlight the methodological approach to solving word problems, presented in the form of an algorithm.

1. Read the statement of the problem so that the student is able to reproduce it on their own.
2. Determine what is specified in the problem and what should be found.
3. Indicate the values in question, mark with letters and write briefly the statement of the problem.

4. Determine which of the unknown values should be marked as the main unknown.
5. The unknowns mentioned in the task should be expressed via the main unknown.
6. Based on the task, we make an equation, inequality or system of unknowns.
7. Solve the resulting equation, inequality or system of equations.
8. Analyze the correctness of the solution.
9. Write the answer to the task.

As examples of word problems for joint work related to the work of the tax service, we offer the following tasks:

Task 1. Department of the Fiscal Service of Ukraine checks the correctness of tax payment of 400 private enterprises. For the first five days, employees exceeded the daily rate by 20%, and in recent days were checking 15 private enterprises above the plan. As a result, in two days before the deadline 405 private enterprises were checked. How many private companies had to be checked by the branch every day according to the preliminary plan.

Let's solve the task according to the mentioned methodical algorithm.

To analyze the task, we construct the following table:

Table1.

	Work (A)	Productivity (p)	End date (t)
Plan	400	p	t_1
Fact	405	p_1 p_2	t_2

We mark the expected productivity p as a main unknown, then the expected end date is $t_1 = \frac{400}{p}$, and actual end date $t_2 = t_1 - 2 = \frac{400}{p} - 2$. According to the task we get $p_1 = 1{,}2p;\ p_2 = p + 15$. We have the following equality $5p_1 + (t_2 - 5)p_2 = 405$.

Task 2. To verify three equivalent regions, three groups of auditors with different capacity were assigned. Each region is audited by one group of auditors. The first group started work $\frac{1}{2}$ hour before the second, and the third – $\frac{1}{3}$ hour later than the second. The inspection of the regions was carried out evenly and without stopping. Some time after the start of the third group of auditors' work, it turned out that by this time each group had performed the same part of the planned work. How long after the end of the work of the second group the first group will finish the work, if the third group finished the check 12 minutes earlier than the second.

It is convenient to perform the analysis in the form of the following Table 2.

Table 2.

Group	Work			Time		Productivity
	All	Till moment	After moment	Till moment	After moment	
I	A	A_1	A_2	x	t_1	p_1
II	A	A_1	A_2	y	t_2	p_2
III	A	A_1	A_2	z	t_3	p_3

We mark the main unknown figure – it is time difference $t_1 - t_2$.

According to the table and problem narrative, prepare the system of equalities:

$$\begin{cases} x - y = \frac{1}{2}, \\ y - z = \frac{1}{3}, \\ t_2 - t_3 = \frac{1}{5}. \end{cases}$$

Express time for each working group till the moment and after it via the work productivity formula:

$$x = \frac{A_1}{p_1}; \;\; y = \frac{A_1}{p_2}; \; z = \frac{A_1}{p_3}; \;\; t_2 = \frac{A_2}{p_2}; \;\; t_3 = \frac{A_2}{p_3}.$$

We have the following system of equalities:

$$\begin{cases} \frac{A_1}{p_1} - \frac{A_1}{p_2} = \frac{1}{2}, \\ \frac{A_1}{p_2} - \frac{A_1}{p_2} = \frac{1}{3}, \\ \frac{A_1}{p_2} - \frac{A_2}{p_3} = \frac{1}{5}. \end{cases}$$

Take the common factor out of brackets, we have:

$$\begin{cases} A_1\left(\frac{1}{p_1} - \frac{1}{p_2}\right) = \frac{1}{2}, \\ A_1\left(\frac{1}{p_2} - \frac{1}{p_2}\right) = \frac{1}{3}, \\ A_2\left(\frac{1}{p_2} - \frac{1}{p_3}\right) = \frac{1}{5}. \end{cases}$$

The solution to the system will answer the question.

An example of a tax word problem for finding the largest and the smallest values can be formulated as follows:

Task 3. Three departments of the fiscal service must verify the accuracy of filling in the declarations. The first department checks 200 declarations per day, the second department checks less declarations per day comparing to the first, and the third department checks 5a more declarations per day comparing to the first. At the beginning the first and the second departments work together, perform $\frac{1}{5}$ of the work, and then all three departments work together and perform $\frac{4}{5}$ of the work. How many fewer declarations per day should be checked by the second department comparing with the first one, so that all the work is done as quickly as possible.

According to the suggested method of solving word problems, the following steps should be performed:

1) Determine the main unknown quantity - a.
2) The analysis of the task narrative is performed in the form of Table 3.

Table 3.

<table>
<tr><td>Departments</td><td colspan="2">Work</td><td colspan="3">Time</td></tr>
<tr><td></td><td>All</td><td>Per day</td><td>I and II</td><td>All 3</td><td>All time</td></tr>
<tr><td>I</td><td>A</td><td>200</td><td rowspan="2">$t_1 = \frac{A/5}{400 - a}$</td><td rowspan="3">$t_2 = \frac{4A/5}{600 + 4a}$</td><td rowspan="3">$t = t_1 + t_2 = \frac{110A}{(400 - a)(150 + a)}$</td></tr>
<tr><td>II</td><td>A</td><td>200 – a</td></tr>
<tr><td>III</td><td>A</td><td>200 + 5a</td><td></td></tr>
</table>

Thus, the time of all work is a function of one argument a. Since the numerator of the obtained fraction does not depend on a, the value of the function is determined by the value of the denominator $(400 - a)(150 + a)$, which has the form of a square trinomial.

It is clear that the time t will be minimal, provided that the denominator of the fraction will be the largest. The study of the quadratic function to the extremum will answer the question.

At the same time, the study (Korostiyanets, 2020)[9] developed a set of educational problems that contribute to the formation of educational activities for future mathematics teachers. As per author's suggestion, methodological tasks for higher education applicants in the educational-professional curriculum "Secondary Education: Mathematics" can be divided into the following types:

1) motivational problems;
2) recognition tasks;
3) comparison tasks;
4) design tasks.

According to the proposed methodology, we suggest considering the finance and tax word problems according to the type of each problem.

Structure motivational finance and tax word problems:

1) The family income is 17 thousand UAH. As a result of wages increase by 15% and the further pandemic-caused decrease by 15%, analyze how family income has changed.

2) Value added tax increased by 5% and then decreased by 5%. Has the tax changed?

3) The customs post increased the allocations to the local budget by 20% and then decreased them by 20%. How have the contributions to the local budget changed?

4) A private enterprise increased the price of goods by 14%, and then by another 14%. By what percentage has the price changed?

5) Salaries of tax officials were increased twice, and the percentage of the 2nd increase was twice higher than that of the first increase in salaries. Determine by what percentage the salary was increased each time, if prior the first increase the salary was 7 thousand UAH, and after the second it became 9240 UAH.

[9] Korostiyanets, T. N. (2020). Pidhotovka majbutnikh vchyteliv matematyky do formuvannya ponyat v uchniv: psykholoho-dydaktychnyj aspekt [Preparation of future mathematics teachers for the formation of concepts among students: psychological and didactic aspect]. In a. o. Tarasenkova N. *Matematychna osvita: mynule, sohodennya, majbutnie [Mathematical education: past, present, future]* (pp. 156-162). Kharkiv: SHNTM "Novyj kurs". [in Ukrainian].

The comparison finance and tax word problems will look like this:

1) The excise duty has been doubled. By what percentage has the excise duty increased?
2) Customs reduced the capacity by 60%. How many times has it decreased?
3) The small business tax increased by UAH 3,000. By what percentage has the tax increased?
4) Tuition fees in private schools increased by 25%. How many times has the tuition fee increased?
5) In some regions of Ukraine, deforestation has been reduced by 45%. How many times has deforestation been reduced?

Recognition finance and tax word problems can look as follows;

Design problems. We offer to choose unambiguous statements among the following.

1) The family's financial income has increased by ½.
2) The family's financial income has increased by 50%.
3) The family's financial income is 150%.
4) The family's financial income has increased 1.5 times.
5) The family's financial income has doubled.

This research is designed to help math teachers not only to fill the school math course with practical finance and tax word problems, but also to share the experience of creating interesting presentations that engage students in studying various topics in class and after school. It is the formation of students' interest in solving practical tasks that is an important tool for increasing interest in mathematics, developing their creative abilities, promoting the development of social and civic competence of students and improving the quality of mathematics education.

The problems are meant to play a major role in achieving educational goals. The elements of educating students, the formation of their civic consciousness, which are developed through the solving of word problems and must be planned and carefully prepared by the teacher. In the process of solving word problems, it is necessary to ensure that students themselves are aware of the appropriate conclusion about the importance of arriving at a correct solution.

Important educational functions of the tasks include the formation of students' cognitive interest and creative abilities, fostering patriotism, aesthetic education and applied direction of mathematical abstractions.

CHAPTER 2 *Mathematical Training at the University*

Technical universities of Ukraine: teaching methods

Tetyana Krylova

Teaching methods

A method is a way to design and substantiate the system of philosophical knowledge; a set of techniques and operations of practical and theoretical comprehension of reality.

Instructional (teaching) methods represent one of the most important structural components of the learning process. Method is the core of the learning process, the link that connects the proposed objective and the end result.

In translation, the word “method” (from the Gr. *methodos*) signifies “a research, a means, a way to achieve a goal”. The etymology of this word leaves a trace on its interpretation as a scientific category.

The philosophical dictionary broadly defines method as “a way to achieve the goal, an activity structured in a specific way”. From the standpoint of modern pedagogical science, teaching methods are characterized as multifaceted, multidimensional, widely attributable, multifunctional didactic phenomenon. The main, most significant features of the method are the following: teaching method is a way for a student to obtain information and master skills and abilities; a mode of teacher-student collaboration, the management of educational and cognitive activity of students; a set of orderly techniques and operations sufficient to obtain the results of joint activities of teachers and students; a way and form of movement of educational content according to the rules of inductive or deductive logic of its deployment; the design and trajectory of cognitive independence and activity of students, method of stimulation and motivation of learning, method of independent experiences, method of formation of evaluative judgments.

The most common is the following definition: "teaching methods are orderly ways of interconnected activities of teachers and students, aimed at solving educational and developmental problems"[1]. Teaching methods perform educational, developmental, fostering, stimulating, communicative, diagnostic and corrective functions.

Y. Komensky, Y. Pestalozzi, A. Disterveg, K. Ushinsky, Y. Horbart, A. Makarenko, V. Sukhomlynsky laid the foundations of the theory of teaching methods.

There were specified such methods as those of research and ready knowledge, passive (verbal) and active (heuristic, motor) methods, 4 groups of methods according to the principle of research approach to teaching: dogmatic, illustrative, heuristic and research methods.

Classifications of teaching methods

Below we consider the main modern classifications of teaching methods.

[1] Encyclopedia of Education. (2008). Head ed. V.H. Kremin. Kyiv, Ukraine : Yurinkom, Inter. [In Ukr.]

Classification 1

Teaching methods can be divided into teaching methods and methods of instruction. The teaching methods include:

- lecture;
- story;
- conversation;
- explanation;
- display (demonstration)

The methods of instruction cover:

- observation;
- listening;
- studying educational and educational-methodical (textbooks, teaching aids, lecture notes etc.) and scientific literature, other materials;
- solving formal and informal tasks;
- research;
- mathematical modeling;
- experiment.

Classification 2

The traditional classification of teaching methods by the source of knowledge includes:

- practical methods (experiment, exercises, training and work practice);
- visual methods (illustration, demonstration, observation of students);
- verbal methods (explanation, clarification, storytelling, conversation, instruction, lecture, discussion, debate);
- work with literature (reading, studying, rewording, cursory review, citing, reproduction, drawing up a plan, taking notes);
- video method (review, teaching, exercises guided by "e-teacher", monitoring).

Classification 3

Classification of methods by purpose according to M.A Danilov and B.P. Esipov consists of the following methods:

- acquisition of knowledge;
- formation of skills and abilities;
- application of knowledge;
- creative activity;
- consolidation;
- verification of acquired knowledge, skills and abilities.

Classification 4

The classification of methods by type of cognitive activity developed by I.Ya. Lerner and M.N. Skatkin, encompasses the following methods:

- explanatory-illustrative or information-receptive method (storytelling, lecture, explanation, work with literature, demonstration of pictures, movies and slides, etc.);
- reproductive method (reproduction of actions allowing the student to apply the knowledge in practice, algorithmic activities, programming);
- problem presentation of educational material;
- heuristic and partial-search method;
- research method, when students solve the problem independently, choosing the necessary methods and relying on teacher’s advice.

Classification 5

Classification according to the main didactic tasks (revised classification 3):

- methods of oral presentation of knowledge by the teacher and boosting students' cognitive activity (storytelling, explanation, lecture, conversation, method of illustration and demonstration accompanying oral presentation of educational material);
- methods of consolidation of the studied material (conversation, work with educational literature);
- methods of independent work of students on comprehension and mastering of new material (work with educational literature, laboratory works);
- methods of educational work on employing the acquired knowledge in practice and development of skills and abilities (exercises, laboratory classes)l;
- methods of test and assessment of acquired knowledge, skills and abilities (daily observation of students' work; oral individual, frontal and compact questioning; lesson scoring, tests, homework check, programmed control).

Classification 6

Classification according to didactic purposes includes two groups of teaching methods:

- methods that promote immediate assimilation of educational material;
- methods that help consolidate and improve the acquired knowledge.

The first group of methods includes:

- information and development methods (teacher’s or professor’s oral presentation of conversation, work with a book);
- heuristic (search) teaching methods (heuristic conversation, debate, laboratory work), research method.

The second group of methods includes:

- exercises (sample exercises, annotated exercises, variable exercises, etc.);
- practical assignments.

Classification 7

Binary classification of M.I Makhmutov is based on a combination of teaching methods (informational, explanatory, instructive-practical, explanatory-motivational, motivational methods) and instructive methods (performative, reproductive, productive and practical, partial-search, search methods).

Classification 8

Binary classification of methods by M.D. Yarmachenko, V.M. Galuzinsky, M.B. Yevtukh[2], which reflects both certain individual aspects and tasks of the educational process.

Classification 9

Polynary classification of teaching methods, covering the sources of knowledge, levels of cognitive activity and types of learning cognition was proposed by V.F. Palamarchuk and V.I. Palamarchuk[3].

Classification 10

Academician Yu.K. Babansky [4] identifies the following traditional teaching methods:

- planning and implementation of educational and cognitive activity;
- stimulating and motivating educational and cognitive activity;
- monitoring and self-control of the effectiveness of educational and cognitive activities.

Methods of planning and implementation of educational and cognitive activity are:

- verbal;
- visual;
- practical;
- inductive;
- deductive;
- reproductive;
- problem-searching;
- methods of independent work of students and work under teacher’s guidance.

The methods of stimulation and motivation of educational and cognitive activities include:

- methods of stimulating and motivating interest in learning;

[2] Aleksiuk, A.M. (1981). General methods of teaching at school. Kyiv, Ukraine. [In Ukr.]
[3] Pedagogy. (1988). Ed. by Yu. K. Babansky. Moscow, Russia : Prosveshchenie. [In Ukr.]

- methods of stimulating and motivating duty and responsibility in learning.

Methods of monitoring and self-control over the effectiveness of educational and cognitive activities are:

- methods of oral control and self-control;
- methods of written control and self-control;
- methods of laboratory control and self-control.

Classification 11

Teaching methods are divided into:

- informative and reporting;
- explanatory and illustrative;
- problematic;
- logical;
- reproductive;
- interactive;
- informative and communicative.

Classification 12

Teaching methods can be divided into traditional and non-traditional.

Traditional methods include:

- analysis and synthesis;
- generalization;
- logical methods of similarity, difference, concomitant changes;
- diagnostic;
- verbal;
- visual;
- explanatory and illustrative;
- reproductive;
- cognitive;
- self-organization of training.

Non-traditional methods include methods of active learning.

Methods of active learning are divided into:

- methods of developmental teaching;
- activating teaching methods;
- training modules.

One of the methods of developmental learning is problem-based learning, which includes heuristic and partial-search learning.

The activating teaching methods (active methods) are:
- educational business games;
- seminar-discussion;
- analysis of specific situations;
- visits of production facilities;
- field trips;
- archiving mail.

Classification 13

In their classification S.I. Perovsky, Ye.Ya. Golant[5] classify teaching methods according to the source of material and the nature of perception.

Methods are divided into:

- verbal (lecture, story, conversation, explanation, discussion, work with books);
- visual (illustrative and demonstrative);
- practical (oral, written, graphic exercises, seminars, laboratory work, essays, research).

In verbal methods the source of knowledge is oral or printed word, in visual methods - observation of objects and phenomena, visual devices, in practical methods the students gain knowledge and develop skills in performing practical actions.

Classification 14

Classification of teaching methods based on data from various levels substantiated by V.P. Bezpalko:

- the first level – knowledge as recognition (the student distinguishes, recognizes a familiar subject, phenomenon, certain information);
- the second level – knowledge as a copy (students are able to retell, reproduce the learned information);
- the third level – knowledge as a skill (the student applies the acquired knowledge, acquired skills and abilities in practice);
- the fourth level – knowledge as a skill (automated skills);
- the fifth level – the category of creativity in a particular field of educational and cognitive activities.

Classification 15

The classification of teaching methods by professor R.S. Pionova [6] includes:
- theoretical;

[5] Didactic methods. Retrieved from http://www.prepodi.ru/praktika-pedagogika/didaktika/1166-didaktika-metod start=3- (accessed 17 June 2021). [In Rus.]

- practical information methods;
- research and creativity methods;
- methods of independent work of students;
- control and evaluation methods.

The theoretical and operational methods include:

-oral presentation;
- oral dialogically constructed statement (conversation);
- explanation;
- storytelling;
- counseling;
- audio-video demonstration;
- demonstration;
- discussion;
- brigade method.

The practical and operational methods include:

- exercises;
- solving problems;
- algorithm;
- "do as I do";
- experience;
- experiment;
- didactic game.

Research and creativity methods include:

- observation;
- experience;
- "socratic conversation";
- labirynth;
- "brainstorm";
- "think, listen, suggest";
- "aquarium";
- creative dialogue;
- analysis of specific situations;
- insight;
- case method.

The methods of independent work of students include:

- reading educational literature;
- video tape;
- expertise;

- listening;
- notes;
- exercises;
- problem solving;
- experience;
- experiment.

The control and evaluation methods cover:

- preparatory exam;
- "chamomile";
- oral presentation;
- control work;
- experience;
- programmable assessment;
- testing;
- the answer from the seat;
- questioning.

Professor I.P. Pidlasiy identifies 6 most reasonable classifications of teaching methods, namely:

- traditional classification;
- classification of methods by purpose (by M.A. Danilov, B.P. Yesipov);
- classification by type (nature) of cognitive activity I.Ya. Lerner, M.N. Skatkin;
- classification by didactic purposes;
- binary and polynary classifications;
- classification by academician Yu.K. Babansky.

The concept of mathematical education of students of higher technical school of Ukraine rests on the following provisions:

1) The high dynamism of modern scientific and technological progress and high expectations about the training of engineers requires appropriate level of mathematical training of students. At the same time, the analysis of the current state of teaching mathematics in technical universities shows that in recent years there has been a significant reduction (up to 50%) in the number of teaching hours for general education, while the traditional course of higher mathematics has remained unchanged. Naturally, the allotted number of hours is not enough to study the specified content thoroughly during lectures and practical classes, so the relevant methodological commissions recommend that 50% of the content be moved to independent work of students.

2) The amount of information on mathematics, which has recently become quite large, cannot be assimilated in a relatively short period of study (2-4 semesters in a technical university), so it must be organized on a fundamentally new basis. This basis can be the management of independent work of students, which is one of the ways to

intensify the educational process and improve the quality of mathematical training of students.

3) In the framework of level system of higher technical education, the only possible way to overcome the difficulties and negative phenomena that have arisen is personalized learning, differentiation and individualization of the educational process.

4) The number of classroom hours allocated by the current curriculum can be considered acceptable if one can reduce the amount of program material for the students, who find an engineering job after graduation, while for those students who continue studies for a master's degree and do research in science and techniques, there will be additional sections and special mathematical courses, in particular, a course of mathematical modeling from the first year in parallel with the general course of mathematics.

5) In order to improve the quality of mathematical training of students of technical specialties, it is necessary to systematically implement the principle of professional orientation of teaching the general course of mathematics, both in the study of theoretical material and in solving the system of exercises.

The professional orientation of training should be based on the principles of professional compliance and continuity, the main means of which are mathematical modeling and the availability of typical applied problems, as well as the principle of fundamental preparation for future professional activities, access to new mathematical ideas, professional unambiguity, applied content. An effective way to promote compliance with these principles and laws is to pklan assignments with special content at the final stage of teaching the disciplines of the mathematical cycle. Special mathematical courses reflecting the future specialists' interests are supposed to ensure the completion of the stage of mathematical training of specialists in the field of technology

6) An effective means of implementing the professional orientation is teaching to students the beginnings of mathematical modeling in the framework of the general course of mathematics and special mathematical courses at the final stage of mathematics for students of master's degrees.

7) The diagnostics of mathematical training and development of students at the beginning of the course of higher mathematics and throughout the study is a necessary condition for differentiation of education ("zero" test work for freshmen, testing, various independent tests, colloquia).

Experimental research has shown that standard calculations, individual homework, laboratory work and module-rating assessment of students' knowledge and skills are effective means of organizing independent work of students.

8) An important way of developing interdisciplinary links in the study of the general course of higher mathematics and special mathematical courses is the participation of teachers of mathematical departments in research work of special departments and involving there student youth.

9) A necessary modern means of improving the mathematical training of students is the systematic use of new information technologies in the study of general mathematics, special mathematics courses, and especially in solving applied problems and research.

10) Effective mathematical training of students of technical universities can be provided only by implementing systematic and integrated approach in the organization of the educational process.

11) One must support students in their desire for knowledge, for skills of independent work, in motivation to learn.

12) Motivate students to develop their general and special competences and competencies.

The objectives of teaching mathematics to students of non-mathematical specialties are:

- ensuring continuity and succession in the study of mathematics throughout the period of study at the university;
- aiming at professional orientation of teaching mathematics by improving the fundamental training of students in mathematics, strengthening the role of numerical methods and their implementation on a computer, the orientation of learning to use mathematical methods in solving applied problems;
- teaching special mathematical disciplines at the current level of development;
- students' progress in acquiring a sufficient stock of mathematical knowledge, familiarity with analytical numerical methods of solving problems of applied content and their implementation on a computer, as well as methods of modeling practical engineering problems;
- boosting educational and cognitive activity of students;
- meeting the needs of special departments in guiding course papers, research and diploma theses;
- students' progress with computers and modern mathematical methods of scientific research;
- students' success in creative application of knowledge and skills to the solution of practical and theoretical issues.

In two-level system of teaching the applied focus mathematical schooling is realized in the study of general mathematics (analytical geometry, elements of linear algebra, mathematical analysis) and especially in the study of special mathematical courses based on differentiation and individualization of the educational process.

The main forms of organization of the educational process are lectures, practical laboratory classes. The general course of mathematics is studied in the first and second years (the first 2-3 semesters), special mathematical courses are studied in the second and third years (1 2 semesters).

The lecture is extremely meaningful in the educational process of higher education. It is expected to meet high requirements not only of scientific but also of methodological nature. Taking into account the need to comply with the principle of professional adaptation of students and the principle of continuity in teaching mathematics in engineering departments, a significant place in the lecture should be occupied by the important principle of applied orientation. The essence of the lecture should reflect the condition of compliance of the lecture with the objective of mathematical training of future specialists. Hence the need for applied orientation of the lecture on mathematics which inevitability results in new mathematical ideas. And this demands fuller application of applied problems and examples in mathematical lectures. Educational lecture for students of non-mathematical specialties includes the following points: selection of examples of applied content, expansion of known interpretations of mathematical concepts, social interpretation of some mathematical concepts, elements of mathematical modeling. Based on this, we propose such a construction of the lecture, which is based on three categories, namely: scientific and substantive, applied orientation, voluntary feedback.

The content of practical classes (exercises, seminars, laboratory work) is determined by the task set for the teachers of mathematics. This may be the acquisition of some skills for solving problems, consolidating the theoretical provisions of the course, demonstration of the use of mathematics in special contexts, consideration of elements of applied nature in solving formal problems. Such a variety of tasks allows one to improve the ways of realization of the professional orientation of teaching mathematics. The content of practical classes in the general course of mathematics should be determined by adhering to four principles of professional orientation of mathematics (fundamentality, preparation for future professional activities, access to new mathematical ideas, professional conformity). In terms of its content, the system of exercises in the general course of mathematics should be designed in such a way that the following rules are met: the rules of sufficient number of formal problems, the rule of professional unambiguity, the rule of applied content.

Independent work of students is an important factor in mastering theoretical material, mathematical methods, consolidating and developing skills of solving formal and applied problems. It consists of systematic efforts in common and individual homework, in the implementation of standard calculations for each topic of the mathematics course, in preparation for laboratory work, colloquia, various types of tests, competitions, seminars, student conferences and ends with course paper focused on future specialty, enhanced by digital tools.

Monitoring of students’ independent work is carried out by questioning in lectures, colloquia, practical classes, by checking homework, laboratory, tests and course papers, standard calculations.

One of the conditions for successful implementation of the concept is scientific and methodological support of the educational process, relying on continuity and succession of mathematical development on the basis of interdisciplinary and interdepartmental relationships, timely publication and updating of methodological literature, coordination of curricula, organization and conduct of research, methodical seminars at departments, student conferences.

Although modern mathematical schooling is impossible without taking into account progress in modern science due to the development of computer technology, it would be deeply mistaken to perceive this fact as a basis for restructuring the whole process of teaching mathematics in technical higher education. It is necessary to abandon the extreme point of view according to which there exist "two mathematics": the theoretical and non-rigorous applied one, the latter of which should be taught in a technical university. Basic mathematical disciplines such as analytical geometry, higher algebra and mathematical analysis cannot sacrifice themselves to non-rigorous applied mathematics, because it is impossible to apply mathematics without learning mathematics itself. However, the teaching of basic mathematical disciplines must still undergo significant changes, the need for which is historically due to the unity of mathematics as a science and due to all the modern requirements of society and production, which challenge the mathematical education of students of higher technical institutions.

The implementation of provisions supporting the concept of mathematical schooling for students of higher technical schools of Ukraine

The process of training and formation of the future engineer deserves careful study and discussion taking into account current trends in society and education. Not everything that is being done now in this direction can be considered satisfactory and impeccable. Let us consider the issues related to the implementation of the concept of mathematical education of technical university students, as well as of other non-mathematical courses that have recently been introduced in technical universities.

Currently, the first serious difficulties in the educational process arise at the beginning of the first semester when mathematics teachers get to know the students and learn about the level of their mathematical training and general development. Unfortunately, a characteristic feature of modern academic groups or lecture flow is that they have obtained different levels of mathematical training. This feature affects the organizational and methodological design of the educational process in higher technical education, and also allows you to make some comments on the organization of the educational process in mathematics in secondary schools.

After conducting the "zero" test, additional classes in elementary mathematics are assigned. These classes can be attended by all students, but those who have low scores on the tasks of this topic – are demanded to attend.

The framework of conducting auxiliary classes in elementary mathematics combines both school and university teaching methods. A characteristic feature of these classes is the strictest possible justification of the most important theoretical issues which were not taught in full during the school year.

Additional classes in elementary mathematics are held 1-2 times a week for one and a half to two months. Questioning allows teachers to operatively check whether the focus material is mastered. Home tasks are regularly offered in order to consolidate the studied material, to develop skills and competencies of solving tasks as well as to plan systematic students' independent work. As a rule, tasks contain assignments tasks that were offered at zero test paper or similar ones. Tasks that are challenging for independent work are analyzed at the blackboard during an additional lesson.

The main element of the feedback system is the repeated "zero" test after a series of classes in elementary mathematics.

Importantly, in order to improve the design of mathematical schooling, it would be appropriate to officially include in the curriculum 8-10 teaching hours per group for elementary mathematics classes, since not all mathematics teachers agree to conduct additional elementary mathematics classes in their personal time.

Recently, there has spread an assumption that students study poorly because high school doesn't provide sufficient background for admission to higher education. The assumption is not incorrect and it is not enough to claim it. Let us analyze some of the reasons behind these facts: pupils (schoolchildren and students), those who teach (teachers, university professors), program requirements and knowledge system, educational literature.

1. Recently, it has been observed that some students are indifferent to classes. They study not for the sake of gaining knowledge but in order to better settle down in life and get a diploma of higher education. In real life it turns out that much of the knowledge is not employed which results in pupils' and students' attitude to learning.

2. A university professor is probably the only job where specialists are not displaying much special training in pedagogy and psychology of higher education. However, this profession requires not only the necessary knowledge but also inspiration, the ability to influence people. With inspiration and devotion professors are supposed to make their best to get the students interested in mathematics, to make them enjoy not only the process of solving the problem but also the chosen rational solution, the logical structure of mathematics, the validity of statements.

3. If you measure the student's workload, provided their conscientious attitude to learning (attending lectures, preparing for practical and laboratory work, homework, etc.) it averages about 14 to 15 hours a day. This explains why students avoid systematic work. In our opinion, the overload of students is the main reason why some students study poorly.

4. A well-known pedagogue, K.D. Ushinsky says that a textbook is the foundation of good learning. This statement fully applies not only to high school but also to higher education. The nature of the textbook, the style of presentation, as well as its content are determined by the purpose that society puts forward before different levels of education. Unfortunately, the analysis of the content of textbooks in higher mathematics showed that mathematical theories are taught in textbooks for both mathematics and for engineers in isolation from the practical activities of the engineer, rather formally and abstractly.

The textbook should not resemble a scientific monograph. It is primarily intended to transfer knowledge in order to prepare students for the job. To do this, special attention should be paid to the elaboration of mathematical models in the technology, production facilities, etc. At the same time, in conditions of the degree system of education it is necessary to know who is going to study with these textbooks: future bachelors or masters.

One of the main reasons why students cannot think critically is the critically reduced class hours for a number of disciplines, including the reduction of hours devoted to the study of mathematics, which does not allow the teacher to consider a phenomenon in its emergence and development, nor analyze its state and prospects.

In order to mend the current situation, professors of the Department of Higher Mathematics at DSTU have been conducting optional classes (for students of lyceums and mathematics classes of high schools in the city), seminars (for students who study well and may become masters in the future), where they discuss some historical aspects of mathematics, their current state, possible prospects of further development etc. The outcome of these seminars, not provided by the program, is preparing abstracts and presentations at scientific student conferences and competitions. In our opinion, holding such seminars clearly represents the differentiation of the educational process and promotes students' success in developing creative critical thinking and increased interest in learning.

To implement the principles of professional adaptation in the general course of mathematics taught to students of technical specialties of higher educational institutions, it is first necessary to ensure the links of mathematics with the study of both general and special technical disciplines.

The implementation of interdisciplinary links in the educational process has become one of the most pressing problems in the methodology of teaching various disciplines in universities.

This problem is studied in broad context as well as in terms of teaching the fundamental material in higher education.

Researchers study the methodology of teaching in higher education and means of management of educational and cognitive activity. Of great importance for this context is interdisciplinary links and links of mathematics with specific subjects for each course. Importantly, higher education is the equipping of students with methods of independent work and research activities. These include the study of methods of science and ways of their application, instilling the ability to independently find the necessary scientific and technical information and supplement their knowledge, creatively conduct scientific research, learn the method of scientific organization of work, the ability to analyze and synthesize facts and phenomena. It is vital for university professors to take into account the laws of the learning process and the sequence of studying the subject:

- propaedeutic study of the subject;
- formulation of patterns of knowledge in the subject structure;
- logically organized algorithmic study of the subject;
- solving some typical problems of a specific subject (physical problem statement, mathematical problem statement, obtaining and analysis of numerical results, technical conclusions).

It is proved that interdisciplinary links in their systematic and purposeful implementation restructure the whole learning process, because they act as a modern didactic principle.

It is worth mentioning that this principle of didactics, especially its organizational and methodological part, can be applied in the work of a university professor, as it will help to smooth the existing contradictions in the teaching of mathematics and special disciplines in technical universities.

Let us note that during a lecture there may be a situation where the student ceases to understand the material or part of the material is moved to independent hours. Such omissions in lectures are usually filled in practical classes and partially supplement it.

Depending on the main purpose of practical classes in the educational process of a university three types of practical classes are most often held:

- exercises;
- seminars;
- laboratory works.

The first type (exercises) is used predominantly when it comes to relation to the course "Higher Mathematics", taught at the faculties of technical profile during the first year of study. Sometimes exercise mode entails elements of the seminar form of conducting practical classes. This happens when students have to explain some theoretical provisions of mathematics, which either they were not aware of in lectures, or they were asked to study some theoretical aspect independently. In recent years, in connection with the use of computer technology some higher educational institutions demonstrate wider use of laboratory form of training. Officially, in addition to exercises, practical classes of this form or another are to be recommended by the mathematics department, considered by the methodical commission of the faculty and approved by the decision of the academic council of the university.

Note that practical classes create convenient conditions for communicating effective feedback. This communication can flow naturally, especially when the students are already prepared for it. For example, when students were offered homework that contains tasks from the lecture course.

Thus, in addition to the main purpose of practical classes there are the following:

- improving the process of mastering, elaborating and supplementing scientific and theoretical material;
- filling the sections allocated by the lecture for independent study;
- testing knowledge of the material;
- implementation of feedback in solving problems, including professional orientation.

The content of practical classes is largely determined by the type of task set by the teacher of mathematics. This can be the formation of certain problem-solving skills, consolidation of theoretical provisions of the course, demonstration of the use of mathematics in special issues, consideration of some applied problems. Such a variety of tasks allows to improve the ways of realization of the professional orientation of teaching mathematics. The content of practical classes of the general course of mathematics should be determined by compliance with the following principles of professional orientation of teaching mathematics.

Compliance with the principle of fundamentality fills the content of practical classes primarily with tasks of purely mathematical nature, which correspond to the topic of a particular lesson. Such tasks are necessary. They help develop the skills to quickly solve the simplest problems which forms elements of mathematical culture.

However, excessive interest in formal problems contributes to the fact that practical classes can become as routine as lectures and will only fulfill the principle of fundamentality in the study of mathematics. Here it is advisable to follow the requirement of a sufficient number of solved formal problems and insist on classical methods of mathematical analysis when it comes to solving such problems.

It’s worth noting that the content of formal tasks hardly takes into account the modern requirements for mathematical education in this specialty. In this case, the principle of professional unambiguity will be used. Such tasks convince students of the importance of mathematics for the study of their chosen technical specialty and increase the motivation to learn. The principle of professional unambiguity means that of primary importance in practical classes are the tasks where the conceptual apparatus corresponds to this specialty and after that students process the tasks that correspond to the mathematical model built on the lecture, they help find solutions by known to students by mathematical methods.

The selection of examples with a focus on future professional activities to be solved in practical classes can entail assignments, the solution of which requires the expansion of the mathematical apparatus beyond the existing program of mathematics. This makes it possible to implement the third principle in the system of methods of teaching mathematics at the faculties of technical profile, which opens new mathematical ideas.

It turns out that the system of exercises in the general course of mathematics should be designed in terms of content in such a way that the following methodological requirements are met:

- sufficient number of formal tasks;
- professional unambiguity;
- applied content.

The requirement for a sufficient number of formal problems means solving in practical classes such a number of tasks from specified textbooks in higher mathematics that would provide the necessary level of knowledge acquisition for the formation of skills and abilities.

The requirement of professional unambiguity means the solution in practical classes of a number of text problems that are directly related to the chosen specialty, as well as tasks where the mathematical models are built on the lecture course.

The requirement of applied content means solving problems related to the future technical specialty of students in practical classes. In senior courses, when studying special mathematics courses in the relevant practical classes, it is necessary to teach students to compose equations, measure the impact of a specific parameter on the solution as well as tasks with incomplete input, when the researcher needs to decide which of the auxiliary hypotheses to choose to obtain numerical results.

It is difficult to recommend the amount of time to be spent on the implementation of each of the requirements in a particular lesson or when studying this topic, because there are no clear recipes. One should be guided primarily by the specific circumstances that may arise when studying this topic. This may be either the lack of a set of appropriate applied tasks with the required professional orientation, or the topic itself requires significant time to master it, or it is merely due to students' unpreparedness for this task, this the solution is more time consuming etc.

Of high importance for improving the professional orientation of teaching mathematics in practical classes are appropriate means which can aid in solving the problem more efficiently. Among them, the first place is occupied by the relevant literature. This should be published series of lectures with a strong professional and applied emphasis, collections of relevant tasks, guidelines for specific topics and syllabi. There are almost no such textbooks. There are many successful collections of formal tasks for the general course of "higher mathematics" and its individual sections, but there are no textbooks that would contain a sufficient number of different applied problems with the annotated solution or with instructions on how to do it independently. Therefore, the primary task of mathematics teachers working at technical faculties should be to create professionally oriented textbooks which, in our opinion, should consist of appropriately developed syllabi of special mathematics courses for students of technical specialties, the texts of lectures of all courses as well as relevant for a specific specialty tasks of applied nature.

Thus, among the three forms of practical training in the study of general course of mathematics exercises are used predominantly. Elements of the seminar form of classes are not excluded and can be used depending on the tasks that are implemented in the practical lesson.

Spurred by the use of information and communication technologies in the educational process (the study of a topic with the help of educational computer programs) laboratory form of classes is used more and more often. This is a relatively new form of practical training, which is very closely related to the course "Computer Science" and can be implemented from the second semester in line with the study of the basics of computer science and programming.

The tools which can be used in practical classes for more efficient work have been identified. In addition to collections of problems in higher mathematics, these include work programs of special mathematics courses for students of technical specialties of universities, textbooks of applied Content, mathematics trainers for personal computers, visual aids and more.

Thus, in the process of training higher education students must acquire skills of independent work, the ability to acquire knowledge independently, use the acquired skills and abilities in practice, acquire general and special competencies and competencies.

Mathematics in technical institutions of higher education: applied orientation

Olha Bondar, Oksana Zadorozhna, Irina Yakunina

Formulation of the problem

The foundation of mathematical education of a specialist is laid in secondary school. Mathematical training in higher school is based on this foundation. Higher mathematics education, in its turn, is the basis for the professional activity of a professional.

Therefore, it is natural to consider the challenges of mathematics education in higher education through this chain of relationships. In other words, the challenges of tertiary mathematics education can be traced to secondary education. They are the problems of the inconsistency of school math education with the needs and requirements of tertiary education. Tertiary education in its turn does not always answer the requirements to the level of mathematical training of a specialist in a developed society[1].

Many young professionals are familiar with the problem when a significant part of the knowledge, skills and abilities they have accumulated in higher education is not used in their further work. Communication with many stakeholders confirms this. Even excellent knowledge of mathematics obtained in the higher education institution is often stalled. That is, the mathematical knowledge of a specialist exists on its own, not finding any application in their professional activity.

Analysis of recent research and publications

Analysis of research and publications shows different approaches to solving the problems of mathematics education in higher education institutions.

Some radically set higher school graduates believe that mathematics should not be taught to students who do not specialize in mathematics. According to these graduates, mathematics is not a discipline in quest; it has no bearing on modernity. They believe that now "everything is automated and optimized", "online calculators help in solving any problem of the math course". Therefore, a school mathematics course is enough to obtain higher education[2].

More moderate graduates consider mathematics to be an important subject designed to lay the foundation for future professional knowledge, to teach logical thinking, and to make connections between concepts. They do not talk about the limitations of academic disciplines, but "about their demand and relevance over time"[3].

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[1] Trius, Y., Baklanova, M. (2021, April 10). *Problems and prospects of higher mathematical education*. Retrieved from https://fi.npu.edu.ua/files/Zbirnik_KOSN/10/32.pdf. [In Ukr.].
[2] Yuzvin, Z. (2021, April 10). *University damn dozen.* Retrieved from https://studway.com.ua/predmeti/ [In Ukr.].
[3]Hadetska, S., Moroz , I. (Eds.). (2019). *Modern and historical problems of fundamental and applied mathematical training in higher education institutions: the view of higher education seekers and young scientists*. Kharkiv: KhNADU. [In Ukr.]

Successful professionals confirm that systematic fundamental mathematical training is a must for the graduates of technical institutions of higher education. But at the same time they emphasize that mathematical knowledge and skills must have an applied nature and be focused on the modern needs of society.

The use of up-to-date tasks of applied nature combined with their proper content involves the technology or methods of their actualization in the process of teaching mathematics. Meeting this challenge is a difficult pedagogical problem, which "requires due mathematical and methodological support"[4].

This approach is expressed in a combination of fundamental and applied mathematics. And the relationship between theoretical and professional orientation should take place in high school. The applied direction in this case provides orientation of the content and methods of teaching on close connection with real life and fundamentals of other sciences, as well as on a wide use of modern information and communication technologies[5].

Therefore, the optimization of the content of the discipline together with effective methods of its teaching should make mathematical education in higher education institutions an important tool of professional activity of a specialist. We are convinced that it is necessary to teach students in such a way that the knowledge received by them should not be stalled, but be one of the prerequisites to their professional growth in the future.

The purpose of the article

Finding ways to implement the applied teaching of mathematics is a very important methodological problem. In this paper we do not have the opportunity to reveal all its aspects in details. We will consider only some of those aspects, which are the objects of our perennial scientific and pedagogical activity. Our goal is to provide those involved in the educational process with practical approaches to solving the above mentioned problems. This concerns, first of all, to the problems of the applied nature of teaching mathematics in a technical higher education institution.

Presenting main material

To ensure high quality modern education in tertiary institutions, new tools are required. They should be focused on professional and personal development. The formation and use of pedagogical methods and means of teaching fundamental disciplines is of primary importance. Higher mathematics is one of the fundamental disciplines in technical institutions of higher education. Therefore, the content of mathematics combined with the methods of its teaching should largely shape the specialists of the new formation.

[4] Radyuk, L. (2021, April 11). *Peculiarities of methodical support of teaching mathematics in the process of professional training of junior specialists.* All-Ukrainian scientific-methodical Internet conference. Retrieved from https://college.nuph.edu.ua/wp-content/uploads/2017/04/T_2_Radiuk.pdf. [In Ukr.].

[5] Kudryavtsev, L., Kirilov, A., Burkovskaya, M., Zimina, O. (2021, April 10). *About tendencies and prospects of mathematical education*. Retrieved from http://www.academiaxxi.ru/Meth_Papers/Paper2.htm [In Rus.].

The quality of education should be determined not only by fundamental scientific training, but also by the ability to apply the acquired knowledge and skills in professional activities. That is, mathematics must have an applied character.

The concepts of applied and professional orientation of a subject are sometimes identified. Scholars and in-service teachers usually separate these concepts[6]. Thus, the professional character of mathematics determines its orientation on a particular specialty or profession. In a technical institution of higher education, the focus is on the professions of engineer, technologist, mechanic, etc.

The applied orientation of a subject, in our opinion, is broader than its professional direction. Thus, the applied nature of mathematics involves the orientation of its content and teaching methods on the close touch with life and the basics of other sciences, on preparing students for the use of mathematical knowledge in future professional activities, on the widespread use of modern information and communication technologies.

The applied direction of mathematics in a higher education institution involves the achievement of certain goals. One of them has to answer the question "What to teach?" Namely, what should be the content of the discipline, which sections and to what extent should be included in the course, with which disciplines and which relationships of mathematics with them are most important to consider. Another goal is to answer the question "How to teach?" Namely, what methods, tools, and organizational methods should be used for this?

There is no unambiguous answer to these questions. The scope of a course of higher mathematics is usually determined by the curriculum of the higher education institution. Teachers sometimes complain that this time is not enough for a quality study of the discipline. This time determines the selection of topics to be studied in the classroom and topics that are intended for independent study by students.

The selection of topics and the sequence of their teaching is an important task in terms of applied mathematics. They depend on the existing mathematical training of students beginning to study special subjects. They also depend on the distribution of disciplines by semesters in the curriculum. For example, the theme "Complex Numbers" must be mastered by students before they begin to study electrical engineering or radio electronics.

The structure of the material is also determined by the internal relationships of the themes of higher mathematics. There should be an optimal ratio of the content of the subject and methods of its teaching, namely the choice of forms, methods and teaching aids.

As it was mentioned above, the amount of time allocated for studying mathematics is usually not enough for a quality study of the discipline. But the ways of presenting the program material and the methods of its teaching can significantly increase not only the volume of the presented material, but also the rate and quality of its acquisition by the students.

Each teacher develops teaching methods due to their own experience and the experience of master teachers. In our opinion, the experience of the outstanding

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[6] Kolyagin, Y., Pikan, V. (1985). On the applied and practical orientation of teaching mathematics. *Mathematics in school,* 6. Retrieved from https://edu-lib.com/periodika/dlya-uchiteley-i-uchashhihsya/matematika-v-shkole/matematika-v-shkole-metodicheskiy-zhurn-125 [In Rus.].

Ukrainian teacher Victor Shatalov, acquired by him in secondary school, is important[7]. The ideology of this experience can also be applied in tertiary education.

Here is an example of one of the elements of the so-called Shatalov system in the teaching of higher mathematics in a technical higher education institution. This is a presentation of educational material in the so-called verbal-graphic form. It is often called Shatalov's reference signals.

A student of a technical higher education institution must know the definitions and be able to apply definite, double, triple, curvilinear and surface integrals of different types. It usually takes a lot of time to study each of these integrals separately. This time can be reduced due to the following material presentation scheme.

At the beginning of the study of the topic "Defined integrals" students are asked to consider a single graphical scheme with organized information about the types of integrals:

- the name of the integral;
- definition of the domain of integration;
- type of integrand function;
- graphic image (if possible) of the integration domain and function;
- integral sum;
- the sign of the image of the integral.

Based on this scheme, the student gets a general concept of the integral. It gives students an understanding of the applied aspects of the topic, such as the calculation of domains, volumes, work, moments of inertia, etc. This way of presenting the topic makes time for the subsequent acquisition of computational skills and completing creative tasks.

Generalization of the concept of integral on the basis of its specific types can be perceived by students in different ways. Depending on the perception of the material, the teacher may choose a different approach. Namely, the definition of the integral in terms of basic science should be considered first and then definitions and methods for calculating specific types of these integrals should be introduced. This method gives students the opportunity to combine the concept of mathematics as a basic science, with its applied aspects.

Another example of combining the fundamentals of mathematics with its applied orientation is the method of studying the topic "Vectors". Students should point out that the definition of a vector can be given in different ways. This can be the definition of a geometric vector as a directed segment of a line of Euclidean space.

The definition of the vector can be introduced axiomatically. Namely, a vector or point of a vector space is an element of linear space over a scalar field. The scalar field is an additive Abelian group in which multiplication by scalars is defined, which satisfies certain axioms. In such a rather abstract form, it is sometimes difficult for a student to see vectors as models of real objects in professional tasks.

So first it makes sense to give different definitions of the vector and point out their relationship. Then it is expedient, from our point of view, to consider the definition

[7] Vinogradov, S. (2009). Shatalov's system - what is the point? *Science and life*, 2, 78-85. [In Rus.].

directed on application by future specialists of a technical, economic, financial direction. The more areas of application of vectors will be considered, the sooner the future specialist will learn to recognize them in their practice.

Thus, the definition of a vector as a directed segment of a line is often used in engineering specialties. There, with the help of vectors, for example, the movement of mechanisms and their technical characteristics are studied.

However, the vector can be considered as an ordered set of n numbers. In other words, a vector is an ordered amount n of numbers that satisfies certain rules of actions with them. Examples of the n-dimensional vector defined in this way are n technical parameters of the mechanism, cost of n material objects, profit of n enterprises, etc. In this case, the actions on the vectors will have an interpretation corresponding to the examples.

It is possible to combine the fundamental foundations of mathematics with its applied direction when studying, for example, the topic "Equations of a line and a plane". Students' attention should be drawn to the existence of different ways of studying these geometric objects. A plane can be considered as a set of two sets that do not intersect. This is a set of points and a set of lines with a symmetric incidence ratio. This approach helps students distinguish between projective, affine, hyperbolic, elliptical, and other planes.

The transition from the abstract form of representation of the plane to its practical applications helps in introducing coordinates and the corresponding operations with them. In this case, the graphic image of a plane or line is transparent in two-dimensional and three-dimensional spaces. But in many applications, n-dimensional objects are considered, where $n > 3$. Their graphic representation may not be necessary to solve the problem.

Therefore, it is advisable for the teacher to draw students' attention to the equation of line and plane, as a description of the relationship between independent variables. This can be a relationship, for example, between distances, speeds, resource costs, and so on. That is, the student, together with knowledge of the origin of the name of the n-dimensional linear equation, must understand it, also as an expression of the interdependence of variables included in the equation. This presentation of the topic is quite clear to students.

A similar transition from the abstract form of representation of mathematical concepts to their applied aspects can be actualized while mastering the theme "Derivatives of functions". First, the concept of differentiation of mapping is considered. It concerns vector functions of points in abstract space. Then the set of complex and real numbers is taken as this space. Finally, the derivatives of real functions from several real variables are discussed. Here, students can already see a clear connection between these derivatives and the derivative of functions of one variable studied at school.

The applied aspect of the derivative is illustrated by examples containing geometric and mechanical meaning of the derivative. These can be the examples of the speed of a material body, acceleration, and so on. These derivatives, considered as the rate of change of one economic or mechanical parameter of the system from another or other parameters, are perceived as being closer to professional activity. Marginal values[8] are a well-known example of such an illustration.

An important chain that combines mathematical theory with practice is the justification and application of empirical formulas. For example, the application of the least squares method allows the student to obtain different types of functional dependence. Based on these dependencies, the student can make predictions of changes in the studied quantities and test these predictions in practice.

The transition from practical problems to theoretical methods of solving them is sometimes quite difficult for students. They have to be able to see a mathematical model in problems. Namely, it is necessary to be able to separate essential variables from insignificant, to define forms of relationship between variables, and to set priority purposes. Together with the creation of a mathematical model, the teacher can outline the essence of the method of mathematical modeling.

As it was noted, the achievement of the applied orientation of teaching mathematics is facilitated by the tasks of practical content. Together with the general requirements for mathematical problems, problems of an applied nature must meet additional requirements. First, the condition of the problem must be clear at the level of scope of its application. Second, it must describe real processes with real input, real question and solution.

The practical content of the task should reveal the application of mathematics not only in technical disciplines, but also in related disciplines, in the organization and technology of modern production. So-called tasks-schemes or stereotypical tasks play a significant role in creating such problems. These are the tasks formulated using mathematical terminology and at first glance they seem to be devoid of any applied nature. But the formulation of these problems in terms of different areas of human activity allows us to see mathematical models in the profession of a graduate of a higher education institution.

Problems of practical content are presented in textbooks mainly in the form of standard or stereotypical formulations[9]. The transition from standard formulations to professionally oriented formulations requires some effort from both the teacher and the student. The teacher should encourage students to make such a transition. The standard wording may be called a task-scheme, which is transformed by the student into a profession-oriented task.

For instance, in the scheme of an elementary problem with tossing a coin, the teacher can first give examples of mechanism operation in one of two equivalent modes, examples of random selection of technical parameters from two identical varieties, and so on. And then the students are asked to come up with original problems of applied direction. Our practice shows that sometimes it is difficult. But the teacher's correction of the content of the problem in the process of creating its formulation helps the student to overcome difficulties.

Another example of standard problem formulation is the problem of testing statistical hypotheses. It gives an arbitrary statistical series for which it is necessary to establish whether this or that statistical hypothesis takes place. This task develops

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[8] Barkovsky, V., Barkovskaya, N. (2010). *Higher mathematics for economists*. Kyiv: Tsentr Uchbovoi Literatury. [In Ukr.].
[9] Shapiro I.M. (2021, April 9). *Use of problems with practical content in teaching mathematics*. Retrieved from https://old.altspu.ru/Journal/pedagog/pedagog_5/a12.html (Last accessed: 9.04.2021). [In Rus.].

students' mathematical skills. For instance, the following formulation can give it an applied direction: "Information on the number of items prohibited for carriage by air and the number of cargo units with these items has been obtained from official statistical sources. According to Pearson's criterion at the specified level of significance, it is necessary to test the hypothesis of the distribution of a random variable (number of objects) according to Poisson's law"[10].

After a mathematical solution of this task, the teacher asks students to make a conclusion about the organization of baggage control and possible ways to improve it.

The applied orientation of mathematics also means the application of interdisciplinary connections in it. These connections exist, first, at the level of like-named concepts. Such concepts in a technical higher education institution are, for example, vector, coordinates, equations, graphs. It should be noted that some concepts require harmonization of interpretation.

Secondly, mathematical means of representing the relationships between the parameters of the technical system are used in the study of other disciplines. Although the teacher of mathematics should not know special disciplines, he should be guided in the application of mathematical concepts. The teacher must have a broad scientific worldview that reflects current trends in science.

Conclusions and prospects for further research

We have considered some aspects of the ways of realization of the applied orientation of teaching mathematics in a technical higher education institution. Examples of a combination of fundamental mathematical concepts and practical methods of their application are given. Some topics of educational material are illustrated with examples of professional orientation. Methodical methods of formation of practical skills and abilities of future specialists in technical specialties in the implementation of the applied direction of mathematics are considered.

The prospect of our research is to further improve practical approaches to solving problems of mathematical training of students of technical institutions of higher education. This should contribute to increasing the level of methodological training of research and teaching staff, to improving the methodology of teaching mathematics in terms of its application. As a result, a graduate of a higher education institution will be ready for successful professional activity.

[10] Zadorozhna, O., Bondar, O., Semenyuta, M. (2020). *Probability and Statistics. Methodical guide to implementation of the laboratory works.* Kropyvnytskyi: Flight academy NAU. [In Eng.].

Integrated teaching of higher mathematics in the educational and scientific institute of automation and electromechanics (National University "Odessa Maritime Academy")

Natalia Orlova, Alla Varynska, Natalia Kornodudova

The National University "Odessa Maritime Academy" (NU "OMA") is one of the leaders among the maritime higher education institutions of Ukraine, thanks to the quality of higher education that thousands of highly qualified specialists received within the walls of the academy.

The academy trains specialists in all educational and qualification levels: bachelor, master in the main areas of maritime specialties. The Academy is accredited at the IV (highest in Ukraine) level of accreditation[1]. Ukraine's entry into the unified international educational process, the creation of competitive educational services increases the number of foreigners wishing to receive international diplomas in the maritime field at OMA. According to the OECD (Organization for Economic Co-operation and Development), the number of foreign students in the world will increase to 5 million by 2025[2].

Engineering majors are accredited by the Institute of Marine Engineering, Science and Technology (IMarEST) UK. The graduates of the academy have long been well known in the world maritime society. The standard of professional competence and registration of engineers, approved by the IMarEST Board in 2004, sets out the basic requirements for the training of marine professionals – *Chartered Engineer (CEng*), which corresponds to a master's degree.

Qualification *(CEng)* involves the skills required for technical and commercial leadership. A charter engineer is characterized by the ability to solve engineering problems using new and existing technologies, applying innovation and creativity.

Incorporated Engineer (lEng) corresponds to a bachelor's degree (national diploma of higher education).

Qualification *(lEng)* involves the possession of effective skills required for technical and commercial management. An incorporated engineer is characterized by the ability to act in accordance with new technologies, using creativity and innovative approaches.

Training of specialists capable of comprehensive solutions to the problems of development and operation (modernization) of new existing automation systems, theoretical research of the object of automation involves:

- ability to apply knowledge of mathematics to the extent necessary for the use of mathematical methods for analysis and synthesis of automation systems;
- basic knowledge of mechanics, heat and power in combination with in-depth training in electronics, control theory and computer technology.

[1] National University "Odessa Maritime Academy" [Electronic resource] Retrieved from http://www.onma.edu.ua/

[2] OECD (2012) / Education at Glance 2010: Add Highlights. - Paris. OECD / Publishing.2012. - 91p.] [Electronic resource] Retrieved from htpp: //www/oecd/org/edu/highlights.pdf.

On the basis of the educational and scientific institute of automation and electromechanics in NU "OMA" the direction of "automation" (151 Automation and computer-integrated technologies) is created, the purpose of which is the training of mechanical engineers for ships with the increased level of automation, electronics and computer management of the ship's power plant. The achievement of the main goal – training a highly qualified maritime specialist of the appropriate level and profile, competitive in the international labor market, who has experience in their profession and is able to work effectively in the specialty at world standards – requires knowledge and mastery of modern tools, including modern mathematical methods.

Higher mathematics is an educational discipline that is the foundation for the study of other general education, general engineering and special disciplines. Search for effective methods of teaching higher mathematics for bachelors and masters majoring in 151 "Automation and computer-integrated technologies" is one of the most important areas of work of teachers of the department "Higher Mathematics".

The curricula for the first (bachelor's) level of higher education (the first cycle of the European Higher Education Area Qualifications Framework) provide for 12 credits (4 semesters of 3 credits), for masters "Mathematical methods of scientific research" – 5 credits. Within a total of 360 hours (for the first bachelor's level); during 184 classroom hours it is planned to master topics – linear and vector algebra, differential and integral calculus, functions of many variables, functional series, differential equations for the function of one and many variables, operational calculus, function theory of complex variables, probability theory and mathematical statistics, random theory processes volume, which will allow the use of mathematical apparatus and methods in the field of automation.

With this amount of research material, an important area of training specialists[3] [4] is the use of modern pedagogical science in the organization of the learning process in tertiary education, development and application of modern professional-oriented teaching technologies, implementation of various didactic methods, forms and tools in the educational process.

The concept of "learning technology" was first mentioned in a report at a UNESCO conference in 1970, where learning technology was seen as a set of ways and means of communication between people arising from the information revolution and used in didactics. Most researchers believe[5] [6] [7] that learning technology is associated with the optimal construction and implementation of the educational process, taking into account the guaranteed achievement of goals, and one of the key points to reveal. The essence of the technological approach to the educational process in higher education is the use of appropriate tools and methods of teaching.

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[3] Savery, J.R., & Duffy, T.M. (1995). Problem based learning: an instructional model and its constructivist framework. *Educational Technology, 35.* p. 31–38.

[4] Orlova, N.D., Krylova, T.V., & Orlova, E.Yu.(2007). Application of professionally oriented learning technology to improve the mathematical training of the master. *Didactics of Mathematics "Problems and Research" International Collection of Scientific Papers, 26, (p.210-217*). Donets′k : DNU [In Ukr.]

[5] Skafa, E.I. (2004). Theoretical and methodological foundations of the formation of methods of heuristic activity in the study of mathematics in the context of the introduction of modern teaching technologies. *Doctor's thesis.* Kiev .[In Ukr.]

[6] Slepkan, Z.I. (2000). *Methods of teaching mathematics*. Kyyiv: Zodiak-EKO. [In Ukr.]

[7] Vilensky, M.Ya., Obraztsov, P.I., & Uman, A.I. (2005). *Technologies of vocational education in higher education*. Moskva: Pedagogicheskoye obshchestvo Rossii. [in Russian].

The criterion of the teacher and student at the technological level is the presence of a clearly defined goal. The material to be studied should be presented in the form of a system of cognitive and practical tasks, using modern means of learning and communication.

The purpose of using different learning technologies (as a process) is to form in a young specialist a willingness to scientifically sound understanding of their professional activities. At the stage of implementing instructional technologies in higher school there are the problems connected with the current features of mathematical education:

– the gap between the level of mathematical knowledge of school graduates and the requirements of higher education (Institution of higher education IHE);
– reducing the number of hours allocated for the course of higher mathematics;
– reduction of classroom classes and increase of hours for independent work;
– the gap between the level of mathematical knowledge of high school graduates and the objective needs of modern science and technology.

Learning technology is the characterized [5 7 8] division of the learning process into interconnected stages; coordination and step-by-step implementation of the actions aimed at achieving the goal set.

Particular attention should be paid to maintaining the continuity in the study of mathematics in the transition from secondary to tertiary education. The content of the new school course should be based on the interaction of school material with the prospect of higher education. The development of analytical skills should be continued during the study of mathematical subjects in the first year of higher education, where school education should expand and deepen not only in content but also in forms and methods of work.

In the process of studying mathematics[9] in high school, and then in a technical institution of higher education, it is necessary to form such a system of teaching mathematical disciplines, through which the knowledge and skills acquired by schoolchildren and students (cadets) can be used in solving applied problems.

The teaching of mathematics should be conducted at a certain level of rigor, be understandable to students and more profound. Nevertheless, one should not indulge in an overly rigorous presentation of complex mathematical material that will never be used in practice, this is especially true of material taught in higher education for technical specialties.

For example, in high school the problem of studying the function of one variable per extreme is solved (the problem of finding the largest or smallest values of volume, the surface area of geometric bodies is the task of physics at the largest and smallest value). In higher education for cadets who receive a qualification *(lEng)*, a similar problem is considered, which leads to the study of the function of many variables to

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[8] Orlova, N.D., & Popova L.K. (2009). Continuity of teaching in the construction of mathematical models in secondary and higher schools. *Didactics of Mathematics: Problems and Research: International Collection of Scientific Papers, 32* (pp. 75–76). Donets′k : DNU[in Russian].

[9] Panchenko, L.L .(2004). On the conceptual apparatus of mathematical modeling in secondary school and pedagogical university. *Scientific journal of NPU named after M.P.Dragomanov, 3,1,* p.89-97.Kyyiv: NPU imeni M.P.Drahomanova. [In Ukr.]

the unconditional or conditional extremum, and undergraduates *(CEng)* solve the problem of the search for the extremum of the functional.

Particular attention should be paid to the sequence of solving the problem of studying mathematics for foreign students studying in higher education institutions (HEI). In many cases, the level of training of foreign students in mathematics does not meet the requirements for HEI students (cadets) of Ukraine due to differences in the curriculums of Ukrainian and foreign schools.

Analyzing various aspects of adaptation and teaching of mathematical disciplines by foreign cadets, we observe the lack of a unified approach to the development of mathematical language of foreign cadets. In our opinion, for successful adaptation it is necessary to unite the efforts of teachers of the preparatory department and teachers of higher mathematics in HEI. Particular attention should be paid to the consistency in the teaching of mathematical language. The use of verbal and nonverbal teaching methods considered in the works of scientists[10] [11] facilitates the dynamic mastery of unfamiliar and complex mathematical material.

At the «OMA» National University, foreign cadets together with the Ukrainian cadets listen to lectures on general education and mathematics.

Thus, teaching foreign cadets mathematics as a general theoretical discipline contributes to the solution of several tasks: the study of the language of communication and the language of the subject; filling gaps in math knowledge; the re-study of a number of math themes.

For foreign cadets the main condition for mastering and understanding the scientific text is to master the terminology and scientific style patterns, which contain the most common semantic categories: subject qualification, qualitative characteristics of the subject, comparison of state and properties, process, purpose, conditions, causes, consequences and others.

The lexical and grammatical basis for the study of scientific style constructions is "The unified standard learning program in the Ukrainian language for foreign students of the basic faculties of the non-philological profile of higher educational institutions of Ukraine of the III-IV levels of accreditation"[12].

The initial stage of teaching the Ukrainian language to foreign students is carried out at the preparatory faculty of the NU "OMA". In the Ukrainian language classes, foreign students study phonetics, vocabulary, grammar and other curriculum material, and in parallel with the study of the language they master the higher mathematics lexicon. To this end, the departments of Ukrainian language and Higher mathematics are working together on educational and methodological materials, which reflect new approaches to working with texts of higher mathematics. Such approaches involve certain stages of presentation of material, which will help to overcome difficulties. Thus, mastering the language of higher mathematics by foreign students has its own characteristics and consists of 2 parts:

[10] Tarasenkova, N.A. (2004). *The theoretic-methodical principles of using of the sign and symbolic means in teaching mathematics of the basic school students. Doctor's thesis,* Cherkasy. [In Ukr.]

[11] Zinonos, N.O. (2015). On the question of learning mathematical language by foreign students of preparatory departments of universities. *Proceedings of the international scientific-methodical conference. Problems of mathematical education (PME-2015).* (pp. 111–112). Cherkas′kyy: natsional′nyy universytet im. B.Khmel′nits′koho . [In Ukr.]

[12] The unified standard learning program in the Ukrainian language for foreign students of the basic faculties of the non-philological profile of higher educational institutions of Ukraine of the III-IV levels of accreditation. (2009). Approved by the Ministry of Education and Science of Ukraine.Kyyiv: NTUU «KPI».[In Ukr.]

1) introduction and practical mastering of the main rules of pronunciation and reading in Ukrainian, learning the general vocabulary ;

2) mastering the Ukrainian language terminology of higher mathematics[13].

At the preparatory department test tasks which are convenient for intermediate, current, and also final control gain popularity. In the structure of test tasks it is necessary to take into account the l vocabulary of a foreign listener, their basic level of knowledge in higher mathematics, the level of mastering the material. The first means that in test tasks it is necessary to pay attention to the understanding of terminology and at the same time in interaction with the language of instruction to show the connection between different parts of the sentence. This contributes to a better understanding of the formulation of the laws of higher mathematics and the definitions of the largest key operations (reduce the fraction, find the percentage, etc.).[14]

Teachers also use interactive learning technologies based on developmental exercises: collective-group (microphone, unfinished sentences, brainstorming), cooperative (work in pairs, in groups; carousel), situational modeling (dramatization, role play), discussion (method -press, case method). Each of these groups of exercises develops separate types of speech, for example, offer during training such tasks as discussion on a certain topic, lexical games, phonetic exercises, etc.[15]

At the main stage of study (I-II courses) foreign students acquire and improve certain skills and abilities that they need at the main faculty in the field of study: foreign students need to understand scientific terminology and constructions in the technical profile of study, master speech in oral and written forms.

In order to intensify the process of language acquisition, the departments of Ukrainian language together with the department Higher mathematics and other technical departments of the NU "OMA" provides coordinated teaching of higher mathematics and the Ukrainian language, which will help optimize the educational process.

Thus, the departments of the Ukrainian language of the NU «OMA» develop methodological materials and manuals, which offer small texts from the basic textbooks of basic technical disciplines.

First, each topic is preceded by new words – key terms and phrases of higher mathematics, which are most common in the following texts. After each text of the test, control questions and tasks for self-examination are offered, among which there are also obligatory examples of problem solving and the tasks for independent study of the material in all sections of higher mathematics. The acquired knowledge of the terminology of higher mathematics contributes to the mathematical and general

[13] Orlova, N.D., & Kornodudova, N.M. (2018). About interrelation of methods of studying a course of mathematics by foreigners at preparatory departments and in tertiary education. *XXXII International Scientific and Practical Conference "Development of Science in the XXI Century" (February 15, 2018). Collection of scientific articles (standard level, academic level) 3,* (pp. 30-36). Kharkiv: Naukovo-informatsiynyy tsentr «Znannya». [In Ukr.]

[14] Pankratov, Cyril.(2020).Odessa National Polytechnic University. Specific features of test tasks in mathematics for foreign students at the initial stage of study. *Topical issues of organization of education of foreign students in Ukraine*: *V International Scientific and Methodological Conference, October 14-16, (pp.48-49).* Ternopil′: Ternopil′s′kyy natsional′nyy tekhnichnyy universytet imeni Ivana Pulyuya. [In Ukr.]

[15] Pometun, O.I., & Pirozhenko, L.V. (2004). *A modern lesson. Interactive learning technologies.* Kyyiv: Vydavnytstvo A.S.K. [In Ukr.]

development of foreign students, abstract and logical thinking, which is essential for future professionals.

As it has been mentioned, one of the ways of solving these problems is to compile manuals for foreign students of the preparatory faculty and foreign students of the Ist – IInd years of study, based on the sections of the Higher Mathematics course. When doing this, educators should try to make use of the vocabulary used in the study of the material, and vice versa, the texts of the manual should be considered as examples of the adapted language of the teacher. At the initial stage of learning mathematics, we also do not recommend the use of additional third-party information, which complicates the perception of the material presented in the manual; We also do not recommend offering foreigners the authentic, non-adapted wording, which is not in the textbooks and which the students are not able to accept due to poor command of the Ukrainian language. Textbooks for international students should contain theoretical material and examples illustrating mathematical concepts and terms.

The manual must include exercises:

− to practice terminological vocabulary, which contributes to the formation of mathematical knowledge and skills;

− be sure to provide a lexical minimum of mathematical terms and symbols;

− solution of standard exercises on this topic with a thorough explanation of the problem;

− at the end of each topic, tasks for self-study with answers to them are offered;

− it is desirable to divide all tasks into three levels of difficulty.

Solving the problems of the first level will reveal the mastery of the material at the level of *D* or *E* (ECTS scale); solving problems of the second level shows the assimilation of material at the analytical level *C* (ECTS scale), which allows you to solve problems in different ways; the solution of the third group of tasks indicates the mastery of the material at the creative level *A* or *B* (ECTS scale).

The professional component is based on the language and speech, in particular, terminological, material, which reflect the features of written and oral texts in the specialty of foreign students, which is the basis for the formation of skills in speech activities that are relevant to the communicative needs of foreign students, implementation of professional communication in the relevant field of education[16].

For example, when studying the topic "Qualification of the subject: the meaning of the predicate and methods of expression" it is necessary to understand the constructions: *what (is) what; that (it) that; bearing (has, received) the name of what; what is what; that is that; by what do we mean that* and others. Particular attention should be paid to the linguistic construction of *what is what*, because in language it has the function of the general qualification of the subject.

We offer foreign students to study mathematical constructions in a small amount of text, where in addition to the common expressions there are also mathematical terms. To do this, it is advisable to perform the following tasks:

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[16] Ushakova, Natalia., & Trostinskaya, Oksana. (2014). Learning the Ukrainian language by foreign students: conceptual principles. *Theory and practice of teaching Ukrainian as a foreign language, 9.* (pp.12–21). L′viv: NU im. Ivana Franka. [In Ukr.]

1). Read the text. Pay attention to the underlined constructions in the text, write them out;

2). Pay attention to the definitions of *experience, hypothesis and theory*.
What constructions express these definitions?;

3). Define these objects and phenomena. Use constructions: what is what, what do we mean by, what is what;

4). Answer the questions to the text: What is the *experience* in...? What is *hypothesis*…? What do we mean by...? and others;

5). Make a sentence from the given words and phrases, using the proper predicate;

6). Find the correspondence (3-4 tables at the level of translation, constructions, semantics);

7). Complete the dialogue, and others.

In the process of higher mathematics acquisition, the main difficulties for its understanding are not only the content, but also the accompanying verbal explanations, because at almost all stages of learning the perception of the material is hindered by the language barrier. Under these conditions, the first thing that matters is the form of presentation of the material, which best ensures its understanding and assimilation. A partial solution to this problem is possible through the use of the language of symbols in mathematics, which in its general basis is an international language, and therefore, foreign students understand it. The use of symbols in the language of mathematics in this sense allows minimizing verbal explanations, reduces the language barrier, and improves the quality of learning. The teacher's language should be legible, well-thought-out, slow, concise and adapted to the level of students’ proficiency in the Ukrainian language. Teachers conducting practical classes should use the notation and mathematical terminology used in lectures. It should be remembered that fluent authentic speech may be perceived by foreigners as a continuous stream of sounds, which causes irritation, and sometimes leads to a complete refusal to attend lectures, listen to the teacher and participate in practical training. Under such conditions, sometimes it is better to remain silent than to say a lot of incomprehensible things. Therefore, the assistant who conducts practical classes should not be cross when students do not show an immediate knowledge of some formulas of elementary mathematics, these formulas are to be written on the board and the teacher should explain how to use them. It may turn out that these formulas are known to students, and it is the insufficient level of the mastery of mathematical terminology in the Ukrainian language makes them "unknown".

Mastering the topic of each lesson involves: mastering foreign language tools by foreign students to build a correct mathematical expression; drawing up a plan and method of solving the problem; ability to use the synopsis; discussion of the topic and method of solving a mathematical problem; correct understanding and use of mathematical terminology; vocabulary development.

Today, we face the challenge of intensifying the teaching of higher mathematics, finding more productive ways of explaining the material, teaching the cadets how to implement the acquired knowledge in further professional activities and to become a creative specialist able to quickly navigate the changing world of high technology. Activation of the educational process consists of two components:

1) activation of the teacher (improvement of scientific knowledge, pedagogical skills, content, forms and methods of teaching) and

2) activation of students. Intensification of the learning process is focused on the cognitive activity of cadets, their psychological abilities to master the material.

The system of mathematics education is becoming contradictory: higher mathematics education is becoming more complicated, and conditions are becoming less favorable, which leads to a deterioration in the level of mathematical training. All this encourages the improvement and search for methods of intensification and intensification of the educational process.

With the reduction of the number of hours allocated to the course of higher mathematics, teachers face the problem of presenting rather large and complex mathematical theories (for example, the theory of functions of complex variables and interdependent operational calculus; series and Fourier transforms, etc.). Note that the operational calculus is successfully used to solve technical problems of automatic control systems and the study of electrical circuits[17].

The solution of this problem is not possible without the introduction into the educational process of methodological complexes in higher mathematics, containing the texts of lectures and practical classes, on the basis of which you can offer a large number of creative tasks.

Educational complexes consist of methodical manuals developed by teachers and issued in the form of electronic and printed methodical instructions.

In this case, an electronic learning model will help as it has some advantages due to the pace of presentation of material with the possibility of self-study of the simplest part of the course, the possibility of simultaneous use of different pedagogical technologies (for example, explanatory lecture with elements of research and practice). Practice shows that in this format it is advisable to consider voluminous and complex mathematical topics (theory of functions of complex variables, operational calculus, Fourier series, stability theory, etc.), which, according to the curriculum, are allocated unreasonably few lecture hours.

Explaining complex theoretical material in the format of a video conference, the teacher (tutor) has the opportunity to fully disclose the topic using models - case technology and correspondence training and the cadet can study it individually, processing complex points again.

Cadets can be asked to keep special workbooks to examine complex mathematical proofs that can be distributed through telecommunications technologies and Internet resources. In this case, the teacher (tutor) has the opportunity to prepare quality manuals, involving the most active cadets in this work. Thanks to various methodological complexes, lectures and practical classes both on-line and off-line are transformed into an active dialogue of the teacher with the cadets.

Changing the structure and expanding the use of mathematical methods requires an organic combination of classical and traditional methods of teaching higher mathematics and will lead to the development of non-traditional forms and new methods and teaching aids.

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[17] Popov, V.G., & Orlova, N.D. (2009). *Mathematical methods of scientific research of automatic control systems.* Odesa: ONMA. [In Ukr.].

Thus, the directions of intensification of the educational process and activation of cognitive activity of Ukrainian and foreign cadets are: creation of educational complexes focused on the performance of independent individual and control works; active participation in conducting and preparing lectures and practical classes; preparation of training modules for gifted students who take part in mathematical olympiads and student conferences; professional orientation of teaching mathematics; formation of mathematical competence and development of creative initiative; application of computer technologies with the use of electronic textbooks, software and methodological complex, training courses, mathematical packages in solving complex mathematical problems.

The greatest difficulties in mastering complex mathematical topics are experienced by foreign students. When lecturing and conducting practical classes, the language and style of presentation of the material, it is desirable to adapt the material in higher mathematics for it to be understood by the foreign students at this stage,. Therefore, when lecturing, it is advisable to select the most necessary vocabulary from the entire volume of mathematical lexicon, to focus on those terms and words of general literary language, without the knowledge of which it is impossible to further master the course of higher mathematics. Vocabulary selected in this way should be activised during practical classes, while the rest of it can remain passive vocabulary.

Experience shows that the texts used in special disciplines cause significant difficulties in reading, learning terminology, understanding the content, namely – texts in higher mathematics (functional series, differential equations for the function of one and many variables, operational calculus, function theory of a complex variable, probability theory and mathematical statistics, the theory of random processes in the volume required for the use of mathematical apparatus and methods in automation) and theoretical mechanics (statics, kinematics, analytical mechanics). As a rule, each topic of the discipline contains a text-block of the fundamental material, which is to be mastered by a foreign student.

Knowledge of higher mathematics terminology contributes to the mathematical and general development of foreign students, their abstract and logical thinking, which is essential for future professionals.

Forms and methods of organizing students' unaided work when mastering "Higher Mathematics" should be given special attention. Increasing the hours for self-study of some sections of the course of higher mathematics presupposes self-education of cadets and systematic, *managed* and *controlled* by the teacher independent activities.

In the pedagogical literature, independent work, as one of the basic principles of learning, has been in the focus of attention since the end of the XVII century. German educator Adolf Disterweg (1760-1866), believed, that[18] "the development and education of more than one person cannot inherit. Everyone who wants to touch them must achieve it on their own, so independent work is a means and result of education'». In European countries and in the United States, increasing the time for independent work compared to the lecture form of classes is considered the most effective for improving the quality of training.

Independent work contributes to the expansion and deepening of knowledge in mathematics, the formation of interest in the professional and cognitive activities of

[18] Diesterweg, (1851). *A school's curriculum.* 4 te Aufl Essen.

cadets, the development of cognitive abilities and mastering the techniques of a complex process of cognition.

From the Curriculum of the discipline "Higher Mathematics" and the Syllabus of the discipline "Higher Mathematics", which are developed in accordance with the educational-professional program (EPP) "Automated control of ship power plants" (table 1) it follows that independent work of cadets is an integral part of a personalized teacher (tutor) program for teaching cadets in higher mathematics, which is updated at the beginning of each academic year. In the conditions of new technologies of training the teacher creates an essentially new complex of educational and methodical documentation based on the program, explaining the essence and the form of a course of higher mathematics, deadlines, procedures, and the assessment principles.

SYLLABUS

SPECIALTY 271 "RIVER AND SEA TRANSPORT"
EDUCATIONAL-PROFESSIONAL PROGRAM
"OPERATION OF SHIP ELECTRICAL EQUIPMENT AND AUTOMATION MEANS"
OF THE FIRST (BACHELOR'S) STAGE

The syllabus of the discipline Higher Mathematics is developed in accordance with the educational-professional program (EPP) Automated control of ship power plants

Developer (s): _Orlova N.D., Associate Professor

The syllabus was approved at a meeting of the Department of Higher Mathematics
Protocol of "___" ______________ "2020 № __

Head of the department Popov V.G.
Secretary of the Department Demidov O.V.

Syllabus was agreed with the guarantor EPP "Operation of ship electrical equipment and automation" Budashko V.V.

1. General information	
Name of educational component:	Higher mathematics
Discipline status (required / optional)	is required
Code of the educational component:	FC1
Type of educational component:	higher mathematics
Year of implementation:	Year of recruitment 2020
Semester:	1,2,3,4.
The department to which the training is assigned discipline	Higher Mathematics

Prerequisites	Knowledge of mathematics in the scope of high school program at a level determined by the regulatory requirements of the UHE, sufficient for further study in the UHE.
Postrequisites	Obtaining knowledge, skills and abilities necessary for mastering general scientific and relevant special disciplines, as well as for the application of mathematical methods in professional activities.
Link to the site of distance learning	http://www.onma.edu.ua/systema-dystantsijnogo-dostupu-do-navch

The language of instruction is – Ukrainian
Number of ECTS credits – 12

The total amount of time to study the discipline	360
Including classroom classes	184
Of these: lectures	102
laboratory / simulation classes	
practical	82
Types of classes and works: (Independent control work (ICW), classroom control work (ACW), abstract (AB), calculation and graphic work (CGW), course work (CW), course project (CP))	*2* CGW
The amount of time for independent work	176
From them: preparation for lectures, independent processing of theoretical material	60
preparation for laboratory classes / classes on simulators	-
preparation for practical classes	56
implementation (CGW)	60
Final form of control (exam, credit)	exam

Table 1.

49% of hours for this discipline is allocated for independent work of cadets, and this work is organized, provided and supervised by teachers of the department "Higher Mathematics". Independent work involves - performance of calculation and graphic tasks (CGW), computational work (CW) in accordance with the curriculum.

The main purpose is to teach cadets methods of independent work with educational material. The material to be studied independently classes is outlined in the development of the work program. The syllabus is approved at the meetings of the department; it is in the calendar plans of lectures and practical classes.

The practice of organizing independent classes in higher mathematics allows us to formulate the requirements that must be met by the material offered for independent work:

- if possible, the material should not contain new mathematical concepts, but it should expand and deepen the already mastered concepts and definitions;
- it should contain information on how to deepen the knowledge gained in class and work with problematic issues;
- the material offered for self-study must meet all the requirements of didactic support of independent work (it is enough to be fully set out in the manual or textbook;
- the availability of a sufficient number of textbooks, teaching materials for CGW, CW).

Independent work of cadets under the guidance of the teacher is carried out for the purpose of the acquisition of skills of work on mathematical literature, fundamental studying of theoretical questions and those subjects of educational programs which are necessary to perform CGW, CW, and writing abstracts. The amount of study material planned for one hour of independent study should not exceed the amount planned for one hour of lectures or practical classes. Particular importance in independent work is given to individual tasks, when the teacher seeks to summarize the experience of cadets to expand the scope of the course of higher mathematics, to prepare the cadet for the Mathematical Olympiad or a speech at a conference.

The use of Internet technologies increases the individual activity of cadets, their initiative grows, the cadet independently searches for the necessary information (work on the Internet with electronic media - electronic textbooks, media library, audio library, etc.), studies, analyzes, compares the results, chooses the most favorable method of solution. The organization of independent work of cadets in such format allows involving practically all cadets in active educational and cognitive process.

It should also be taken into account that the "tutor - cadet" contact is necessary not only to explain unclear learning topics, but also to support cooperation between both "tutor - cadet" and "cadet - cadet". One of the ways to organize independent work with theoretical material (a group of cadets over 25) is to work on the type of triads[19]. Practical classes should be designed to improve the skills of finding the best answers, calculations, solutions.

Independence of thinking develops only when cadets offer their solutions to problems (both right and wrong). Group work enhances the factor of motivation and mutual intellectual activity, increases the effectiveness of cognitive activity through mutual control.

[19] Jarvis, R., Dempsey, K., Gutierrez, G., Lewis, D., Rouleau, K., & Stone, B. (2017). Peer coaching that works: The power of reflection and feedback in teacher triad teams. Denver, CO: McREL International. https://files.eric.ed.gov/fulltext/ED588635.pdf

The participation of the partner significantly restructures the psychology of the cadet. In the case of individual training, the cadet subjectively assesses his activity as complete and complete, but such an assessment may be erroneous.

At group individual work there takes place a group self-check with the subsequent correction by the teacher. With a fairly high level of independent work, the cadet can perform the individual part of the work and show it to a classmate.

Distance learning changes the forms of consultations (question-answer) and creates the possibility of a new type of consultations - discussion and lectures. The position of the teacher is the position of the consultant, which creates an atmosphere of "freedom of choice in teaching" the subject of higher mathematics: to prepare and adjust the cadets to the information and the process that will be offered in the next stages of work; to offer cadets who have good mathematical training for in-depth study of special sections of the course "Higher Mathematics" and indicate the possibility of their use in future specialties.

When controlling cadets' knowledge, the basic didactic principles of knowledge testing and assessment should be followed and observed[20], "the main ones are six didactic principles of knowledge testing and assessment: activity, systematicity, individuality, differentiation, objectivity and unity of requirements."Taking these principles as a system of knowledge control, it is necessary to establish a minimum that must be mastered to ensure an adequate perception of higher mathematics. Current control should be carried out systematically, at each lesson in higher mathematics.

An important task is to close the gap between the level of mathematical knowledge of freelance graduates and their ability to solve engineering problems using new and existing technologies, applying innovation and creativity.

The methodological basis in teaching the creation of mathematical models[21] is to master first a simplified scheme for creating a model (school course), and then extended schemes (for bachelors) and extended schemes with subsequent forecasting of results (for masters). Cadets who solve applied problems are more aware that mathematical disciplines integrated into computer technology are a powerful tool for the study of natural phenomena.

Models of real objects, modeling phenomena have long been used in science and engineering practice to test ideas, test hypotheses, and obtain experimental material. The future master must effectively use and implement in his subject activity new information and communication technologies, process experimental data using modern mathematical methods of scientific research, use telecommunication means. To create a holistic view of the possibilities of using new mathematical methods in practical research for undergraduates, special disciplines "Mathematical modeling in mechanics", "Mathematical methods of research" were introduced.

[20] Krylova, T.V. (2021). Pedagogical control at the technical university. *Materials IX of the International Scientific and Methodological Conference "Problems of Mathematical Education""Problems of Mathematical Education" PME-2021, April 9-10.* (P.40-41) ..Cherkasy: Vydavets FOP Hordiyenko YE.I . [In Ukr.].

[21] Orlova, N.D. (2011). Implementation of the elements of student-centered learning in the scientific activities of master's cadets. *Materials s II of the вy Interuniversity scientific-practical conference. Scientific activity as a way of formation of professional competencies of the future specialist. December 1-2.* (p.51-53). Sumy: SumDPU imeni A.S. Makarenka. [In Ukr.].

Within studying these disciplines it becomes possible to use the method of projects[22], which helps to teach cadets to think independently, solve technological problems, involving knowledge from different fields (mathematics, physics, theoretical mechanics, etc.), the ability to predict results and possible solutions, establish causal -hereditary connections.

Thus, using modern teaching methods, maintaining continuity in education "secondary school → institution of higher education → practical activities" can train qualified professionals, competitive in the labor market, capable of competent, responsible and effective activities in their specialty and form the engineering elite of the future.

[22] Polat, E.S. , Bukharkina, M. Yu., Moiseeva, M.V., & Petrov, A.E. (2005). *New pedagogical and information technologies in the education system.* Moskva: Izdatel'skiy tsentr «Akademiya».[in Russian].

CHAPTER 3 *Math Teacher Training in Graduate and Postgraduate Education*

Teacher moments as tools for fostering mathematic education students' teacher knowledge in geometry

Olha Matiash, Liubov Mykhailenko

Introduction

The effective teaching of high geometry may be a powerful tool for the development of spatial representation abilities of a person and their (a person's) constructive skills[1]. Also, geometric education can become a powerful means of the formation of students' logical thinking with its characteristic features of validity, consistency, fullness, criticality, and rationality[2] [3] [4] [5]. We are convinced that the improvement of the quality of high school geometric education should be among the vital tasks of the evolution of modern pedagogical theory and practice.

Literature review and research questions

We have analyzed a large volume of literature on teaching and studying geometry at school. In particular, in the paper[6] the authors put emphasis on the knowledge of mathematics and specific methodical knowledge in teaching school geometry. M. J. Driscoll et al.[7] (2007) state that in American schools geometry did not have a significant influence on the development of students' thinking, in particular — in the middle school. The book supports the idea that math teachers should master the ability of cultivating pupils' thinking potential in geometry class. The paper[8] highlights the efforts of the authors', aimed at the development of practical theory of the meaningful knowledge necessary for teaching. The aim of the study was to scrutinize the nature of the professionally-oriented math knowledge of math students in real class and determine the scope of "must-have" math knowledge for teaching via the analysis of the emerging problems. In the context of our research we were interested in the results of the publication[9], regarding the processes and tools, used for teaching math

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[1] Steele, M. D. (2006). *Middle grades geometry and Measurement: Examining change in knowledge needed for teaching through a practice-based teacher education experience*. (Doctoral Dissertation), University of Pittsburgh. Proquest Dissertations and Theses. (UMI № 305248105) Retrieved from http://d-scholarship.pitt.edu/7351/

[2] González, G. & Herbst, P. (2013). *An Oral Proof in a Geometry Class: How Linguistic Tools Can Help Map the Content of a Proof.* Cognition and Instruction, (31:3, 271-313), doi: 10.1080/07370008.2013.799166

[3] Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal,* 33(2), 455-488. doi:10.3102/00028312033002455

[4] Herbst, P.G. (2002). Establishing a custom of proving in American school geometry: evolution of the two-column proof in the early twentieth century. *Educational Studies in Mathematics*, 49(3), 283–312 doi: 10.1023/a:1020264906740

[5] Cirillo, M., & Herbst, P. (2012). Moving toward more authentic proof practices in geometry. *The Mathematics Educator*, 21 (2), 11–33. Retrieved from https://files.eric.ed.gov/fulltext/EJ961514.pdf

[6] Herbst, P., Kosko, K. (2014). Mathematical Knowledge for Teaching and its Specificity to High School Geometry Instruction. *Research Trends in Mathematics Teacher Education*, 23-45. doi:10.1007/978-3-319-02562-9_2

[7] Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5-10*. Portsmouth, NH: Heinemann.

[8] Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content Knowledge For Teaching. *Journal of Teacher Education*, 59 (5), 389-407. doi:10.1177/0022487108324554

[9] Markovits, Z., & Smith, M. (2008). *Cases as Tools in Mathematics Teacher Education*. In Tools and Processes in Mathematics Teacher Education. Leiden, The Netherlands: Brill | Sense. doi:10.1163/9789087905460_004

education students. The papers[10] [11] illustrate how the psychological analysis of communication in the modern Internet space is carried out. We have long-term experience of teaching geometry, hands-on experience in the methodology of teaching mathematics to the students of pedagogical Universities, and in our publications we address the problems of the geometry teaching at school and training of the math education students for teaching school geometry[12] [13] [14]. Given this, it can be stated that the efficiency of the geometry teaching at school greatly depends on the methodical competence of the teacher. In our opinion, the methodical system of training math education students to teach geometry at school requires much more attention than it is given to. Our idea, aimed at the improvement of the quality of methodical training is based on the concept of the creation of special conditions, needed for the formation of the teacher knowledge in geometry teaching.

Purpose of the research

The paper aims at explaining the content of teacher moments used for the formation of teacher knowledge, highlighting the benefits of using teacher moments, and proving their positive impact on the formation of methodological skills necessary for teaching geometry.

Presenting main material

Methodical competence (teacher knowledge) is manifested in readiness and ability to methodically correctly meet the complex of methodical challenges when developing high-school students' geometry competence. We view the case-study method as it allows of having a real field-trip into the school geometry class. School teachers have to solve the infinite variety of methodical and educational challenges of various types and levels[15] [16] [17]. We view the process of the high-school students' personality development in geometry class as the aim of the methodical activity of math teacher[18]

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[10] Lazarenko, N. (2017). Symbiosis Of Methodological Approaches To The Development Of Education In The Information Society. *Science and Education*, 30(4), 107-112. doi:10.24195/2414-4665-2017-4-18

[11] Lazarenko, N. I., Kolomiets, A. M., & Palamarchuk, O. M. (2018). Communication in the Internet Space: Psychological Aspect. *Information Technologies and Learning Tools*, 65(3), 249. doi:10.33407/itlt.v65i3.2036 [in Ukr.]

[12] Matiash, O. (2013). *Theoretical and methodological bases of formation of method competence of the future teacher of mathematics for teaching geometry to students*: monograph. Vinnytsia: FOP Lehkun V.M. Retrieved from https://library.vspu.edu.ua/polki/akredit/kaf_01/matyash2.pdf [in Ukr.]

[13] Shvets, V.O., Bevz, V.G., Shkolnyi, O.V., & Matiash, O.I. (2020) Ukraine: School Mathematics Education in the Last 30 Years. *International Studies in the History of Mathematics and Its Teaching Eastern European Mathematics Education in the Decades of Change*, 229-274. doi:10.1007/978-3-030-38744-0_6

[14] Matiash, O., & Mykhailenko, L. (2020) Opportunities for Method Competence Development of Mathematics Teachers: The Role of Participation in Competitions with Colleagues. *Universal Journal of Educational Research*, 8(3), 747 - 754. doi: 10.13189/ujer.2020.080303.

[15] Cirillo, M. (2018). Engaging Students with Non-routine Geometry Proof Tasks. *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs*, 283-300. doi:10.1007/978-3-319-77476-3_16

[16] Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *Zdm*, 44(5), 601-612. doi:10.1007/s11858-012-0438-6

[17] Kuzniak, A., Nechache, A. (2021). On forms of geometric work: A study with pre-service teachers based on the theory of Mathematical Working Spaces. *Educational Studies in Mathematics*, 106(2), 271-289. doi:10.1007/s10649-020-10011-2

[18] Smith, S. (2018) Minding the Gap: A Comparison Between Pre-service and Practicing High School Teachers' Geometry Teaching Knowledge. *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs*, 163-180. doi:10.1007/978-3-319-77476-3_10

[19] [20] [21]. In our study a teacher moment is a task, used in the methodical training of teacher on the level of comprehension, design and practical realization of the methodical activity necessary for developing teacher knowledge. Teacher moments differ from other problems by it that in the process of their solution the students are involved in the methodical activity. Teacher moments are the tools for developing the creative potential of pre-service geometry teachers.

The idea of creating special conditions for the formation of the math education students' teacher knowledge in geometry teaching has been implemented in Vinnytsia Mykhailo Kotsyubynskyi State Pedagogical University in the following way:

- a selective discipline "Selected problems of the methods of teaching geometry at school" was introduced in the Bachelors' training curricula;
- a compulsory subject "Theory and methods of teaching geometry at school" was introduced in the Masters' curriculum;
- a number of educational-methodical resource books have been developed and introduced in the methodological training class: recommendations on geometry teaching from the methodological heritage of the prominent Ukrainian mathematicians and methodical experts (2012); methodical guidelines for the formation of knowledge and skills on the subject "Quadrilaterals" (2012); development of the spatial representation of the pupils by means of computer technologies in the professional-oriented school (2013); methodical guidelines for the systematization and generalization of facts and methods of planimetry in the process of studying geometry in higher school (2013);
- under the teachers' guidance students prepare the publications for the annual thematic collection of the student works "Methodical Search": "One task – ways of solution are different" (2011); "Technologies of the introduction of mathematical notions in the process of teaching mathematics" (2012); "Application of the mathematical knowledge and skills" (2013); "Geometric exercises" [22] (2014).

In 2012 we worked-out the collection of teacher moments on the methods of geometry teaching for the methodological training of math education students (more than 1000 tasks)[23]. All educational-methodical problems are distributed there in accordance with the competencies of the mathematics teacher. The solution of the series of teacher moments that requires the realization of certain methodical actions, promotes the formation and development of each methodical skill. In 2012-2019, the collection of teacher moments on the methods of teaching geometry was field-tested in the

[19] Herbst, P., Boileau, N., & Gürsel, U. (2018). Examining the Work of Teaching Geometry as a Subject-Specific Phenomenon. *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs*, 87-110. doi:10.1007/978-3-319-77476-3_6

[20] Ko, I., & Herbst, P. (2020). Subject Matter Knowledge of Geometry Needed in Tasks of Teaching: Relationship to Prior Geometry Teaching Experience. *Journal for Research in Mathematics Education*, 51(5), 600-630. doi:10.5951/jresematheduc-2020-0163

[21] Bulf, C. (2019). Professional actions of novice teachers in the context of teaching and learning geometry. *Eleventh Congress of the European Society for Research in Mathematics Education,* Utrecht University, Feb 2019, Utrecht, Netherlands. Retrieved from https://hal.archives-ouvertes.fr/hal-02402133/document

[22] Department of Algebra and Mathematics Teaching Methods. (nd). Retrieved from http://amnm.vspu.edu.ua [in Ukr.]

[23] Matiash, O., Voievoda, A., Mykhailenko, L. & Nakonechna, L. (2012). *Collection of educational and methodical problems on methods of teaching geometry at school.* Vinnytsia: FOP Lehkun V.M. (n.d.). Retrieved from https://docplayer.net/73715254-Zbirnik-navchalno-metodichnih-zadach-z-metodiki-navchannya-geometriyi.html [in Ukr.]

conditions of real educational process. Four different teachers of mathematics used this collection. By the results of the pedagogical experiment, the collection of teacher momentswas updated and republished as "Teaching geometry at school. Practicum". Composition of educational-methodical problems of the collection is performed by the teacher in accordance with the specific aims and tasks of the methodical competence for teaching geometry, taking into account the following: form of the methodical training (lectures, practical or laboratory classes, individual work of the students, pedagogical internship); level of the students' geometric and methodical training, level of their personal qualities development and pedagogical abilities; availability of the corresponding material-technical base for the organization of the education activity of the students (sufficient quantity of various geometry textbooks, modern technical teaching aids, etc.); level of the interdisciplinary links in the process of specialized training.

All the problems of the given complex can be conventionally divided into four groups. The first group will include teacher moments, aimed at comprehension and application of the theoretical block of the methodical training of the math teachers. The examples of such teacher moments are as follows:

- ✓ What age peculiarities of the pupils of the 5-6 grades are to be taken into consideration in the process of the first explanation of the problem solution: Draw the triangle ABC. Measure the length of the side AB and mark its middle point by the letter D. Draw a line across point D, parallel to the line AC. Make sure that the line divides the side BC in two.
- ✓ Using handbooks on psychology define and pinpoint the characteristic features of memory, imagination, and thinking of the pupils, gifted in geometry. Suggest several test tasks for the diagnosis of the pupils' abilities to learn geometry.

The second group contains teacher moments, aimed at the development of practical block of the methodical training content of the teacher. The examples:

- ✓ At what grade and studying what theme is it worth considering Bramagupta's problem (Prove, that the product of two sides of the triangle equals the product of the altitude, traced to the third side and the diameter of the circle, described around the given triangle) to stimulate the cognitive interest of the pupils? What additional information may influence the increase of pupils' interest?
- ✓ What intra-subject links in studying geometry can be applied when solving the problem: Find the area of the trapezium if its bases equal 2 cm and 7 cm and the diagonals – 10 cm and 17 cm? Substantiate the answer.

The third group includes the teacher moments, convenient for the development of the analytical block of the methodical training content of the teacher. The examples:

- ✓ When studying the theme "Prism", high-school students answered different questions in the following way:

a) Prism is a polyhedron which has two faces – equal n-gon and the other n-faces – parallelograms.

b) Prism is regular if all its edges are equal.

c) Perpendicular, drawn from any point of the base to the plane of the second base, is called the altitude of the prism.

d) Prism, lateral edges of which are perpendicular to the planes of the bases, is called the right prism.

e) If the lateral edges of the prism are not perpendicular to the planes of the bases, such prism is called an oblique prism.

f) Segments, connecting the corresponding points of the prism bases, are equal and parallel.

g) Prism is called n-gonal prism, if its bases are simple polygons.

h) Prism is right, if all the edges are equal.

i) In the arbitrary prism all the lateral edges are equal rectangles.

j) Separate case of the prism is parallelepiped.

Which of these statements are false? Indicate the methods for correcting and preventing the corresponding mistakes.

✓ Perform the comparative quantitative and qualitative analysis of the selections of the problems from three alternative school geometry textbooks on the theme "Geometrical constructions" in the 7 grade.

The fourth group comprises teacher moments, suitable for the organization of quasi-professional activities of pre-service math teachers. The following task may be an example of such methodical problems:

✓ Prepare and conduct a fragment of the geometry lesson (the 7^{th} grade) on the theme "Third feature of the triangles equality", using various methods for the consolidation of the learning material in the conditions of the level differentiation of the geometry teaching at school (type of the lesson: combined).

The developed set of teacher moments is aimed at enhancing teacher knowledge in geometry, development of students' teacher thinking, provision of the theoretical and practical readiness for teacher activity.

The aim of the experiment: to test the options of developing geometry teacher skills through the set of teacher moments for the methodical training of pre-service math teachers. We performed the diagnostics of developing geometry teacher skills and made the correction of the methodical activity. The development of students' teacher skills can be diagnosed both in the process of teacher moments and as their result. The pragmatic indices were the visible results of how the students dealt with teacher moments: correctness of the solution, correspondence to the requirements to the particular stage of education, originality (own subjective vision of the solution of the problem of the teacher`s methodical activity that meets the requirements of the theory and practice of teaching).

The psychological-pedagogical indices of the educational-methodical problem solution are: the activity, confidence, independence of students in the teacher moment activity, participation in the discussion of the obtained solutions. Diagnostic and control assessments (cross-sections) of the teacher knowledge in geometry has been done in the form of written tests, comprising 4 teacher problems (diagnostic cross section) or 8 teacher problems (control cross section). Teacher problems consisting of four components are proportionally presented in the written test: theoretical, analytical, practical, and a teacher moment.

Teachers involved in the pedagogical experiment, regularly performed diagnostic and control cross sections of the development of math education students' geometry teacher skills in the experimental and control groups. In each part of the experiment 6 diagnostic and 2 control cross sections of teacher skills formation were carried out. In the first part of the experiment the diagnostic cross sections were made on the material of such geometry teacher knowledge as: the ability to formulate and develop geometric representations of the 5-6 graders on the propaedeutic level of the systematic geometry course; the ability to form and develop geometric knowledge and skills concerning geometric figures on the plane; the ability to form and develop the pupils' mathematic competence concerning the proof of the geometric statements; the ability to form and develop the pupils' mathematical competence concerning the measurement and calculation of the geometric variables; the ability to form and develop the pupils' mathematical competence concerning geometrical constructions; the ability to form and develop the pupils' mathematical competence concerning various methods and ways of the solution of planimetric problems.

In the second part of the experiment the diagnostic cross sections are made on the material of the following teacher competences: the ability to form and develop mathematical competence of the secondary school pupils concerning the representation of the geometrical figures on the plane; the ability to form and develop geometric knowledge and skills of pupils according to the aim of their profile training; the ability to form and develop the integral, systematic knowledge and skills of the pupils in geometry; the ability to form and develop pupils' competences in applying knowledge and skills in geometry for the solution of practical and applied problems; the ability to form and develop positive personal attitude of pupils and students to geometry and the process of its learning; the ability to form and develop the competence in the field of geometry of the pupils in the classes of the advanced study of geometry.

As the quantitative criterion of the knowledge formation the coefficient $k_i = \frac{a}{b}$ is used, a – is the sum of all the points, accumulated by the student, as a result of performing diagnostic or test work, b – maximally possible sum of points after performing the work. The average value of the K coefficient of the methodical skills formation in the experimental groups was determined as the arithmetic mean of k_i values. The level of teacher skills formation in the experimental groups was determined by the value of K coefficient: 0 - 0.5 – low level; 0.51 - 0.7 – intermediate level; 0.71 - 0.85 – upper intermediate level; 0.86 -1 – high level of the teacher skills formation. After the fourth cross section of the teacher skills formation of the students of the experimental groups, a greater part of the students achieved an upper intermediate level in mastering the necessary methodical skills.

The results of performing complex test works were estimated, applying the same technology as for the diagnostic cross sections of the teacher skills. Quantitative results of performing complex test works by the students of the experimental and control groups are shown in Table 1.

Table 1 Results of the analysis of the methodical skills formation of the Bachelor's program students and Master's program students after performing the pedagogical experiment.

Average value of the coefficient	Bachelor's program students		Master's program students	
	Experimental group	Control group	Experimental group	Control group
K	0,71	0,42	0,79	0,46

We will consider the distribution of the students of the experimental and control groups by the levels of the methodical skills formation (Table 2).

Table 2 Distribution of the Bachelor's program students and Master's program students by the levels of the methodical skills formation after the experiment.

Final level	Bachelor's program students		Master's program students	
	Experimental group (%)	Control group (%)	Experimental group (%)	Control group (%)
Low	14	63	9	56
Intermediate	39	21	43	25
Upper intermediate	32	16	27	19
High (advanced)	15	-	21	-

It is seen from Table 2 that the considerable growth of all the indices occurred in the experimental group in the first part of the experiment and in the second one, they exceeded the corresponding indices of the students in the control group. Each student in the control group achieved a high level of teacher skills formation. Thus, by the results of the experiment, it can be stated that by means of teacher moments the teacher skills of the pre-service geometry teachers can be purposefully formed.

Conclusions

Teacher moments, aimed at teaching pupils geometry we constructed in accordance with the tasks of the methodical activity of the teacher, that is why, we consider them as the pedagogical objectives, which are the tools for teaching, development, education, and specialized training of the future teachers in the conditions of the educational process. Teacher moments are characterized by certain contradictions between available and needed level of the formation of teacher knowledge and skills. Teacher moments help in the organization of student's activity, aimed at mastering the necessary teacher knowledge, formation of the corresponding teacher skills and obtaining certain methodical principles. Teacher moments give students the possibility to try themselves in future professional activities, it helps them to reveal and develop their professional qualities. The advantages of teacher moments are that they integrate theoretical and practical aspects of teacher training, strengthening its practical objectives. By the results of our studies we formulated basic requirements, regarding the application of teacher moments in developing geometry teacher knowledge of math education students:

- Teacher moments, used for training geometry education students must reflect real methodical problems, occurring in the professional activity of the teacher.
- Teacher moments must be used taking into account the principle of accessibility, i.e., the selection of the problems for the specific form of teacher training envisages various levels of information and activity readiness of the students to deal with teacher moments.
- In the sequencing of the teacher moments, the succession principle must be obeyed, i.e., each previous problem is to prepare the student for the solution of the next problems.
- The system of teacher moments must contain the possibility of increasing the level of student's independence in dealing with them.
- When developing teacher moments it is necessary to take into consideration the fact that they must be of heuristic character and presuppose various approaches for dealing with them.
- Types and structure of teacher moments should allow no conventionality in approaching them and dealing with them.

Developing instructional materials for hands-on MLIL (Tertiary Education)

Nina Tarasenkova, Iryna Akulenko, Iryna Kulish, Iryna Nekoz

Introduction

Nowadays, educators and teachers emphasize that the development of such personal qualities as independence, criticality, the ability to work in a team, to determine the goals of activities and the ways to achieve them, are the key ones to a successful human life in the 21st century. As noted by the authors of the international study[1], these 21st century competencies are associated with growth in the cognitive, interpersonal, and intrapersonal domains. Among the competencies related to growth in the cognitive, interpersonal and intrapersonal domains, skills related to relationship-building and communication occupy a special place. These are core competencies [2] [3] [4] [5] [6] as they make a measurable contribution to education, relationships, employment, health and well-being. They are key ones for absolutely all people, not just those with a particular profession, occupation, or lifestyle (Ananiadou, & Claro[7], 2009; Rychen[8], 2003). Communication in the context of the 21st century refers not only to the ability to "communicate effectively, orally, in writing, and with a variety of digital tools" but also to "listening skills" (Fullan[9], 2013, p. 9). Some frameworks[10] include information

[1] *21st Century Competencies: Foundation Document for Discussion. Phase 1: Towards Defining 21st Century Competencies for Ontario* (2016). Edition Winter. 70 p. Retrieved from http://www.edugains.ca/resources21CL/About21stCentury/21CL_21stCenturyCompetencies.pdf (accessed September 2021).

[2] European Commission, Directorate-General for Education and Culture, European Reference Framework. (2007). Key competences for lifelong learning – a European framework. Luxembourg: Office for Official Publications of the European Communities. Retrieved from: www.alfa-trall.eu/wp-content/uploads/2012/01/EU2007-eyCompetencesL3-brochure.pdf. (accessed Sept. 2021).

[3] Finnish National Board of Education. (2015). Learning and competence 2025. Retrieved from: www.oph.fi/download/164907_learning_and_competence_2025_finnish_national_board_of_education.pdf. (accessed Sept. 2021).

[4] OECD (Organisation for Economic Co-operation and Development). (2003). Key competencies for a successful life and well-functioning society. The definition and selection of key competencies: Executive summary. Summary of report by S.D. Rychen & L.H. Salganik (Eds.). Göttingen: Hogrefe and Huber Publishers. Retrieved from: www.oecd.org/dataoecd/47/61/35070367.pdf (accessed Sept. 2021).

[5] P21 (Partnership for 21st Century Skills). (2011). Framework for 21st century learning. Retrieved from: www.p21.org/our-work/p21-framework. (accessed Sept. 2021).

[6] Pellegrino, J.W., & Hilton, M.L. (Eds.). (2012). Education for life and work: Developing transferable knowledge and skills in the 21st century. National Research Council. Committee on Defining Deeper Learning and 21st Century Skills, Board on Testing and Assessment and Board on Science Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

[7] Ananiadou, K., & Claro, M. (2009). 21st century skills and competences for new millennium learners in OECD countries. OECD Education Working Papers, No. 41. Paris: OECD Publishing. Retrieved from: http://dx.doi.org/10.1787/218525261154

[8] Rychen, D.S. (2003). Key competencies: Meeting important challenges in life. In D.S. Rychen & L.H. Salganik (Eds.), Key competencies for a successful life and a well-functioning society (pp. 63–107). Göttingen: Hogrefe & Hube Publishers.

[9] Fullan, M. (2013). Great to excellent: Launching the next stage of Ontario's education agenda. Toronto: Ontario Ministry of Education. Retrieved from: www.edu.gov.on.ca/eng/document/reports/FullanReport_EN_07.pdf. (accessed Sept. 2021).

[10] British Columbia Ministry of Education. (2013). Defining cross-curricular competencies: Transforming curriculum and assessment. Draft. Retrieved from: www.bced.gov.bc.ca/irp/docs/def_xcurr_comps.pdf. (accessed Sept. 2021).

and digital literacy in the concept of communication. Other frameworks, such as P21[11], have distinct information, media, and technology skills. Some jurisdictions[12] include information and communications technology (ICT) skills with literacy and numeracy as foundational curriculum. We will consider the development of key communication skills in the context of mastering mathematical content, i.e. in the context of integrated mathematics and foreign language learning (MLIL). Our focus will be on the integrated study of elements of number theory and congruence theory by pre-service mathematics teachers for reasons that have been analyzed in detail in the paper of Tarasenkova, Akulenko, Kulish, & Nekoz[13] (2020).

Background of the study

The application of the CLIL approach in the study of Mathematics and a foreign language (MLIL) has been explored (Dale & Cuevas[14], 1987; Clarkson[15], 1992; Clarkson & Dawe[16], 1994; Ellerton & Clarkson[17], 1996; Abedi[18], 2001; Neville-Barton & Barton[19], 2005; Barwell[20], 2009) in different countries in the context of different problems: features of planning and designing teaching/learning; general pedagogical laws of construction of the educational process aimed at mastering the mathematical content by means of English; creating curricula and programs for teaching Mathematics by means of English; policy of assessing learning outcomes and achievements of students (pupils), etc. Belgian researchers (Surmont, Struys, Noort, & Craen[21], 2016) noted the positive impact of the approach on improving the level of mathematical competence of students in a relatively short period of time.

Czech researchers (Novotna and Hofmannova [22], 2021) paid special attention to such aspects of integrated teaching of mathematical content and English as: the

[11] P21 (Partnership for 21st Century Skills). (2011). Framework for 21st century learning. Retrieved from: www.p21.org/our-work/p21-framework. (accessed Sept. 2021).

[12] European Commission, Directorate-General for Education and Culture, European Reference Framework. (2007). Key competences for lifelong learning – a European framework. Luxembourg: Office for Official Publications of the European Communities. Retrieved from: www.alfa-trall.eu/wp-content/uploads/2012/01/EU2007-keyCompetencesL3-brochure.pdf. (accessed Sept. 2021).

[13] Tarasenkova, N., Akulenko, I., Kulish, I., Nekoz, I. (2020) Preconditions and Preparatory Steps of Implementing CLIL for Future Mathematics Teachers. *Universal Journal of Educational Research* 8(3): 971-982, DOI: 10.13189/ujer.2020.080332. Retrieved from: http://www.hrpub.org/download/20200229/UJER32-19514908.pdf

[14] Dale, D. C., & Cuevas, G. J. (1987). Integrating language and mathematics learning. In C. JoAnn (Ed.), *ESL through content-area instruction* (pp. 9-23). Regents, New Jersey: Prentice Hall.

[15] Clarkson, P. C. (1992). Language and mathematics: A comparison of bilingual and monolingual students of Mathematics. *Educational Studies in Mathematics*, 23(4), 417-430.

[16] Clarkson, P. C., & Dawe, L. (1994). Problem solving in two languages: A longitudinal study of bilingual students in Melbourne and Sydney. In G. Bell, B. Wright, N. Leeson & J. Geage (Ed.), *Challenges in mathematics education: Constraints on construction* (Vol 1, pp. 173-178). Sydney: MERGA.

[17] Ellerton, N. F., & Clarkson, P. C. (1996). Language factors in mathematics teaching and learning, In A. J. Bishop et al (Ed.*), International handbook of mathematics education* (pp. 987-1033). Dordrecht, The Netherlands: Kluwer Academic Publishers.

[18] Abedi, J. (2001). The language factor in mathematics tests. *Applied Measurement in Education*, 14(3), 219–335.

[19] Neville-Barton, P., & Barton, B. (2005). The relationship between English language and mathematics learning for non-native speakers. Final Report. Wellington. Teachers learning & Research Initiative. Retrieved from: http://www.tlri.org.nz/tlri-research/research-completed/school-sector/relationship-between-english-language-and-mathematics (accessed September 2021).

[20] Barwell, R. (2009). Multilingualism in mathematics classrooms: global perspectives. Clevedon: Multilingual Matters.

[21] Surmont, J., Struys, E., Noort, M., & Craen, P. (2016). The effects of CLIL on mathematical content learning: A longitudinal study. *Studies in Second Language Learning and Teaching*, 6(2), 319-337.

[22] Novotna, J., Hofmannova, M. (2021). Context-dependent learner comprehension strategies Mathematics taught in English to Czech learners. Retrieved from: https://www.researchgate.net/publication/267830680_Context-dependent_learner_comprehension_strategies_Mathematics_taught_in_English_to_Czech_learners (accessed Sept. 2021).

interaction of three “languages” in teaching Mathematics (native language, foreign language and the language of Mathematics), as well as the advantages and disadvantages of using CLIL in Mathematics. We will start our work based on the findings obtained by Czech researchers. We’ll take into account the assertion of Czech scientists that learners always combine perceived stimuli with the knowledge and skills they have accumulated previously. The perception process depends a lot on the context. Learning isolated vocabulary items is unacceptable. The cognitive strategies are universal and not domain dependent. An analysis of the process of learning Mathematics and complementary language (English) by researchers from the UK (Barwell[23], 2005) shows that a clearer reflexive model of the relationship between content, language, and learning should be created.

Some researchers (Martínez[24], 2019) point to shortcomings in the implementation of MLIL and argue that “research on mathematics and language education in bilingual classrooms rarely maintains the interdisciplinary and interconnected nature of this phenomenon. Instead, research has tended to focus on language learning as a prerequisite for mathematics learning. Moreover, each of these two bodies of work is disseminated in its corresponding field, with few opportunities for interdisciplinary analysis” (Martínez[25], 2019, p. 187). In the course of our research, we also identified some difficulties and reservations in the integrated teaching/learning Mathematics and foreign language. They are primarily related to the fact that the laws of cognitive processes that lead to the acquisition and successful use of language and mathematical content, being to some extent common, however, have their own specifics related to the subject area and the degree of abstractness of learning objects (mathematical and linguistic constructs). However, compliance with the laws of mastering the elements of mathematical content (formation of mathematical concepts, work with theorems and methods of mathematical activity) and elements of linguistic content (foreign language vocabulary, grammar, etc.) should lead to better results along with their coordination in learning and methodological support. To confirm this opinion, we rely on research (Akbarov, Gonen, Aydogan[26], 2018) at the Kazakh National University, which studied the experience of students in studying Mathematics using the CLIL approach. These studies (Akbarov, et al., 2018) analyzed the experience, results and opinions of students, their attitudes, preferences, and perceptions of the educational process aimed at content and language integrated learning. The results of the study generally indicate an average level of satisfaction with this approach in the educational process, but there is an increase in the level of foreign language competence of students. At the same time, as the researchers (Akbarov, et al., 2018) note, a positive and significant correlation of the level of English competence with the perception of Mathematics and other disciplines taught at the university in English was found.

[23] Barwell, R. (2009). *Multilingualism in mathematics classrooms: global perspectives.* Clevedon: Multilingual Matters.
[24] Martínez, J.M. (2019). An Integrated Approach to Mathematics and Language Theory and Pedagogy. In: Robinson K., Osana H., Kotsopoulos D. (eds) Mathematical Learning and Cognition in Early Childhood. Springer, Cham. Retrieved from: https://doi.org/10.1007/978-3-030-12895-1_11
[25] Martínez, J.M. (2019). An Integrated Approach to Mathematics and Language Theory and Pedagogy. In: Robinson K., Osana H., Kotsopoulos D. (eds) Mathematical Learning and Cognition in Early Childhood. Springer, Cham. Retrieved from: https://doi.org/10.1007/978-3-030-12895-1_11
[26] Akbarov, A., Gonen, K., Aydogan, H. (2018). Content and (English) language integrated learning (CLIL) applied to math lessons. Acta Didactica Napocensia, 11(2), 1-10. Retrieved from: https://www.researchgate.net/publication/326479259_Content_and_English_language_integrated_learning_CLIL_applied_to_math_lessons, (accessed Sept. 2021).

Analyzing various aspects of integrated teaching/learning language and Mathematics within the CLIL approach, the researchers point out the advantages of using a foreign language as the language of teaching Mathematics, as well as the advantages of using Mathematics as a means of teaching a foreign language. Possibilities of focusing on the linguistic and mathematical components provide different variations of CLIL-lessons (Tejkalova[27], 2013). Pointing to the advantages of the integrated learning of Mathematics and foreign languages, scientists (Miqdadi and Dina al-lamal[28], 2013) emphasize that the main purpose of teaching mathematics is the formation of such qualities of personal thinking as consistency of argumentation, provability, alternative opinions (complete induction), system, structure, consistency in the process of expressing ideas and solving problems. These characteristics of mathematical thinking, according to experts, are enhanced if teaching Mathematics is combined with language learning. We share this vision of scientists. For example, the proof of a theorem (verbal or written) involves the construction of not only mathematical but also linguistic constructions for the logical ordering of a chain of purely mathematical arguments and conclusions. The logical action of defining mathematical concepts teaches students such a way of expressing an opinion, when it is necessary to indicate only the essential properties of a certain class of objects, distinguishing them from insignificant ones. Students' thinking in this case is aimed at performing such mental operations as comparison (in the form of comparison and opposition), analogy, classification, generalization, etc. The results of these operations are crystallized in the form of a concept definition, which uses only the necessary and appropriate verbal constructions (necessarily logically consistent, devoid of descriptiveness, inaccuracy, and metaphor).

Studying the role of a foreign language in teaching Mathematics, Italian scholars (Favilli, Maffei, & Peroni[29], 2013) proceeded from a textual linguistic approach. Distinguishing four main types of mathematical discourse, namely: dialogic, descriptive, argumentative, and regulative-directive discourse (Benveniste[30], 1966); Weinrich[31], 2001; MacWhinney[32], 1995; Searle[33], 1969) the researchers emphasize that the first three types can be used to analyze mathematical discourse in classroom and, at the same time, represent the main linguistic aspects of communicative competence, namely: dialogue, description, narration. For example, the reasoned language in mathematics belongs to the category of narrative, but has its own mathematical specificity. The fourth type of discourse, regulatory and directive, is associated with a certain algorithmization in the formation and application of a certain

[27] Tejkalova, L. P. (2013). Mathematics for language, language for mathematics European Journal of Science and Mathematics Education. 1(1), 23-28. Retrieved from: https://files.eric.ed.gov/fulltext/EJ1108199.pdf, (accessed Sept. 2021).
[28]Miqdadi, R., Dina al-lamal, (2013). Difficulties in Content and Language Integrated Learning: The Case of Math. Jordan Journal of Educational Sciences, 9(4), 449-459. Retrieved from: https://www.researchgate.net/publication/306314476_Difficulties_in_Content_and_Language_Integrated_Learning_The_Case_of_Math (accessed Sept. 2021).
[29] Favilli, F., Maffei, L., & Peroni, R. (2013). Teaching and Learning Mathematics in a Non-native Language: Introduction of the CLIL Methodology in Italy. *US-China Education Review*, 3(6), 374-380. https://files.eric.ed.gov/fulltext/ED543810.pdf, (accessed Sept. 2021).
[30] Benveniste, E. (1966). *Problèmes de linguistique générale* (Vol. 1). Paris: Gallimard.
[31] Weinrich, H. (2001). *Tempus: Besprochene und erzaehlte welt*. Muenchen: Beck.
[32] MacWhinney, B. (1995). *The CHILDES project: Tools for analysing talk*. Hillsdale, N. J.: Erlbaum.
[33] Searle, J. R. (1969). *Speech acts: An essay in the philosophy of language*. Cambridge: Cambridge University Press.

method of mathematical activity (Favilli, Maffei, & Peroni[34], 2013). In our opinion, these aspects must be taken into account when designing learning materials for MLIL.

A New Zealand researcher Latu[35] (2006) studied the processes of students' comprehension of mathematical text, the influence of students' language difficulties related to the English language acquisition and the level of understanding and mastering mathematical content, the influence of language competence on students' mathematical competence in native language and foreign language. The main conclusions reached by New Zealand scientists as a result of their research are that students who use their mother tongue along with English, applying the phenomenon of "switching" from language to language, show better learning outcomes in learning Mathematics in English and demonstrate top scores. The researchers emphasize the importance of teachers' awareness that switching from language to language is a common practice for bilingual students. The results of this study formed the basis for our scientific research and practical development of educational support for integrated learning of Mathematics and English.

An Iranian researcher Clarkson[36] (2008) has also focused on the process of "switching" students from one language to another while solving mathematical problems. The experts believe that the reasons for the difficulties in switching, for example, are due to the fact that earlier, students used the terms in their native language, and now they need to spend some time correlating these terms with English equivalents. The researchers (Clarkson, 2008, p. 52) suggest that during the lesson, bilingual students will continue to use the translation of certain concepts while performing mathematical tasks in order to understand and comprehend them better. Obviously, these problems are the basis for the use of native language in the CLIL model, as a kind of scaffolding for students at the stage of introducing new material or while proving the theorem.

Belgian educators Surmont, Struys, Van Den Noort, & Van De Craen[37] (2016) study the efficiency of teaching Mathematics based on the CLIL technology at secondary school. Their statistical data indicates that CLIL positively affects not only the language learning but also content acquisition, and has a positive impact on the mathematical performance of students even after a short period of time. Their studies prove that CLIL possibly influences pupils' cognitive development, and more specifically, their metalinguistic awareness. The increased metalinguistic awareness can lead to a better understanding of Mathematics and insight into the abstract language of it. To improve Mathematics performance, bridges between learning a language and Mathematics (and science for that matter) should be created. Their results indicate that the combination of a language and content has a positive effect on cognitive development.

[34] Favilli, F., Maffei, L., & Peroni, R. (2013). Teaching and Learning Mathematics in a Non-native Language: Introduction of the CLIL Methodology in Italy. US-China Education Review, 3(6), 374-380.
[35] Latu, V.F. (2006). Language Factors that Affects Mathematics Teaching and Learning of Pasifika Students. Semantic Scholar.
[36] Clarkson, Z.P. (2008). Iranian bilingual students reported use of language switching when doing mathematics. *Mathematics Education Research Journal*, 20(1), 52–81.
[37] Surmont, J., Struys, E., Noort, M., & Craen, P. (2016). The effects of CLIL on mathematical content learning: A longitudinal study. *Studies in Second Language Learning and Teaching*, 6(2), 319-337. Retrieved from: https://doi.org/10.14746/ssllt.2016.6.2.7, (accessed Sept. 2021).

The participants of the Polish project supported by the European Union "Two Highways of Life – Mathematics and English" make generalizations, conclusions and recommendations[38] as to the improvement of Mathematics education on the basis of CLIL. They stress the importance of CLIL as the approach that helps students to develop the skills of both the native and foreign language. This approach provides students with a different educational experience compared to the traditional English language learning since applying the CLIL approach, subject material and a foreign language are learnt together as inseparable parts of each other. In addition, general educational skills and logical thinking are developed. The CLIL technology can involve alternative teaching methods (of both Mathematics and a foreign language). This approach presents new objectives and challenges for teachers and students.

Ukrainian scientists Vlasenko, Lovyanova, Chumak, Sitak, & Kalashnykova[39] (2019) study the formation of the foreign language competence of engineering students by means of CLIL. The researchers apply the CLIL method on the example of an Elementary Mathematics course integrated with the English language learning. They confirm that the integrated elective course of English and Elementary Mathematics has a positive impact on forming the foreign language competence of engineering students, as well as on the improvement of their motivation to study.

Jordan researchers Miqdadi, & Al-Jamal[40] (2013) study the difficulties (epistemological, pedagogical, personal and discourse) encountered by the integrated teaching and learning of Mathematics and English. They found that students who come to the university with different, sometimes insufficient knowledge of many mathematical concepts in Arabic, have discourse problems. Pedagogical problems are relevant to the teaching process in terms of presentation methods, teacher's preparation, assessment, the ways of material presentation and mostly knowledge proficiency. Researchers identify epistemological problems as the problems that may be attributed to the difficulty of the nature of mathematics and the abstractness of mathematical concepts. The authors recommend that it is necessary to reinforce the foundation of English and later to lay the basis of CLIL with the help of English for Specific Purposes.

The CLIL implementation needs appropriate educational and methodological support. Bystray, Belova, Vlasenko, Zasedateleva, & Shtykova[41] (2018) study the means of the CLIL approach, particularly, the features of special texts and systems of exercises that become the basis for reaching the CLIL educational goals. They state that the text as a source of information presents a specific topic to a reader, but it also serves as a basis of the lexical and grammar module that provides acquisition of the scientific technology and certain grammar and structural-stylistic constructs. The text in CLIL should serve as a starting point for the discussions and the expansion of the

[38] Teaching Mathematics through English – Teaching English through Mathematics. Comenius Project "Two Highways of Life – Maths and English". Retrieved from https://clilmaths.jimdo.com (accessed Sept. 2021).

[39] Vlasenko, K., Lovyanova, I., Chumak, O., Sitak, I., & Kalashnykova, T. (2019). The formation of foreign language competence of engineering students through CLIL method. *Revista Dilemas Contemporáneos*: Educación, Política y Valores. Year VII, Special Edition, November 2019.

[40] Miqdadi, R., & Al-Jamal, D. (2013) Difficulties in Content and Language Integrated Learning: The Case of Math. *Jordan Journal of Educational Sciences,* Vol. 9, No. 4, 2013, pp 449-459

[41] Bystray, Y., Belova, L., Vlasenko, O., Zasedateleva, M., Shtykova, T. (2018). Development of second-language communicative competence of prospective teachers based on the CLIL Technology (From the experience of a pedagogic project at a Department of History). *Revista Espacios*, 39 (52), 12. Retrieved from http://www.revistaespacios.com/ /a18v39n52/18395212.html (accessed Sept. 2021).

vocabulary on the given topic, thus simultaneously stimulating the development and activation of communicative skills of dialogue and monologue speech.

In our research[42] we came to the conclusion that the MLIL technology should be based on students' experience gained at mathematical disciplines, and learning ESP during the first and the second year of their study at the University. The MLIL-related teaching process should be student-centered and interactive. The introduction of new mathematical content by the Mathematics teacher should be previously regulated and agreed with the English teacher.

Education should provide the implementation of four types of class activities: reading, listening, speaking, and writing in English. These activities should be combined with the methods of mathematical activity appropriate to the mathematical component and comprehensive feedback. So, MLIL needs special appropriate learning materials (LM).

Methodology

Educational and methodological support of the discipline (instructional materials (IM), teaching materials (TM), learning materials (LM)) according to Lewis[43] (2018) is a set of materials that involves both objects and human resources applied by the teacher with the aim of achieving the planned educational objectives. That is, instructional materials in the educational context (Mehisto[44], 2012) can be defined as knowledge and information presented in various means and formats that contribute to the achievement of the planned learning outcomes. At the same time, learning materials can help [45] a student to concretize a learning experience, and make learning more interesting, interactive, and exciting[46]. Quality TM motivate students to study, solve problems, search for new learning resources in collective and independent work, contribute to the creation of a learning environment, in which students learn both content and language simultaneously (Mehisto[47], 2012). As students face more complex tasks and challenges, it is learning material that should act as scaffolding to support students. Researches (Mehisto[48], 2012) offer certain principles for the creation of quality learning materials for the educational process, which use the CLIL methods, namely: clarity of educational intentions and process (language, content, learning skills); systematic assistance in mastering academic language; promoting the development of learning skills and student autonomy; various forms of assessment and self-assessment; creation of a safe educational environment; promoting cooperation;

[42] Tarasenkova, N., Akulenko, I., Kulish, I., & Nekoz, I., (2020). Preconditions and Preparatory Steps of Implementing CLIL for Future Mathematics Teachers. *Universal Journal of Educational Research*, 8(3), 971-982. Retrieved from: https://www.hrpub.org/download/20200229/UJER32-19514908.pdf (accessed Sept. 2021).
[43] Lewis, B. (2018). TLM or Teaching Learning Materials Definition. Retrieved from: https://www.thoughtco.com/tlm-teaching-learning-materials-2081658 (accessed Sept. 2021).
[44] Mehisto, P. (2012). Criteria for producing CLI learning material. *Encuentro*, 21, 15-33, Retrieved from: https://files.eric.ed.gov/fulltext/ED539729.pdf (accessed Sept. 2021).
[45] What is Instructional Materials In Global dictionary
[46] Monsuru Babatunde Muraina (2015) Relevance of the Use of Instructional Materials in Teaching and Pedagogical Delivery: An Overview. In Handbook of Research on Enhancing Teacher Education with Advanced Instructional Technologies
[47] Mehisto, P. (2012). Criteria for producing CLI learning material. *Encuentro*, 21, 15-33, Retrieved from: https://files.eric.ed.gov/fulltext/ED539729.pdf (accessed Sept. 2021).
[48] Mehisto, P. (2012). Criteria for producing CLI learning material. *Encuentro*, 21, 15-33, Retrieved from: https://files.eric.ed.gov/fulltext/ED539729.pdf (accessed Sept. 2021).

search for ways to use authentic language; promoting critical thinking; widespread use of scaffolding; promoting the meaningful nature of the educational process.

Researches (Neville-Barton, Barton[49], 2005) show that students have some problems while learning mathematics and English due to their language difficulties. Researchers argue that students do not always realize their difficulties; and teachers do not always anticipate these difficulties, students face some difficulties with "switching" languages in the classroom, and rely on the texts or handouts. They mostly focus on the process and approach mathematical problems in tests, trying to recognize a familiar procedure without trying to understand the context.

Results and discussion

Our experience argue that planning and development of LM for the MLIL should:

1) ensure the implementation of different approaches to the introduction and consideration of previously known and new (to students) mathematical concepts, facts and methods, domestic and foreign scientific school;

2) consider and combine the patterns of perception and assimilation of mathematical content (concepts, facts, and methods of mathematical activities) and patterns of formation of foreign language competence through listening, reading, speaking, and writing;

3) implement new trends in teaching/learning caused by modern global processes and rapid changes in modern society, such as the spread of distance or blended learning, development of new educational services, mathematical software, etc. Elements of LM focus on the optimal planning and design of all stages of teaching/learning and the relevant types of educational activities.

One of the main components of the LM for the MLIL is materials for the practical part of the MLIL. The design of LM for the practical part should take into account the general psychological and didactic patterns of perception and assimilation by students:

1) mathematical concepts (illustration of concepts, examples and counterexamples to concepts, establishing the fact that a certain mathematical object belongs to the content of the specified mathematical concept, primary consolidation of the concept, application of the concept in familiar and changed situations);

2) mathematical facts (theorems) and their proofs, the formation of students' ability to prove theorems (through the processing and reproduction of ready-made proofs for the production of independently invented proofs of mathematical facts);

[49] Neville-Barton P., Barton B. (2005) The Relationship between English Language and Mathematics Learning for Non-native Speakers. Teaching and Learning Research Initiative. Wellington, New Zealand. Downloaded from: http://www.tlri.org.nz/pdfs/13909.pdf (accessed Sept. 2021).

3) methods of mathematical activity, which are generalized in the rules and algorithms (such as, for example, the decomposition of a composite number into prime factors, Euclid's algorithm for calculating GCD, etc.).

The design of LM for the practical part of the MLIL should also provide the implementation of four types of activities in classes: reading, listening, speaking, and writing in English. If the TM for lectures are aimed at providing students with the perception of oral (listening) and written speech (reading) during the lecture, then the practical classes have their own specific focus. The anticipated result in the practical classes is not only listening comprehension or reading comprehension, and one's own lingual and non-lingual behavior, but the mastery of mathematical and linguistic components in perception, reading, writing, but also speaking. These linguistic activities should be combined with the ways of the appropriate mathematical activity.

We have designed [50] the following expected results of integrated learning of the elements of the theory of divisibility and English Language (Table 1).

Table 1 Mathematical and foreign language components of expected results.

Mathematical component of expected results	**Foreign language component of expected results**
The student *formulates* definitions, *gives* examples and counter-examples of the basic concepts of the course, *formulates* the basic theorems of the course. The student *finds* an incomplete fraction and the remainder of the division of an arbitrary integer *a* by an arbitrary non-zero integer *b*; *calculates* GCD of two numbers using Euclid's algorithm; *represents* an arbitrary natural number in the form of a canonical decomposition; *calculates* GCD, LCM $\tau(n)$, $\sigma(n)$, $\varphi(n)$. The student *determines* the simplicity (complexity) of a natural number based on the canonical decomposition of it. The student *reduces* ordinary fraction and determines approximate values of rational and irrational numbers with predetermined accuracy using the properties of an infinite simple continued fraction and the convergents of the continued fraction. The student *proves* the properties of a Pythagorean triple, the remainder division theorem, the theorem on finding GCD of two numbers using the Euclid's algorithm; the	The student *understands* the meaning of the words in a lexical minimum of the foreign language (FL); forms new words using affixes of FL; *forms* sentences of different structural types of FL; *paraphrases* utterances for a given purpose; *distinguishes* between synonyms of the learnt concepts; *distinguishes* between true and false statements; *forms* necessary grammatical forms of the words in FL; *writes* words and sentences using spelling and syntax rules of FL; *analyzes* the word formation in FL; *analyzes* sentence structure of FL; *uses* necessary units of lexical and grammatical minimum of FL. The student *comprehends* materials that are read and heard in FL; *evaluates* and *analyzes* what is read and heard in FL; *reproduces* what is read and heard in FL; *plans* and *formulates* future utterances in FL; *implements* the idea of speaking in the process of speech

[50] Tarasenkova, N., Akulenko, I., Kulish, I., & Nekoz, I. (2020). Preconditions and Preparatory Steps of Implementing CLIL for Future Mathematics Teachers. Universal Journal of Educational Research, 8(3), 971-982. Retrieved from: https://www.hrpub.org/download/20200229/UJER32-19514908.pdf (accessed Sept. 2021).

theorem on the decomposition of a natural number into prime factors; the theorem on the structure of divisors of a natural number, the theorems related to co-prime numbers. The student *devises* formulas for finding values of numerical functions *τ(n), σ(n), φ(n).* The student *uses* the properties of congruencies *to find* the reminders after dividing a numerical expression by a given number. The student *solves* linear congruencies using the properties of congruence, using an infinite simple continued fraction and modular multiplicative inverse of a congruence class; quadratic congruence by addition to perfect square; *uses* the properties of Legendre and Jacobi symbols to solve quadratic congruence; congruence of higher degrees on tables and properties of indexes; linear Diophantine equations by means of the infinite simple continued fraction and congruence. The student *gives* the examples of encryption methods using the properties of linear, quadratic congruence, finding quadratic root prime and composite modulo. The student *proves* Euler and Fermat theorems, theorems about the number of linear congruence solutions; the number of quadratic residues and non-residues, and the Eulerian criterion for quadratic residues and non-residues.	activity in FL; *corrects* own speech and the speech of others; *pronounces* certain results in FL; *reproduces* information content in FL; *uses* professional and interdisciplinary knowledge in speech activity in FL. The student *joins* the problem discussion in FL; *supports* discussion on a given topic; *uses* language means in the communication appropriately; *gives* arguments to them; *is oriented* in the situation of communication in FL; *establishes* contact with an interlocutor; *changes* strategy and speech behaviour depending on the communicative situation; *has* discussion in FL; *supports or denies* the views of an interlocutor.

TM for practical part of the MLIL should facilitate the students' mastering mathematical content and performing additional assignments with foreign language load. So, they need to involve a special system of multilevel bilingual assignments.

To systematize bilingual tasks, Tarasenkova's[51] (2014) concepts and Tarasenkova's & Borkach's[52] (2016) ideas were used. As our previous research shows, this set of exercises should involve:

1) a system of the bilingual tasks for cooperative performance by students during practical classes (individually with further discussion in a group, in team work with the use of interactive learning forms)

2) a system of the bilingual tasks for independent performance by students in individual independent work. The system of bilingual tasks is structured in several groups (Figure 1).

[51] Tarasenkova, N. (2014). Peculiar Features of Verbal Formulations in School Mathematics. Global Journal of Human-Social science : G : Linguistics & Education, 14 (3). 61-67.
[52] Tarasenkova, N., Borkach, E. (2016). The system of bilingual tasks for students of natural and mathematical specialties of universities with Hungarian language of instruction. Science and education a new dimension, IV (40), Issue: 81. 66-69.

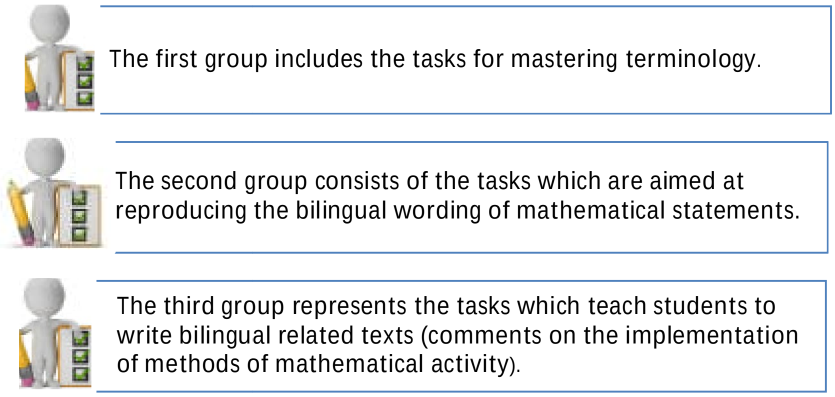

Figure 1 A system of the bilingual tasks.

The first group includes the tasks for mastering terminology. The second group consists of the tasks which are aimed at reproducing the bilingual wording of mathematical statements. The third group represents the tasks which teach students to write bilingual related texts (comments on the implementation of methods of mathematical activity). We will focus on the features of the first group of tasks and provide the relevant examples.

Terminological tasks in MLIL LM

The first level tasks are related to the formation of students' ability to establish bilingual terminological pairs of concepts within a specific content module or topic study. The tasks of the second level require creating bilingual comparative conceptual terminological tables, terminological clusters that reflect the links between the concepts studied. We associate the tasks of the third level in this group with students' training to formulate bilingual definitions of a concept independently. We offer examples of such tasks in the study of the topic "The ratio of divisibility in the ring of integers. The remainder theorem. The greatest common divisor (GCD) and the least common multiple (LCM) of numbers. Euclid's algorithm. Coprime integers".

Task 1.1 (*first level*). Give the Ukrainian equivalents to the following concepts: The number *d* divides the number *n*, *d* is a divisor of *n*, *d* is a factor of *n*, or n is a multiple of *d*, Divisibility Properties: Reflexivity property, Transitivity property, Multiplication property, Cancellation property, Linearity property, Comparison property, an integer *n* is even, an integer *n* is odd, an integer is prime, an integer is composite, the divisors of an integer, a common divisor of two integers, the quotient and the remainder on division of a by *b*, the quotient and the remainder pair are unique, the greatest common divisor of two nonzero integers *a* and *b*, the GCD operator, two numbers are relatively prime (mutually prime, coprime numbers), a method to compute GCD (*a*, *b*) where *a* and *b* are nonnegative, the Euclidean Algorithm, the lowest common multiple (LCM).

Task 1.2 (*first level*). Give the English equivalents to the following concepts: the divisibility ratio in the ring of integers; the number *a* is completely divisible by the number *b*, the number *a* is divisible by the number *b* with the remainder, the greatest common divisor (NCD) and the least common multiple (LCM) of two or more natural numbers; linear representation of NCD of two natural numbers; mutually prime numbers; Prime number; compound number, canonical decomposition of a natural number.

In order to ensure switching between the three languages (English, Ukrainian and mathematical-symbolic), special exercises are required, such as Task 1.3 (first level)

Task 1.3 (first level) Write symbolically: *d* is a divisor of *n*, Divisibility Properties: Reflexivity property, Transitivity property, Multiplication property, Cancellation property, Linearity property, Comparison property, the greatest common divisor of two nonzero integers *a* and *b* is *d*, *a* is relatively prime to *b*, *a* and *b* are coprime numbers, an integer *n* is even, an integer *n* is odd, the number *a* when divided by 5 gives the remainder 2, the number *b* when divided by 7 gives the remainder 4. The formula of the number *t*, which when divided by 3 gives the remainder 2 and when divided by 4 gives the remainder 3 is…

Task 1.4 (*first level*). Find the corresponding definition or explanation to the concepts (Table 2).

Table 2 The Concepts and Definition.

Notion	**Definition**
factor	_____is a number that can be divided by another number without a remainder
multiple	_____is a part of something that is left over when other parts have been completed.
dividend	_____is a number or quantity that when multiplied with another produces a given number or expression.
divisor	_____if the only positive integer that is a divisor of both of them is 1.
quotient	_____which are not all zero, is the largest positive integer that divides each of the integers.
remainder	_____is a number by which another number is to be divided.
the greatest common divisor (GCD) of two or more integers__	_____is an efficient method for computing the greatest common divisor (GCD),
the lowest common multiple of two integers *a* and *b*	_____is a number to be divided by another number.
the Euclid’s algorithm	_____is the smallest positive integer that is divisible by both *a* and *b*.

two integers *a* and *b* are coprime, relatively prime or mutually prime if__	_____is a result obtained by dividing one quantity by another.
Prime number	_____is any positive integer greater than 1 that is divisible only by itself and 1

Task 1.5 (*second level*). Construct the concept table (Table 3) on the theme “The ratio of divisibility in the ring of integers. The remainder theorem. The greatest common divisor (GCD) and the least common multiple (LCM) of numbers. Euclid’s algorithm. Coprime integers”.

This table should reflect new concepts, basic concepts, basic mathematical facts and the ways of proving them. In this table, terms for basic and new concepts should be provided in English. The wording of the basic mathematical facts and the ways of proving them should be represented in the native language.

Table 3 Conceptual Table for the Topic Study.

New concepts	Basic concepts	Basic mathematical facts	Methods and techniques used in proving

Task 1.6 (*second level*). Create a cluster displaying the links between the main concepts and theorems on the theme “The ratio of divisibility in the ring of integers. The remainder theorem. The greatest common divisor (GCD) and the least common multiple (LCM) of numbers. Euclid’s algorithm. Coprime integers”.

Task 1.7 *(third level)*. Read the given terms in Ukrainian (in English). Give analogues in English (in Ukrainian). Define the concepts in Ukrainian and in English. Complete Table 4.

*Table **4*** Define the Concepts in Ukrainian and in English.

Term in Ukrainian	Term in English	Definition in Ukrainian	Definition in English

Task 1.8 *(third level*). ***Data adequacy tasks***. In this type of task, it is necessary to find out if there is enough data to solve the problem. It is not necessary to solve the problem itself. The answer options are as follows:

Data 1) are enough to solve the problem, and data 2) - not enough;

A) Data 2) are enough to solve the problem, and data 1) – not enough;
B) Data 1) and 2) are enough to solve the problem;
C) Data 1) and data 2) only together are enough to solve the problem;
D) Data 1) and data 2) even together are not enough to solve the problem.

Given: 1) $a \vdots b$, $a > 0$, $b > 0$; 2) $a \vdots b$, $a < 0$, $b < 0$. Compare *a* and *b*.

Given: 1) $a, b \in \mathrm{N}$, $\frac{2}{a+b} \geq 1$; 2) $2 < a+b < 4$. Find *a* and *b*.

Given: 1) $\tilde{N}$ – a prime number equal to the sum of two prime numbers; 2) a digital recording of numbers can be shown as $BB + BB = 1C4$. Find one-digit number $\tilde{N}$.

Given: 1) $a \vdots b$; 2) $c \vdots d$. Determine if $a + c \vdots b + d$.

Given: 1) $a \vdots b$; 2) $c \vdots d$. Determine if $ac \vdots bd$.

Given: 1) $a > 0$, $b > 0$; 2) $\frac{a}{b}$ is a shortcut fraction. Determine if $\frac{a-b}{a+b}$ fraction is a shortcut.

Given: 1) $a > 0$, $b > 0$; 2) $\frac{a-b}{a+b}$ is a shortcut fraction. Determine if $\frac{a}{b}$ fraction is shortcut.

Task 1.9 (*third level*). Translate the tasks. Solve them in Ukrainian. Build a "cluster" using graphical interpretation for the given tasks. To do this, select the basic task, auxiliary tasks, as well as tasks of theoretical, generalizing nature, establish links between tasks:

1. Prove that of *n* + 1 natural number, there are at least 2 that have the same remainder when divided by *n*.
2. Prove that of *n* + 1 natural number, there are at least 2, the difference of which is divisible by *n*.
3. Prove that from an arbitrary set containing *n* numbers, you can choose several ones whose sum is divisible by *n*.
4. Prove that among arbitrary *n* + 1 natural numbers, each of which does not exceed *2n*, we can choose two such numbers, one of which is divisible by the other.
5. Prove that among arbitrary *n* + 1 natural numbers, each of which is less than *2n,* we can choose three such numbers, one of which is the sum of the other two.
6. Prove that among arbitrary *n* + 1 natural numbers, each of which does not exceed *3n*, we can choose two such numbers, the ratio of which is the power of 3.

Conclusions

MLIL needs special appropriate instructional materials combining mathematics and language practical activity for classroom and independent work. These learning materials should contribute to the achievement of the planned learning outcomes. Quality instructional materials are intended to motivate students to study, solve problems, search for new learning resources in the classroom and during independent work. Our design of quality teaching materials for the practical part of the MLIL activity is based on the developed Mathematics and English components of the expected results. The system of bilingual tasks is structured in several groups. The first group includes the tasks for mastering terminology. The second group consists of the tasks which are aimed at reproducing the bilingual wording of mathematical statements. The third group represents the tasks that teach students to write bilingual related texts (comments on the implementation of methods of mathematical activity).

BIBLIOGRAPHY

21st Century Competencies: Foundation Document for Discussion. Phase 1: Towards Defining 21st Century Competencies for Ontario (2016). Edition Winter. 70 p. Retrieved from http://www.edugains.ca/resources21CL/About21stCentury/21CL_21stCenturyCompetencies.pdf (accessed September 2021).

A model curriculum "Mathematics. Grades 5-6" for institutions of general secondary education (Authors – S. Skvortsova, N. Tarasenkova). (2021) : Retrieved from: https://drive.google.com/file/d/1ykOgcS2OiQbBxXAfxFoW-SxykuwZMlFm/view [In Ukr.]

Abedi, J. (2001). The language factor in mathematics tests. *Applied Measurement in Education*, 14(3), 219–335.

Akbarov, A., Gonen, K., Aydogan, H. (2018). Content and (English) language integrated learning (CLIL) applied to math lessons. Acta Didactica Napocensia, 11(2), 1-10. Retrieved from: https://www.researchgate.net/publication/326479259_Content_and_English_language_integrated_learning_CLIL_applied_to_math_lessons, (accessed Sept. 2021).

Aleksiuk, A.M. (1981). General methods of teaching at school. Kyiv, Ukraine. [In Ukr.]

All-Ukrainian Experiment "New Ukrainian School", grades 5 – 9 (2021) : Retrieved from: https://imzo.gov.ua/osvita/nush/vseukrains-kyy-eksperyment-nush-5-9-kl/ [In Ukr.]

Ananiadou, K., & Claro, M. (2009). 21st century skills and competences for new millennium learners in OECD countries. OECD Education Working Papers, No. 41. Paris: OECD Publishing. Retrieved from: http://dx.doi.org/10.1787/218525261154

Aristotle (IV Century BC). (1978). Topica, Chapter 5. Russian edition: Aristotle, Works, vol.2, chpt. 5, Myslj, Moskva, p 354. [In Rus.]

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content Knowledge For Teaching. *Journal of Teacher Education*, 59 (5), 389-407. doi:10.1177/0022487108324554

Barkovsky, V., Barkovskaya, N. (2010). *Higher mathematics for economists*. Kyiv: Tsentr Uchbovoi Literatury. [In Ukr.].

Barwell, R. (2009). *Multilingualism in mathematics classrooms: global perspectives*. Clevedon: Multilingual Matters.

Bashchuk, O. Yu., S. M. Kuchmenko, L. V. Skaskiv, та O. B. Chernobai. (2019). *Vyshcha ta prykladna matematyka: zbirnyk vprav ta zadach [Higher and applied mathematics: collection of exercises and tasks]*. Irpin: Universytet DFS Ukrainy. [in Ukr.].

Benveniste, E. (1966). *Problèmes de linguistique générale* (Vol. 1). Paris: Gallimard.

British Columbia Ministry of Education. (2013). Defining cross-curricular competencies: Transforming curriculum and assessment. Draft. Retrieved from: www.bced.gov.bc.ca/irp/docs/def_xcurr_comps.pdf. (accessed Sept. 2021).

Bulf, C. (2019). Professional actions of novice teachers in the context of teaching and learning geometry. *Eleventh Congress of the European Society for Research in Mathematics Education*, Utrecht University, Feb 2019, Utrecht, Netherlands. Retrieved from https://hal.archives-ouvertes.fr/hal-02402133/document

Bystray, Y., Belova, L., Vlasenko, O., Zasedateleva, M., Shtykova, T. (2018). Development of second-language communicative competence of prospective teachers based on the CLIL Technology (From the experience of a pedagogic project at a Department of History). *Revista Espacios,* 39 (52), 12. Retrieved from http://www.revistaespacios.com/ /a18v39n52/18395212.html (accessed Sept. 2021).

Chernobai, O.B. (2020). Pro deyaki typy poshukovykh zadach [About certain types of search tasks]. In M. I. Burda, *Matematychna osvita: mynule, sohodennya majbutnie do 100-richchya vid dnya narodzhennya O.F. Semenovycha*: monohrafiya [Mathematical education: past, present, future to the 100th anniversary of the birth of O.F. Semenovich: monograph] (pp. 102-107). Kharkiv: SHNTM "Novyi kurs", 2020.-203c. [in Ukr.].

Chernobai, O. (2019). On the use of Algorithms in Teaching Probability Theory. In *Curent Issues in Ensuring the Quality of Mathematical Education*, by N. Tarasenkova, 138-154. Budapest: SCASPEE.

Chernobai, O. (2021). Praktychne napovnennya matematyky zadachamy z finansovym ta podatkovym zmistom [Practical filling of mathematics with tasks having financial and tax content]. *Materialy IX mizhnarodnoji naukovo-praktychnoji konferentsiji "Problemy matematychnoji osvity"(PMO-2021) [Proceedings of the IX International Scientific and Practical Conference "Problems of Mathematical Education" (PMO-2021)* (pp. 176-177). Cherkasy: FOP Hordiienko I.I. [in Ukr.].

Cirillo, M. (2018). Engaging Students with Non-routine Geometry Proof Tasks. *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs*, 283-300. doi:10.1007/978-3-319-77476-3_16

Cirillo, M., & Herbst, P. (2012). Moving toward more authentic proof practices in geometry. *The Mathematics Educator*, 21 (2), 11–33. Retrieved from https://files.eric.ed.gov/fulltext/EJ961514.pdf

Clarkson, P. C. (1992). Language and mathematics: A comparison of bilingual and monolingual students of Mathematics. *Educational Studies in Mathematics*, 23(4), 417-430.

Clarkson, P. C., & Dawe, L. (1994). Problem solving in two languages: A longitudinal study of bilingual students in Melbourne and Sydney. In G. Bell, B. Wright, N. Leeson & J. Geage (Ed.), *Challenges in mathematics education: Constraints on construction* (Vol 1, pp. 173-178). Sydney: MERGA.

Clarkson, Z.P. (2008). Iranian bilingual students reported use of language switching when doing mathematics. *Mathematics Education Research Journal*, 20(1), 52–81.

Dale, D. C., & Cuevas, G. J. (1987). Integrating language and mathematics learning. In C. JoAnn (Ed.), *ESL through content-area instruction* (pp. 9-23). Regents, New Jersey: Prentice Hall.

Department of Algebra and Mathematics Teaching Methods. (nd). Retrieved from http://amnm.vspu.edu.ua [in Ukr.]

Didactic methods. Retrieved from http://www.prepodi.ru/praktika-pedagogika/didaktika/1166-didaktika-metod start=3- (accessed 17 June 2021). [In Rus.]

Diesterweg. (1851). *A school's curriculum.* 4 te Aufl Essen.

Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5-10*. Portsmouth, NH: Heinemann.

Ellerton, N. F., & Clarkson, P. C. (1996). Language factors in mathematics teaching and learning, In A. J. Bishop et al (Ed.*), International handbook of mathematics education* (pp. 987-1033). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Encyclopedia of Education. (2008). Head ed. V.H. Kremin. Kyiv, Ukraine : Yurinkom, Inter. [In Ukr.]

European Commission, Directorate-General for Education and Culture, European Reference Framework. (2007). Key competences for lifelong learning – a European framework. Luxembourg: Office for Official Publications of the European Communities. Retrieved from: www.alfa-trall.eu/wp-content/uploads/2012/01/EU2007-eyCompetencesL3-brochure.pdf. (accessed Sept. 2021).

Favilli, F., Maffei, L., & Peroni, R. (2013). Teaching and Learning Mathematics in a Non-native Language: Introduction of the CLIL Methodology in Italy. *US-China Education Review*, 3(6), 374-380. https://files.eric.ed.gov/fulltext/ED543810.pdf, (accessed Sept. 2021).

Favilli, F., Maffei, L., & Peroni, R. (2013). *Teaching and Learning Mathematics in a Non-native Language: Introduction of the CLIL Methodology in Italy*. US-China Education Review, 3(6), 374-380.

Finnish National Board of Education. (2015). *Learning and competence 2025*. Retrieved from: www.oph.fi/download/164907_learning_and_competence_2025_finnish_national_board_of_education.pdf. (accessed Sept. 2021).

Fullan, M. (2013). *Great to excellent: Launching the next stage of Ontario's education agenda*. Toronto: Ontario Ministry of Education. Retrieved from: www.edu.gov.on.ca/eng/document/reports/FullanReport_EN_07.pdf. (accessed Sept. 2021).

Giouvanakis, T., Evaggelos, K., Mpakavos, A. & Samaras, H. (2016). Creative thinking through concept mapping. *International Journal for Cross-Disciplinary Subjects in Education, Volume 7, Issue 1*. pp 2705-2711.

González, G. & Herbst, P. (2013). *An Oral Proof in a Geometry Class: How Linguistic Tools Can Help Map the Content of a Proof.* Cognition and Instruction, (31:3, 271-313), doi: 10.1080/07370008.2013.799166

Gozli Davood G. https://www.researchgate.net/publication/342502764_Imagination_in_General_Psychology_Thin king_wi th_Luca_Tateo's_A_Theory_of_Imagining_ Knowing_and_Understanding http://philosophy.niv.ru/doc/dictionary/encyclopedic/.

Hadetska, S., Moroz , I. (Eds.). (2019). *Modern and historical problems of fundamental and applied mathematical training in higher education institutions: the view of higher education seekers and young scientists*. Kharkiv: KhNADU. [In Ukr.]

Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *Zdm*, 44(5), 601-612. doi:10.1007/s11858-012-0438-6

Herbst, P., Boileau, N., & Gürsel, U. (2018). Examining the Work of Teaching Geometry as a Subject-Specific Phenomenon. *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs*, 87-110. doi:10.1007/978-3-319-77476-3_6

Herbst, P., Kosko, K. (2014). Mathematical Knowledge for Teaching and its Specificity to High School Geometry Instruction. *Research Trends in Mathematics Teacher Education*, 23-45. doi:10.1007/978-3-319-02562-9_2

Herbst, P.G. (2002). Establishing a custom of proving in American school geometry: evolution of the two-column proof in the early twentieth century. *Educational Studies in Mathematics*, 49(3), 283–312 doi: 10.1023/a:1020264906740

Jarvis, R., Dempsey, K., Gutierrez, G., Lewis, D., Rouleau, K., & Stone, B. (2017). *Peer coaching that works: The power of reflection and feedback in teacher triad teams*. Denver, CO: McREL International. https://files.eric.ed.gov/fulltext/ED588635.pdf

Kitano, M., Montgomery, D., Vantassel-Baska, J. & Johnsen, S. (2008). *Using the National Gifted Standards for PreK-12 Professional Development*. NAGC&CEC-TAG. Corwin Press, Thousand Oaks, California, p. 124.

Ko, I., & Herbst, P. (2020). Subject Matter Knowledge of Geometry Needed in Tasks of Teaching: Relationship to Prior Geometry Teaching Experience. *Journal for Research in Mathematics Education*, 51(5), 600-630. doi:10.5951/jresematheduc-2020-0163

Kolyagin, Y., Pikan, V. (1985). On the applied and practical orientation of teaching mathematics. *Mathematics in school,* 6. Retrieved from https://edu-lib.com/periodika/dlya-uchiteley-i-uchashhihsya/matematika-v-shkole/matematika-v-shkole-metodicheskiy-zhurn-125 [In Rus.].

Korostiyanets, T. N. (2020). Pidhotovka majbutnikh vchyteliv matematyky do formuvannya ponyat v uchniv: psykholoho-dydaktychnyj aspekt [Preparation of future mathematics teachers for the formation of concepts among students: psychological and didactic aspect]. In a. o. Tarasenkova N. *Matematychna osvita:*

mynule, sohodennya, majbutnie [Mathematical education: past, present, future] (pp. 156-162). Kharkiv: SHNTM "Novyj kurs". [in Ukr.].

Koslov, N. I. nkozlov.ru/library/samorazvit/d4147/#.VaEFBvamqqko. [In Rus.].

Krylova, T.V. (2021). Pedagogical control at the technical university. *Materials IX of the International Scientific and Methodological Conference "Problems of Mathematical Education""Problems of Mathematical Education" PME-2021, April 9-10.* (P.40-41) ..Cherkasy: Vydavets FOP Hordiyenko YE.I . [In Ukr.].

Kudryavtsev, L., Kirilov, A., Burkovskaya, M., Zimina, O. (2021, April 10). *About tendencies and prospects of mathematical education*. Retrieved from http://www.academiaxxi.ru/Meth_Papers/Paper2.htm [In Rus.].

Kuzniak, A., Nechache, A. (2021). On forms of geometric work: A study with pre-service teachers based on the theory of Mathematical Working Spaces. *Educational Studies in Mathematics*, 106(2), 271-289. doi:10.1007/s10649-020-10011-2

Latu, V.F. (2006). *Language Factors that Affects Mathematics Teaching and Learning of Pasifika Students*. Semantic Scholar.

Law of Ukraine "On Complete General Secondary Education". (2020) : Retrieved from: https://zakon.rada.gov.ua/laws/show/463-20#Text [In Ukr.].

Law of Ukraine "On Education". (2017): Retrieved from: https://zakon.rada.gov.ua/ /laws/show/2145-19#Text [In Ukr.].

Lazarenko, N. (2017). Symbiosis Of Methodological Approaches To The Development Of Education In The Information Society. *Science and Education*, 30(4), 107-112. doi:10.24195/2414-4665-2017-4-18

Lazarenko, N. I., Kolomiets, A. M., & Palamarchuk, O. M. (2018). Communication in the Internet Space: Psychological Aspect. *Information Technologies and Learning Tools*, 65(3), 249. doi:10.33407/itlt.v65i3.2036 [in Ukr.]

Lazarov B. (2018). Topic-oriented Upgrade of Subject-oriented Educational System. In Tarasenkova, N. (Eds). *Current Status and Prospects of Mathematical Education*: Monograph. SCASPEE, Budapest, Hungary. pp 37-53. https://www.thefamouspeople.com/profiles/hristo-botev-5649.php (Access Date: 26 March, 2021).

Lazarov, B. (2013). Developing Synthetic Competence along Individual Educational Trajectory. *Proceedings of the International Conf. on Math. Edu. on Creativity & Giftedness*. The Korean Society of Mathematical Education, (August 9–10, 2013), pp 251-262.

Lazarov, B. (2020). *Developing Synthetic Competence via Socratic Style Teaching in Secondary School Extracurricular Activities*. Institute of Mathematics and Informatics, Sofia, pp 147-162.

Lewis, B. (2018). *TLM or Teaching Learning Materials Definition*. Retrieved from: https://www.thoughtco.com/tlm-teaching-learning-materials-2081658 (accessed Sept. 2021).

MacWhinney, B. (1995). *The CHILDES project: Tools for analysing talk*. Hillsdale, N. J.: Erlbaum.

Markovits, Z., & Smith, M. (2008). Cases as Tools in Mathematics Teacher Education. In *Tools and Processes in Mathematics Teacher Education*. Leiden, The Netherlands: Brill | Sense. doi:10.1163/9789087905460_004

Martínez, J.M. (2019). An Integrated Approach to Mathematics and Language Theory and Pedagogy. In: Robinson K., Osana H., Kotsopoulos D. (eds) *Mathematical Learning and Cognition in Early Childhood*. Springer, Cham. Retrieved from: https://doi.org/10.1007/978-3-030-12895-1_11

Matematyka. (2020). Retrieved from [Mathematics] Ministerstvo osvity i nauky Ukraiiny [Ministry of education and science of Ukraine]: https://mon.gov.ua/ua/osvita/zagalna-serednya-osvita/navchalni-programi/navchalni-programi-dlya-pochatkovoyi-shkoli [in Ukrainian]. [In Ukr.].

Matiash, O. (2013). *Theoretical and methodological bases of formation of method competence of the future teacher of mathematics for teaching geometry to students*: monograph. Vinnytsia: FOP Lehkun V.M. Retrieved from https://library.vspu.edu.ua/polki/akredit/kaf_01/matyash2.pdf [in Ukr.]

Matiash, O., & Mykhailenko, L. (2020) Opportunities for Method Competence Development of Mathematics Teachers: The Role of Participation in Competitions with Colleagues. *Universal Journal of Educational Research*, 8(3), 747 - 754. doi: 10.13189/ujer.2020.080303.

Matiash, O., Voievoda, A., Mykhailenko, L. & Nakonechna, L. (2012). *Collection of educational and methodical problems on methods of teaching geometry at school.* Vinnytsia: FOP Lehkun V.M. (n.d.). Retrieved from https://docplayer.net/73715254-Zbirnik-navchalno-metodichnih-zadach-z-metodiki-navchannya-geometriyi.html [in Ukr.]

Mehisto, P. (2012). Criteria for producing CLI learning material. *Encuentro*, 21, 15-33, Retrieved from: https://files.eric.ed.gov/fulltext/ED539729.pdf (accessed Sept. 2021).

Mikaelian H. S. (2017). Values as significant component in the National Educational Curriculum of Armenia (with presentation in mathematics education). *Ethische Bildung für eine nachhaltige, dialogische Zukunftp*. Gratz, Austria, p. 171-182.

Mikaelian H. S. (2019). *Aesthetic foundations of mathematical education*. Yerevan-Cherkasy. Edith Print. p.220. [In Rus.],

Mikaelian H. S. (2019). Aesthetic needs and mathematics education. *Bulletin of Cherkasy University series pedagogical sciences.* Issue № 1. Cherkasy, Ukraine. p. 263-269. [In Rus.].

Mikaelian H. S. (2020). Aesthetic emotions in the process of teaching mathematics. *Bulletin of Cherkasy Universit, series pedagogical sciences*. Issue № 3. Cherkasy, Ukraine, p. 176-187. [In Rus.].

Mikaelian H. S. (2020). Aesthetics of Mathematical Education and the Mental Phenomenon of Attention. *Bulletin of Cherkasy University series pedagogical sciences*. Issue № 1. Cherkasy, Ukraine. p. 67-74. [In Rus.].

Mikaelian, H. S. (2015). *Beauty and educational potential of mathematics*. Yerevan: Edith Print. [in Arm.].

Ministry of Education and Science. (2015). Regulation No 5 of 30.11.2015 for the general education. Darzhaven Vestnik, I. 95 of 08.12.2015. [in Bulg.]

Miqdadi, R., & Al-Jamal, D. (2013) Difficulties in Content and Language Integrated Learning: The Case of Math. *Jordan Journal of Educational Sciences,* Vol. 9, No. 4, 2013, pp 449-459

Monsuru Babatunde Muraina. (2015). Relevance of the Use of Instructional Materials in Teaching and Pedagogical Delivery: An Overview. In *Handbook of Research on Enhancing Teacher Education with Advanced Instructional Technologies.*

National University "Odessa Maritime Academy" [Electronic resource] Retrieved from http://www.onma.edu.ua/

Neville-Barton P., Barton B. (2005). *The Relationship between English Language and Mathematics Learning for Non-native Speakers. Teaching and Learning Research Initiative*. Wellington, New Zealand. Downloaded from: http://www.tlri.org.nz/pdfs/13909.pdf (accessed Sept. 2021).

Neville-Barton, P., & Barton, B. (2005). *The relationship between English language and mathematics learning for non-native speakers. Final Report.* Wellington. Teachers learning & Research Initiative. Retrieved from: http://www.tlri.org.nz/tlri-research/research-completed/school-sector/relationship-between-english-language-and-mathematics (accessed September 2021).

Novak, J. & Gowin, D. (1984). *Learning how to learn*. Cambridge University Press, Cambridge, UK, pp 15-54.

Novotna, J., Hofmannova, M. (2021). *Context-dependent learner comprehension strategies Mathematics taught in English to Czech learners*. Retrieved from: https://www.researchgate.net/publication/267830680_Context-dependent_learner_comprehension_strategies_Mathematics_taught_in_English_to_Czech_learners (accessed Sept. 2021).

OECD (2012) / Education at Glance 2010: Add Highlights. - Paris. OECD / Publishing.2012. - 91p.] [Electronic resource] Retrieved from   htpp: //www/oecd/org/edu/highlights.pdf.

OECD (Organisation for Economic Co-operation and Development). (2003). *Key competencies for a successful life and well-functioning society. The definition and selection of key competencies: Executive summary*. Summary of report by S.D. Rychen & L.H. Salganik (Eds.). Göttingen: Hogrefe and Huber Publishers. Retrieved from: www.oecd.org/dataoecd/47/61/35070367.pdf (accessed Sept. 2021).

Order of the Cabinet of Ministers of Ukraine of 14.12.2016 № 988-r "On approval of the Concept of Implementation of State Policy in the Field of Reforming General Secondary Education “New Ukrainian School” for the period up to 2029". (2016): Retrieved from: https://zakon.rada.gov.ua/laws/show/988-2016-%D1%80#Text [In Ukr.].

Order of the Ministry of Education and Science of 12.07.2021 №795. (2021): Retrieved from: https://imzo.gov.ua/osvita/nush/vseukrains-kyy-eksperyment-nush-5-9-kl/ [In Ukr.].

Orlova, N.D. (2011). Implementation of the elements of student-centered learning in the scientific activities of master's cadets. *Materials s II of the ву Interuniversity scientific-practical conference. Scientific activity as a way of formation of professional competencies of the future specialist. December 1-2.* (p.51-53). Sumy: SumDPU imeni A.S. Makarenka. [In Ukr.].

Orlova, N.D., & Kornodudova, N.M. (2018). About interrelation of methods of studying a course of mathematics by foreigners at preparatory departments and in tertiary education. *XXXII International Scientific and Practical Conference "Development of Science in the XXI Century"* (February 15, 2018). Collection of scientific articles (standard level, academic level) 3, (pp. 30-36). Kharkiv: Naukovo-informatsiynyy tsentr «Znannya». [In Ukr.]

Orlova, N.D., & Popova L.K. (2009). Continuity of teaching in the construction of mathematical models in secondary and higher schools. *Didactics of Mathematics: Problems and Research: International Collection of Scientific Papers, 32* (pp. 75–76). Donetsʹk : DNU[in Rus.].

Orlova, N.D., Krylova, T.V., & Orlova, E.Yu.(2007). Application of professionally oriented learning technology to improve the mathematical training of the master. *Didactics of Mathematics "Problems and Research" International Collection of Scientific Papers, 26, (p.210-217*). Donetsʹk : DNU [In Ukr.]

P21 (Partnership for 21st Century Skills). (2011). Framework for 21st century learning. Retrieved from: www.p21.org/our-work/p21-framework. (accessed Sept. 2021).

Panchenko, L.L. (2004). On the conceptual apparatus of mathematical modeling in secondary school and pedagogical university. *Scientific journal of NPU named after M.P.Dragomanov, 3,1,* p.89-97.Kyyiv: NPU imeni M.P.Drahomanova. [In Ukr.]

Pankratov, Cyril.(2020).Odessa National Polytechnic University. Specific features of test tasks in mathematics for foreign students at the initial stage of study. *Topical issues of organization of education of foreign students in Ukraine*: *V International Scientific and Methodological Conference, October 14-16, (pp.48-49).* Ternopilʹ: Ternopilʹsʹkyy natsionalʹnyy tekhnichnyy universytet imeni Ivana Pulyuya. [In Ukr.]

Pedagogy. (1988). Ed. by Yu. K. Babansky. Moscow, Russia : Prosveshchenie. [In Rus.]

Pellegrino, J.W., & Hilton, M.L. (Eds.). (2012). *Education for life and work*: Developing transferable knowledge and skills in the 21st century. National Research Council. Committee on Defining Deeper Learning and 21st Century Skills, Board on Testing and Assessment and Board on Science Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Penkov, B. & Chobanov, I. (1958). *Botevite „Urotsi za parvite chetiri aritmeticheski pravila“* [Botev’s “Lessons about the first four arithmetic rules”]. Fiziko-matematichesko spisanie. №1. pp 61-67. (in Bulg.)

Philosophy Encyclopedic Dictionary. (2004). Edited by A.A Ivin. Moscow, Russia. [In Rus.].

Polat, E.S. , Bukharkina, M. Yu., Moiseeva, M.V., & Petrov, A.E. (2005). *New pedagogical and information technologies in the education system.* Moskva: Izdatel'skiy tsentr «Akademiya». [in Rus.].

Pometun, O.I., & Pirozhenko, L.V. (2004). *A modern lesson. Interactive learning technologies*. Kyyiv: Vydavnytstvo A.S.K. [In Ukr.]

Professional standards by professions "Primary school teacher of general secondary education", "Teacher of general secondary education", "Primary education teacher (with a diploma of junior specialist)".(2020) : Retrieved from: https://zakon.rada.gov.ua/rada/show/v2736915-20#Text [In Ukr.].

Radyuk, L. (2021, April 11). *Peculiarities of methodical support of teaching mathematics in the process of professional training of junior specialists*. All-Ukrainian scientific-methodical Internet conference. Retrieved from https://college.nuph.edu.ua/wp-content/uploads/2017/04/T_2_Radiuk.pdf. [In Ukr.].

Rudenko, I. B., & Chernobai, O. B. (2017). *Vyshcha ta prykladna matematyka [Higher and applied mathematics]*. Irpin: Universytet DFS Ukrainy. [in Ukr.].

Rychen, D.S. (2003). *Key competencies*: Meeting important challenges in life. In D.S. Rychen & L.H. Salganik (Eds.), Key competencies for a successful life and a well-functioning society (pp. 63–107). Göttingen: Hogrefe & Hube Publishers.

Savery, J.R., & Duffy, T.M. (1995). Problem based learning: an instructional model and its constructivist framework. *Educational Technology, 35.* p. 31–38.

Searle, J. R. (1969). *Speech acts: An essay in the philosophy of language*. Cambridge: Cambridge University Press.

Shapiro I.M. (2021, April 9). *Use of problems with practical content in teaching mathematics*. Retrieved from https://old.altspu.ru/Journal/pedagog/pedagog_5/a12.html (Last accessed: 9.04.2021). [In Rus.].

Shvets, V.O., Bevz, V.G., Shkolnyi, O.V., & Matiash, O.I. (2020) Ukraine: School Mathematics Education in the Last 30 Years. *International Studies in the History of Mathematics and Its Teaching Eastern European Mathematics Education in the Decades of Change*, 229-274. doi:10.1007/978-3-030-38744-0_6

Skafa, E.I. (2004). Theoretical and methodological foundations of the formation of methods of heuristic activity in the study of mathematics in the context of the introduction of modern teaching technologies. *Doctor's thesis.* Kiev. [In Ukr.]

Slepkan, Z.I. (2000). *Methods of teaching mathematics*. Kyiv: Zodiak-EKO. [In Ukr.]

Smith, S. (2018). Minding the Gap: A Comparison Between Pre-service and Practicing High School Teachers' Geometry Teaching Knowledge. *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs*, 163-180. doi:10.1007/978-3-319-77476-3_10

State Standard for Primary Education. (2018) : Retrieved from: https://zakon.rada.gov.ua/laws/show/87-2018-%D0%BF#Text [In Ukr.].

State Standard of Basic Secondary Education. (2020) : Retrieved from: https://zakon.rada.gov.ua/laws/show/898-2020-%D0%BF#Text [In Ukr.].

Steele, M. D. (2006). *Middle grades geometry and Measurement: Examining change in knowledge needed for teaching through a practice-based teacher education experience*. (Doctoral Dissertation), University of Pittsburgh. Proquest Dissertations and Theses. (UMI № 305248105) Retrieved from http://d-scholarship.pitt.edu/7351/

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal,* 33(2), 455-488. doi:10.3102/00028312033002455

Stoyanov, Z. *Hristo Botyov. Opit za biografia*. BZNS Pblsh, Sofia, (10th Edition 1960) pp 391-392. Initially published in 1888. [in Bulg.].

Surmont, J., Struys, E., Noort, M., & Craen, P. (2016). The effects of CLIL on mathematical content learning: A longitudinal study. *Studies in Second Language Learning and Teaching*, 6(2), 319-337.

Surmont, J., Struys, E., Noort, M., & Craen, P. (2016). The effects of CLIL on mathematical content learning: A longitudinal study. *Studies in Second Language Learning and Teaching*, 6(2), 319-337. Retrieved from: https://doi.org/10.14746/ssllt.2016.6.2.7, (accessed Sept. 2021).

Tarasenkova, N. (2014). Peculiar Features of Verbal Formulations in School Mathematics. Global Journal of Human-Social science : G : Linguistics & Education, 14 (3). 61-67.

Tarasenkova, N., Akulenko, I., Kulish, I., & Nekoz, I. (2020). Preconditions and Preparatory Steps of Implementing CLIL for Future Mathematics Teachers. *Universal Journal of Educational Research, 8(3),* 971-982. Retrieved from: https://www.hrpub.org/download/20200229/UJER32-19514908.pdf (accessed Sept. 2021).

Tarasenkova, N., Borkach, E. (2016). The system of bilingual tasks for students of natural and mathematical specialties of universities with Hungarian language of instruction. *Science and education a new dimension, IV (40), Issue: 81*. 66-69.

Tarasenkova, N.A. (2004). *The theoretic-methodical principles of using of the sign and symbolic means in teaching mathematics of the basic school students. Doctor's thesis*, Cherkasy. [In Ukr.]

Teaching Mathematics through English – Teaching English through Mathematics. Comenius Project "Two Highways of Life – Maths and English". Retrieved from https://clilmaths.jimdo.com (accessed Sept. 2021).

Tejkalova, L. P. (2013). Mathematics for language, language for mathematics *European Journal of Science and Mathematics Education. 1(1),* 23-28. Retrieved from: https://files.eric.ed.gov/fulltext/EJ1108199.pdf, (accessed Sept. 2021).

The Concept of the New Ukrainian School. (2016) : Retrieved from: http://mon.gov.ua/activity/ education/zagalna-serednya/ ua-sch-2016/ konczepcziya.html [In Ukr.].

The New Encyclopedia of Philosophy. (2001). In 4 vols, edited by V.S. Stepin. Moscow, Russia. [In Rus.].

The unified standard learning program in the Ukrainian language for foreign students of the basic faculties of the non-philological profile of higher educational institutions of Ukraine of the III-IV levels of accreditation. (2009). Approved by the Ministry of Education and Science of Ukraine.Kyyiv: NTUU «KPI».[In Ukr.]

Trius, Y., Baklanova, M. (2021, April 10). *Problems and prospects of higher mathematical education*. Retrieved from https://fi.npu.edu.ua/files/Zbirnik_KOSN/10/32.pdf. [In Ukr.].

Ushakova, Natalia., & Trostinskaya, Oksana. (2014). Learning the Ukrainian language by foreign students: conceptual principles. *Theory and practice of teaching Ukrainian as a foreign language, 9.* (pp.12–21). L′viv: NU im. Ivana Franka. [In Ukr.]

Vilensky, M.Ya., Obraztsov, P.I., & Uman, A.I. (2005). *Technologies of vocational education in higher education.* Moskva: Pedagogicheskoye obshchestvo Rossii. [in Rus.].

Vinogradov, S. (2009). Shatalov's system - what is the point? *Science and life*, 2, 78-85. [In Rus.].

Vlasenko, K., Lovyanova, I., Chumak, O., Sitak, I., & Kalashnykova, T. (2019). The formation of foreign language competence of engineering students through CLIL method. *Revista Dilemas Contemporáneos*: Educación, Política y Valores. Year VII, Special Edition, November 2019.

Weinrich, H. (2001). *Tempus: Besprochene und erzaehlte welt.* Muenchen: Beck.

Yuzvin, Z. (2021, April 10). *University damn dozen.* Retrieved from https://studway.com.ua/predmeti/ [In Ukr.].

Zadorozhna, O., Bondar, O., Semenyuta, M. (2020). *Probability and Statistics. Methodical guide to implementation of the laboratory works.* Kropyvnytskyi: Flight academy NAU. [In Eng.].

Zadorozhnia, T., Kharenko, S., Kuchmenko, S., Chernobai, O., Bashchuk, O., & Skaskiv, L. et al. (2016). Zadachi pro podatky [Tasks about taxation]. *Matematyka v ridnii shkoli [Mathematics in home school], 10,* cc. 16-21. [in Ukr.].

Zinonos, N.O. (2015). On the question of learning mathematical language by foreign students of preparatory departments of universities. *Proceedings of the international scientific-methodical conference. Problems of mathematical education (PME-2015).* (pp. 111–112). Cherkas′kyy: natsional′nyy universytet im. B.Khmel′nits′koho . [In Ukr.]

OUR AUTHORS

Akulenko Iryna – Doctor (habil.) of Science in Pedagogy, Full Professor,
Department of Automation and Computer-Integrated Technologies,
Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: akulenkoira@ukr.net

Bondar Olha – PhD in Physical and Mathematical Sciences, Associate Professor,
Department of Physics and Mathematics,
Flight Academy of the National Aviation University, *Ukraine*
e-mail: bondarkla@ukr.net

Chernobai Olga – PhD in Physical and Mathematical Sciences, Associate professor,
Department of Higher Mathematics,
University of State Fiscal Service of Ukraine, *Ukraine*
e-mail: chernobai.olga@gmail.com

Kornodudova Natalia – Senior lecturer of Ukrainian Studies Department,
Ukrainian Studies Department,
National University "Odessa Maritime Academy", *Ukraine*
e-mail: marina7@te.net.ua

Krylova Tetyana – Doctor (habil.) of Science in Pedagogy, Full Professor,
Department of Higher Mathematics,
Dniprovsk State Technical University, *Ukraine*
e-mail: lebmax@ukr.net

Kulish Iryna – PhD in in Educational Theory, Associate Professor,
Head of the Foreign Languages Department,
Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: irinakulish@ukr.net

Lazarov Borislav – PhD in Pedagogy and Education, Professor,
Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, *Bulgaria*
e-mail: lazarov@math.bas.bg

Matiash Olha – Doctor (habil.) of Science in Pedagogy, Professor,
Department of Algebra and Mathematics Teaching Methods,
Vinnytsia Mykhailo Kotsiubynskyi State Pedagogical University, *Ukraine*
e-mail: matyash_27@ukr.net

Mikaelian Hamlet – Doctor (habil.) of Science in Pedagogy, Full Professor,
PhD in Physical and Mathematical Sciences,
Head of laboratory "Axiology of Mathematical Education"
Armenian State Pedagogical University after Kh. Abovyan, *Armenia*
e-mail: h.s.mikaelian@gmail.com

Mykhailenko Liubov – Doctor (habil.) of Science in Pedagogy, Associate Professor,
Department of Algebra and Mathematics Teaching Methods,
Vinnytsia Mykhailo Kotsiubynskyi State Pedagogical University, *Ukraine*
e-mail: mikhailenkolf@gmail.com

Nekoz Iryna – PhD in Professional Education, Associate Professor,
Foreign Languages Department,
Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: nekoz@email.ua

Orlova Natalia – PhD in Technical Sciences, Associate Professor,
Department of Higher Mathematics,
National University "Odessa Maritime Academy", *Ukraine*
e-mail: natorl2969@gmail.com

Skvortsova Svitlana – Doctor (habil.) of Science in Pedagogy, Full Professor,
Corresponding Member of the National Academy of Pedagogical Sciences of Ukraine,
Department of Mathematics and Methods of Teaching Mathematics,
South Ukrainian National Pedagogical University named after K. D. Ushynsky, *Ukraine*
e-mail: skvo08@i.ua

Tarasenkova Nina – Doctor (habil.) of Science in Pedagogy, Full Professor,
Department of Mathematics and Methods of Teaching Mathematics,
Bohdan Khmelnytsky National University of Cherkasy, *Ukraine*
e-mail: ntaras7@ukr.net

Varynska Alla – PhD in Philology, Associate Professor,
Head of Department of Ukrainian Studies,
National University "Odessa Maritime Academy", *Ukraine*
e-mail: varinskay1@gmai.com

Yakunina Iryna – PhD in Technical Sciences, Associate Professor,
Department of Physics and Mathematics,
Flight Academy of the National Aviation University, *Ukraine*
e-mail: yakunina_irina @ukr.net

Zadorozhna Oksana – PhD in Pedagogy and Education, Associate Professor,
Department of Physics and Mathematics,
Flight Academy of the National Aviation University, *Ukraine*
e-mail: ks.zadorozhnaya1@gmail.com

For Notes

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Competentization and mathematical education: Monograph

2021, 126 p., Hardcover, 500 copies
ISBN: ISBN 978-615-01-3972-2

DOI: http://doi.org/10.31174/SEND-MON004

Managing editors:
Barkáts, D.
& Diadyk, A.

Készült a Rózsadomb Contact Kft nyomdájában.
1022 Budapest, Balogvár u. 1.
www.rcontact.hu

BUDAPEST