Visualization of the decisions of applied problems in multiprocessor computing systems

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Abstract. The article is devoted to distributed simulation of visualization of decision vectors of applied problems on the basis of schemes of increased accuracy order. The higher computational speedup in comparison with the finite-difference approach is illustrated by analytical solutions that allow simultaneous and parallel computations in all temporary layers. It is shown that the most promising approach to mathematical simulation of applied problems is the one that is based on numerical-analytical solutions.

Keywords: *multiprocessor computing system, speedup, visualization, distributed simulation, numerical- analytical solution.*

Target setting. Significant computation speedup of applied problems is achieved by means of finite-difference schemes due to the parallelization effect. However, the numerical-analytical algorithms for solving applied problems deserve special attention. Greater computational speedup compared to the finite difference approach can be achieved through analytical solutions that allow simultaneous and parallel computing for all temporary layers and, in this case, do not use combined memory. Thus, the most perspective approach to the mathematical simulation of applied problems should be the one that is based on numerical-analytic solutions.

Effective means during the processing of heat and mass transfer tasks in metallurgical industry are considered to be the application of parallel computing technologies on distributed cluster-type systems that have relatively low cost and are easily scaled both by the number of processors and by the amount of RAM [2, 12]. Consequently, the distributed simulation of the vector visualization of applied problems solutions on the basis of schemes of the raised accuracy order is an essential and relevant task.

Analysis of recent research and publications. Heat and mass transfer processes of metallurgical production should be considered as large systems [8-10]. Today, solving complex, large-scale tasks requires powerful computers and is characterized by 'parallel' term, that is, there are parallel computers, computing systems, parallel computing methods, etc. [3 − 5]. In broad terms, this term entered almost immediately after the appearance of the first computers, or rather, after realizing the fact that the computers created were not able to solve, during the optimal term, many practical tasks. The emergence in computing systems of new and expensive communication tools, a more advanced elemental base, stimulated the development of high-performance computations based on multiprocessor computing systems [1, 7].

In addition, the class of problems in question is usually solved through set of finite-difference equations, which essence is to replace the derivatives by difference relations. In this case, from the numerical algorithm point of view, the solution of finite-difference equations is divided into explicit and implicit schemes [11]. In an explicit scheme, the values of the desired function are determined sequentially, layer by layer. However, despite the apparent simplicity and ease of computing, such a scheme has one significant drawback. If the size of the grid $l > h$, the rounding errors can become so large that the resulting solution becomes meaningless. It is known that for the explicit scheme there must be met the condition: $l / h^2 \le 0{,}5$. But the following empirical rule is fair: if we reduce the values of l and h, then the error of approximation of partial derivatives with finite-difference derivatives also decreases. However, the smaller the grid, the more computations need to be made, which means larger rounding errors. Implicit circuits allow to compute with a large step without significantly degrading accuracy, but such an approach requires a larger amount of computation.

The considered analysis shows that the solution methods of this class of problems should be not only diverse, but also must combine quantitative assessments with the qualitative analysis possibilities. Nowadays, there have been some trends in development of numerical-analytic methods with complex logical structure, but unlike the piecewise difference methods they are of a higher accuracy order and with possibility of making algorithms with adaptation according to the approximation methods [13, 14]. In terms of computation, this approach is somewhat lengthy, but it shows a peculiar benchmark for comparison with other practical methods. At the same time, given that the computational experiment is carried out on a multiprocessor system, it can be asserted that the circumstance that constrained the development of the numerical-analytical approach is now losing its relevance. In this regard, this research has further developed the idea of making schemes of increased accuracy order on the basis of a numerical-analytical approach to the computations of a wide class of the studied problems.

Unresolved parts of the issues. Numerical solution of a typical problem of metallurgical thermophysics, especially multidimensional and non-stationary, generates a huge amount of data. Therefore, the systematizing and interpreting this information, giving it physical characteristics gains special importance. For example, plotting or isolating is a fairly common way of presenting information. However, the service packs used are based on the data arrays processing, arranged in relation to nodes in the grid area. As a rule, they do not apply a priori information about construction methods. This can be explained only by the fact that in practice of using methods of finite-difference approximation there is a stable opinion about the change uncertainty of the desired function in the intervals between nodal points of net area.

According to the authors [6, 15], simple ideas that underlie the primitive replacement of derivatives by finite differences can not be successful without analyzing and taking into account specific properties of solutions of a particular class of problems. While processing the computational algorithm it is necessary to use a priori information about the problem, and first of all, about its membership to one or another class of functions smoothness. The above-mentioned approach became the basis for distributed vectors simulation of applied problems. This research illustrates its importance through example of the initial-boundary problem.

The purpose of the research is to develop a numerical solution to the problem of metallurgical thermophysics based on the application of multiprocessor systems. Particular attention should be paid to numerical-analytical algorithms for solving the set problems. Higher computation speedup comparing with the finite difference approach can be accomplished through the analytical solutions that allow simultaneous and parallel computing for all temporary layers without use of combined memory. To offer a package of application programs (PPP) that implements the solution of coefficient inverse heat conduction problems by mathematical simulation for carrying out computational experiments, based on the application of a multiprocessor computer system. The PPP should be developed covering requirements of object-oriented programming. At the same time, the solution of the coefficient problems reduces to the problems of optimal control, computing algorithms of which include in the package. Also, the PPP must include a data visualization block.

Main research results. The solution of the boundary value problem for the heat conduction equation is considered. Let us find a function that is described by the equation of the form:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, \quad (1)$$

herein

$$u(0,x) = 0,\ u(t,0) = 1,\ u(t,2) = 1., \quad (2)$$

Let us make a uniform grid, the step of which, respectively:

$$Dx1 = 0{,}01;\ Dt1 = 0{,}001, \quad (3)$$

Let a sequential algorithm be implemented by an implicit scheme by a sweep method. Then after the sampling of equation (1) the following system of linear algebraic equations (SLAE) is obtained:

$$U_{p,1}-U0_{p,1}=\left(\frac{Dt1}{Dx1^2}\right)\left[U_{p+1,1}+U_{p-1,1}-2U_{p,1}\right], \quad (4)$$

at the same time, the numbers of internal network nodes correspond to the expression: $p=\overline{1,2m-1}$; desired network functions $-U_{0,1}=1,\ U_{2m,1}=1,\ U_{p,1}$; values of the variable $U0_{p,1}$ are taken from the previous temporary layer.

The system of linear algebraic equations (4) has a three-diagonal structure, particularly:

$$C_pU_{p+1,1}-U_{p,1}+D_pU_{p-1,1}=f_p, \quad (5)$$

herein

$$\left.\begin{aligned} C_p = B_p &= \frac{Dt1/Dx1^2}{\left(1+Dt1/Dx1^2\right)},\\ f_p &= \frac{-U0_{p,1}}{\left(1+Dt1/Dx1^2\right)}, \quad if\ \ p=\overline{1,2m-1} \end{aligned}\right\}, \quad (6)$$

A fairly simple and convenient sequential method for solving the difference boundary value problem (4) - (6) is one of the methods for exclusion of unknown Gauss scheme and is called the sweep method. A small number of arithmetic operations, as well as a rather weak sensitivity to computational errors, make the sweep method a very convenient means for implementing sequential computational algorithms.

Here are several aspects of computational nature in computing simulation. When solving non-stationary problems by implicit (or explicit) methods the computations are always carried out according to the emporary layers consistently. If all the information about the adjacent layer is located in the RAM, then no special complications arise. However, if the problem is so big that it does not meet the stated above condition, then there should be used the combined memory. The information transfer time from slow memory to the operational is proportional to the number of points in the layer. The task solution finding time on the next layer is also proportional to the number of points in the layer. But one operation execution period is much less than the average time value of sending a unit of information from the slow memory to the operational one. Therefore, with such a computation, most of the time is spent on the transfers' organization, that is, spent nonproductively. Hence, the following question arises: can there be any increase of the efficiency of using computer memory when solving a given class of problems? And if there is a possibility, then how? The answers to the questions can be obtained with a more detailed research of the graph algorithm problem solution. Firstly, it is obvious that such a problem can be solved by a parallel processor. And secondly, the features of the parallelization of the problem should be such that the time of the corresponding calculations and data processing in the RAM becomes greater than the time spent on data transfer. Finally, in order to eliminate the use of combined memory in solving the problem (1) for such an equation there must either be applied a numerical-analytical approach, or one of the methods of mathematical physics, for example, Laplace's integral transformation in time.

Numerical-analytical approach. For each nod ($x=x_p$) of the network area the solution of a given equation is found in the analytic functions class that allow its representation in the form of a Taylor series method, that is

$$u_{p+\varepsilon_{x,1}}(t,x)=\sum_{n=0}^{\infty}\varepsilon_x^n u_{p,n+1}(t), \quad (7)$$

herein normalized variable

$$\varepsilon_x=\frac{x-x_p}{x_{p+1}-x_p}\in[-1,1], \quad (8)$$

unknown Taylors components of the desired function u are defined as follows:

$$u_{p,n+1}(t) = \frac{(x_{p+1} - x_p)^n}{n!} \frac{\partial^n u}{\partial x^n}_{|x=x_p}, \quad (9)$$

After substituting the series (9) into the relation (7), using the indeterminate coefficients method, we obtain a system of differential equations in the form of a ordinary differential equations system (ODES). Considering the obtained relation as recurrent by value of n, we can write the corresponding consequences. Then the equation (3.1) general solution gets the following form:

$$u_{p+\varepsilon_{x,1}}(x,t) = \left\{ u_{p,1}(t) + \sum_{n=0}^{\infty} \frac{\varepsilon_x^{2n}}{(2n)!} \left(\frac{Dx1^2}{a}\right)^n \frac{\partial^n u_{p,1}(t)}{\partial t^n} \right\} - \quad (10)$$

$$- \frac{\varepsilon_x}{\lambda} \cdot \left\{ u_{p,2}(t) + \sum_{n=0}^{\infty} \frac{\varepsilon_x^{2n}}{(2n+1)!} \left(\frac{Dx1^2}{a}\right)^n \frac{\partial^n u_{p,2}}{\partial t^n} \right\}.$$

It should be noted that the computing system can be used to increase the amount of available memory. For instance, with an increase by N times the number of processors, the available memory increases the same. This circumstance becomes very significant when solving multidimensional problems when there are problems with the computing environment memory (swapping, etc.). Therefore, for a more complete analysis of the developed multiprocessor system efficiency, the computational experiments were carried out in the simulation of multidimensional problems.

Consider the peculiarity of constructing splitting schemes for distributed simulation of applied problems. To have the ability of switching to significantly more complex algorithms, it is necessary to put the developed methodology on a fundamental theoretical basis. To do this, difference schemes of splitting can be used as one of the most important means for modeling multidimensional nonstationary problems of mathematical physics. The difference scheme of splitting is one of the important means of computing multidimensional non-stationary problems of mathematical physics. The point is that the difference schemes, where the number of arithmetic operations required for the transition between temporary layers is proportional to the number of unknown values of the desired functions, is called *economic*. It is known that the computation under explicit schemes is very simple. The quality of arithmetic operations in them is not subject to improvement. However, being economical, an explicit scheme is stable only with its strict limitation on the step of the grid in time. The difference schemes of splitting based on the set of not quite equivalent to each other sentences, but with a stereotypical goal to reduce the three-dimensional propagation problem of the domain of dependence on the sequence of schemes include unknown variables, which act alternately in coordinate directions and reduce the solution of such problems to scalar sweep method. Therefore, the difference scheme of splitting is considered an economic and, of course, stable, that is, as if combining the benefits of explicit and implicit schemes.

Moreover, there should be noted that the greatest effect from the use of up-to-date systems for processing information with a high level of parallelism is likely to be achieved when the described schemes are applied to perform matrix calculations in linear algebra or in methods for solving differential equations with partial derivatives. If there was an opportunity to use one processor on one computation node during the solution of the mentioned equations, then one can perform computations in all nodes in parallel and simultaneously. Surely, it is unreal. A typical finite-difference grid is composed of 50x50 or 100x100 nodes, so its computation in such architecture requires a system with 2,500 or 10,000 processors.

The use of numerical-analytical solutions allows for each temporary layer to perform computations simultaneously at any time, and, consequently, it does not require organization of information transfer from slow memory to the operational, which means, the interprocessor data exchange is excluded. This explains the significant solution speedup of the problems that were simulated by numerical-analytic methods.

Today, there are various software products, often called packages or software complexes. This research considers the applications package, intended for thermophysical experiments processing by inverse methods. The main purpose of the PPP making is to provide practical assistance to the researcher in all stages of the thermal-physical experimentation using inverse methods by a personal computing cluster.

Hence, the class of inverse thermal problems of metallurgical thermophysics is examined. Their formulation is done in terms of "cause-effect" relationship. According to the accepted model, the boundary conditions and their parameters, initial conditions, thermophysical properties and etc, are related to causal characteristics of the heat exchange process. In this interpretation, the establishment of causal relationships is the goal of direct heat transfer problems. Conversely, if certain information about the temperature field needs to be restored to causal characteristics, then we have one or another formulation of inverse heat transfer problems (IHTP) belonging to the class of problems that are incorrect from the positions of Hadamard.

The identification method of heat conduction equations according to the data of the thermophysical experiment is based on the interpretation of IHTP as the optimal control problems [5]. In this case, the mathematical model (MM) is considered to be guided by the set of input parameters given by the vector *R*. These include the coefficients of the heat equation. For given values of the components of the vector *R*, the solution of the heat equation with initial and boundary conditions is not only the function of spatial coordinates and time, but also the input parameters. Assuming that at certain moments of time the thermal state of the sample, and some parameters of the vector *R* are unknown, we arrive at the problem of optimal control. Introduction of the functional allows us to formulate the method of identifying the algorithm of IHTP solution. The MM's structure in this case is reduced to two controlled models:

- temperature (*model* 1),
- streaming (*model 2*).

Such a statement allowed to divide the control parameters for the vector *R*. Thus, by model 1 it is possible to involve control in the form of thermal conductivity coefficient, and the model 2 - the of thermal conductivity coefficient.

Due to the chosen method of approximation, the solution of the heat equation (direct method) is reduced to standard computational procedures - the sweep method and use of differential effects of analytic solutions on the nodes of the grid, provided that $p = \overline{1,2m_x - 1}, m_x \in Z$.

The domain of vector *R* parameters admissible values in MM is selected on the basis of a priori information about the model. In the PPP, this procedure is formalized by introducing several conditions in the algorithm. The methodology of the approach is reduced to the construction of a minimizing functional sequence, particularly:

$$J(R) = (T_e - T_p)^2, \quad (11)$$

wherein T_e, T_p are the temperature values, they are known from the experiment and computation obtained as a result of the MM solutions.

In this formulation the IHTP coefficient computation is reduced to the problem of optimal control, which solution algorithm is implemented in this PPP.

The experimental data visualization results are shown in Fig. 1.

The visualization results analysis demonstrate that the isolines are smoothed out, which most accurately reflects the computational algorithm. Thus, when making service programs for processing and issuing results to print in the form of charts and isolines, the proposed approach allows to minimize work on input and output data of the studied problems class.

Since the values of the base nodes are arranged in a grid area, then for each temporary layer, the operations shown in formula (10) are not related to each other. Therefore, the computations when making graphs or isolines can be performed in parallel and simultaneously.

The PPP is used for planning and processing of the thermophysical experiment results by the inverse methods. The developed algorithms used in the PPP can quite simply be rebuilt for the solution of other coefficient and boundary IHTPs.

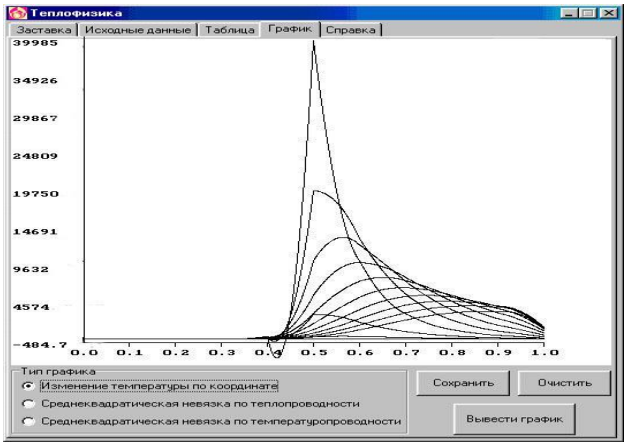

Fig. 1. Processing of the test problem solution results in graphs

Conclusions. In this research, the approach to the numerical-analytical concept of visualizing vectors in solutions allows us to obtain any necessary data for making smooth graphs or isolines on the corresponding grids. The algorithm maximum parallel forms are a subject of special interest, since they determine the minimum possible time for visualization algorithm implementation.

For the computing experiments based on application of a multiprocessor computing system there was developed a package of applied programs that implements the solution of coefficient inverse heat conduction problems by the method of mathematical modeling. The PPP is made to meet requirements of object-oriented programming. In this case, the computation of the IHTP coefficient reduces to optimal control problems, which solution algorithms are implemented in this PPP. Note that the PPP also includes a data visualization unit.

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Визуализация решений прикладных задач в многопроцессорных вычислительных системах

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Аннотация. Статья посвящена распределенному моделированию визуализации векторов решений прикладных задач на основе схем повышенного порядка точности. Более высокое ускорение вычислений по сравнению с конечно-разностным подходом иллюстрируется использованиям аналитических решений, которые позволяют проводить вычисление одновременно и параллельно по всем временным слоям. Показано, что наиболее перспективным подходом к математическому моделированию прикладных задач следует считать тот, который основывается на численно-аналитических решениях.

Ключевые слова: *многопроцессорная вычислительная система, ускорение, визуализация, распределенное моделирование, численно-аналитическое решение.*