MATHEMATICS

Mathematical models on the basis of fundamental trigonometric splines

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Abstract. The paper considers some systems of basic functions which are easy to be used in the interpolation problems. The most well-known today systems of basic functions include Lagrange system of basic functions and the systems of complete, even and odd basic trigonometric polynomials. However, there are some other systems of basic functions on analytical grid, including the systems of polynomial basic splines and trigonometric complete, even and odd basic splines. Some of these systems are considered in this paper.
Keywords: *basic functions, Lagrange functions, interpolation, simple polynomial splines, trigonometric splines.*

Introduction. Modern science is characterized by soaring amounts of scientific information. In turn, the ability of computers exceed human capabilities in the field of data processing. It is the combination of these factors and have proven their computerization.

Computerization of science allows when building mathematical models use a more complex mathematical objects and methods. Earlier problems of approximation of functions used mainly polinomials approaching; applying the same powerful means of computing has provided opportunities for use in such tasks other system functions, including polynomial and trigonometric splines [2].

In the problems of applied theory of approximation in the role of close function is convenient to use the generalized polynomials for some systems functions, coefficients, which are themselves value close function. System functions allow you to get this view is called **the fundamental**.

The importance of this approach is that when using linear processing methods close features processing subject to only the most basic functions. This fact in most cases allows to carry out the necessary calculations processing experimental data in two stages. The first stage is carried out the calculations related to the handling of the fundamental functions (these calculations can be carried out previously), in the second stage are calculations that take into account the value of the close function.

Holding the 2-step treatment has a number of advantages, among which we mention the possibility to calculate the experimental data in real time scale that often is important in many of the problems of data processing in real time scale.

To the systems of fundamental functions, the most prominent at present belong to the system of fundamental interpolation features Lagrange system full, odd and even fundamental interpolation trigonometric polynomials. These system functions to draw attention to the fact that algebraic and trigonometric polynomials are linearly dense sets in spaces under continuous and periodic continuous functions. However, there are other fundamental system functions, including systems of polynomial and trigonometric fundamental splines. [2]. Consider the following system of fundamental functions in more detail.

Main part

Statement of the problem. Let the segment $[0,T]$, set to some grid Δ_N, $\Delta_N=\{t_i\}_{i=0}^{N}, 0\le t_0<t_1<...<t_N\le 1$. Also along the specified segment $[0,T]$ set the function $f(t)$, and it is known values as $f(t_j)=f_j$, $j=0,1,\ldots,N$ of this function in the grid. The generalized polynomial needs to be build

$$\Phi_N(t)=c_0\varphi_0(t)+c_1\varphi_1(t)+...+c_N\varphi_N(t)$$

On function system $\varphi_0(t),\varphi_1(t),...,\varphi_N(t)$ of the given class, which depends $N+1$ st options $c_0, c_1,\ldots,c_N$ and satisfies the conditions

$$\Phi_N(t_j)=f_i,\quad j=0,1,\ldots,N.$$

The formulated task called the task of interpolation and polynomial $\Phi_N(t)$ is called the generalized interpolation polynomial.

It is clear that the definition of functions $\varphi_0(t),\varphi_1(t),...,\varphi_N(t)$ not be narrower than period specification interpolation functions. In many cases, the problems of interpolation system functions $\varphi_0(t),\varphi_1(t),...,\varphi_N(t)$ on grind Δ_N it is advisable to choose in such way so so that the coefficients $c_k, k=0,1,..., N$, of generalized interpolation polynomial are the value of the interpolation polynomial $\Phi_N(x_j)$ function $f(t)$ in the nodes of the grid. In this case the fucntion $\Phi_N(t)$ takes the form

$$\Phi_N(t)=f(t_0)\varphi_0(t)+f(t_1)\varphi_1(t)+...+f(t_N)\varphi_N(t)\,. \quad (1)$$

The (1) follows that in this case, the function $\varphi_0(t),\varphi_1(t),...,\varphi_N(t)$ must be carried out in relation:

$$\phi_j(t_i)=\begin{cases}1, & j=i;\\ 0, & j\ne i.\end{cases} \quad (2)$$

Function for those in the grind Δ_N take place such relations (2), are being called fundamentals in the grind [1].

The system of fundamental functions. Lets consider some of the system's fundamental functions in more detail.

1. The system of fundamental functions of Lagrange

The system of functions $\varphi_0(t),\varphi_1(t),...,\varphi_N(t)$ consist of algebraic polynomials $l_{Nk}(t)$, $k=0,1,\ldots,N$, the degree N, which are of the form

$$l_{Nk}(t)=\frac{(t-t_0)(t-t_1)...(t-t_{k-1})(t-t_{k+1})...(t-t_N)}{(t_k-t_0)(t_k-t_1)...(t_k-t_{k-1})(t_k-t_{k+1})...(t_k-t_N)}=\frac{\omega_N(t)}{(t-t_k)\omega'(t_k)},$$

where, $\omega_N(t)=(t-t_0)(t-t_1)...(t-t_N)$,

referred to as the system of fundamental functions of Lagrange on the grid Δ_N.

It is easy to check that,

$$l_k(t_i)=\begin{cases}1, & k=i;\\ 0, & k\neq i,\end{cases}$$

$$k,i=0,1,\ldots,N.$$

Using the basic functions of Lagrange, the interpolation polynomial of Lagrange that interpolates the function $f(t)$ in the grid Δ_N can be written as:

$$L_N(t)=\sum_{k=0}^{N} f_k l_{Nk}(t).$$

The disadvantage of Lagrange fundamental functions is that they behave quite irregular; in particular, between the nodes of the grid they take values larger than one unit, i.e. modules values of these polynomials in points their bends do not fall monotonically with increasing distance from the site of interpolation.

2. The system of complete trigonometric polynomial of the fundamental

When considering trigonometric functions we will assume that the $T\equiv 2\pi$. On the $[0,2\pi]$ the uniform grid

$\Delta_N=\{t_i\}_{i=1}^{N}$ $t_i=\frac{2\pi}{N}(i-1)$ has been set.

The system of fundamental trigonometric polynomials on the grid Δ_N consists of polynomials $t_k(t)$, $k=1,..N$, $N=2n+1, n=1,2,...)$, where N - the number of interpolation nodes, and the n – the order of the trigonometric polynomial.

On this grid fundamental trigonometric polynomials $t_k(t)$, $k=1,..N$, in order n can be written in such way:

$$t_k(t)=\frac{1}{N}\left[1+2\sum_{j=1}^{n}\cos j(t-t_k)\right].$$

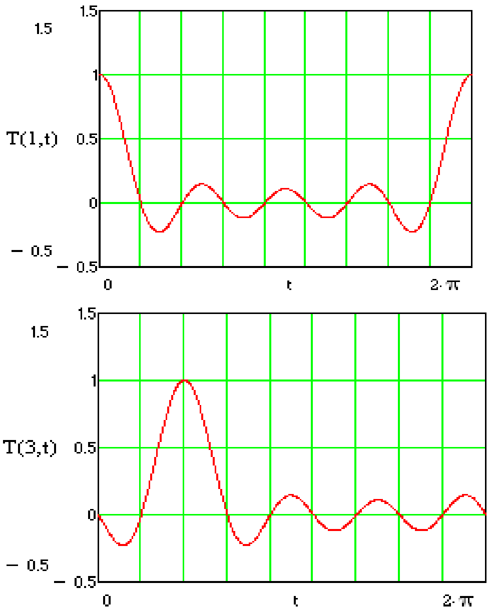


Fig. 2. The fundamentals on the grid Δ_N trigonometric polynomials $t_1(t)$ and $t_3(t)$ ($N=9$).

The system of basic trigonometric functions, from some grid Δ_N is the only in the sense that there is something a single system of trigonometric polynomials of order n that satisfies this grid conditions

$$t_k(t_j)=\begin{cases}1, & k=j;\\ 0, & k\neq j,\end{cases}\quad (k,j=1,\ldots,N).$$

Using a system of basic trigonometric polynomials $t_k(t)$, $k=1,..N$, fundamental trigonometric polynomials can be written in such way:

$$T_n(t)=\sum_{k=1}^{N} f_k t_k(t).$$

3. Systems of fundamental polynomial splines. Among other classes of polynomial functions attention classes of simple polynomial splines, which, as is well known, is the best linear approximation of vehicle classes differentiated features. In addition, it is worth noting the possibility to control the differential properties of such spline, which is very convenient in many tasks.

It is easy to build a system of fundamental polynomial splines. On the segment $[0,T]$ has been set the grid Δ_N. Label through $C^m_{[0,T]}$ class real continuous functions that have continuous derivatives up to the $m-$th order, aand through $P_{2m-1}[0,T]$ class of algebraic polynomials of degree $2m-1, m=1,2...,$ specified on the segment $[0,T]$. Define $N+1$ the sequence $\Theta_{Nk}=\{\zeta_{ki}\}_{i=0}^{N}$, that defined this way:

$$\zeta_{ki}=\begin{cases}1, & i=k;\\ 0, & i\neq k.\end{cases}$$

Lets define system of fundamental polynomial splines of degree $2m-1, m=1,2...,$ with defect $d, 1\leq d\leq m$ (if $d=1-$ polynomial splines is called simple)

$$S_{2m-1,d}(k,\Delta,t), k=0,1,..., N,$$

So, $S_{2m-1,d}(k,\Delta,t)=S_{2m-1,d}(\Theta_{Nk},\Delta,t)$.

Therefore, the fundamental polynomial spline $S_{2m-1,d}(k,\Delta,t), k=0,1,..., N,$ takes the value 1 in k-th node of the grid Δ_N and a value of 0 in all other nodes of the grid.

Using a system of the fundamental polynomial splines, the spline that interpolates the function $f(t)$ in the grid Δ_N can be written in the form

$$S_{2m-1,d}(f,\Delta,t)=\sum_{k=0}^{N} f_k S_{2m-1,d}(k,\Delta,t).$$

We do not provide graphs of basic simple polynomial splines based on uniform grid because they coincide with the fundamental trigonometric splines, graphics which will be below [3].

The class of polynomial splines have a number of shortcomings, which greatly hampered their use in many problems of science and technology. One of those disadvantages is the substantial growth of the difficulties their building with the growth of the degree of the spline; as a result, in practice mainly used only polynomial splines of the third order. The other principal drawback of polynomial splines is their structure; in turn, this leads to the fact that the operations of integration and differentiation splines need to carry over the polynomial approximations that make these splines.

So some interest cause other types of interpolation features that have certain properties of smoothness. One of these classes of functions are trigonometric interpolation splines [2] [3], which have the advantage of polynomial splines and serves only analytical expression throughout the interval approximation; In addition, the complexity of their construction does not increase with increasing orders of splines.

4. The system of fundamental trigonometric splines. The system of fundamental trigonometric splines will have mark $St(r,t)$. Unlike fundamental trigonometric polynomials $t_k(t)$, that are shown in the [3] depend more on setting $r, r=1,2,...;$ his option determines the differential properties of trigonometric splines. So when any value r $St(r,t)\in C^{r-1}_{[0,2\pi]}$.

In the paper [4] was investigated the influence of differential properties of fundamental trigonometric interpolation splines interpolation accuracy on both ends and the middle section of the interpolation.

Fundamental trigonometric interpolation splines, which we will examine on uniform grids Δ_N have a look

$$St(r,t)=\frac{1}{N}\left\{1+2\sum_{j=1}^{\frac{N-1}{2}}\alpha_j^{-1}(r)[C_j(r,t)\cos jt_k+S_j(r,t)\sin jt_k]\right\},$$

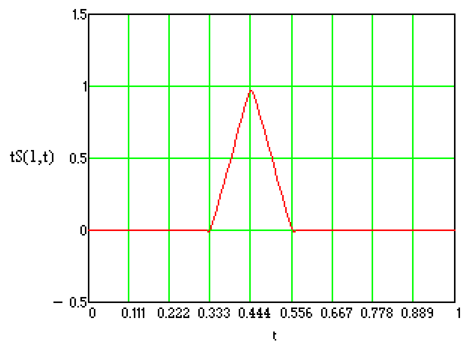
Where $C_j(r,t)=\frac{\cos jt}{j^{r+1}}+\sum_{m=1}^{\infty}\left[\frac{\cos(mN+j)t}{(mN+j)^{r+1}}+\frac{\cos(mN-j)t}{(mN-j)^{r+1}}\right]$

$$S_j(r,t)=\frac{\sin jt}{j^{r+1}}+\sum_{m=1}^{\infty}\left[\frac{\sin(mN+j)t}{(mN+j)^{r+1}}-\frac{\sin(mN-j)t}{(mN-j)^{r+1}}\right],$$

$$\alpha_j(r)=\frac{1}{j^{r+1}}+\sum_{m=1}^{\infty}\left[\frac{1}{(mN+j)^{r+1}}+\frac{1}{(mN-j)^{r+1}}\right]$$

$j=1,2,...,n$, $k=1,...,N$, r - spline's degree

The look of some of the fundamental trigonometric splines with different values of the parameter *r* are given in Fig. 3.



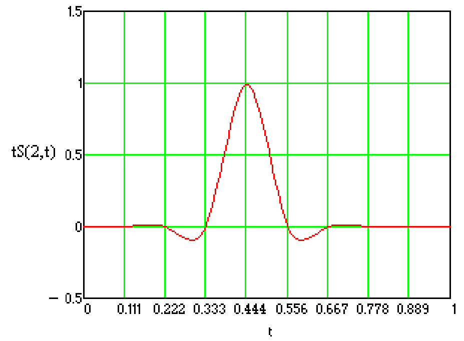


Figure 3. Graphs of basic trigonometric splines with *r* =1,2

It is easy to see that fundamental trigonometric splines module highs come from the removal from the site of interpolation. Also for the fundamental trigonometric splines at any values *r* of condition

$$\sum_{k=0}^{N}tS(r,t)=1$$

A certain weakness for fundamental trigonometric splines is their frequency. However, they can be applied to non-periodic functions, applying this special methods of periodic continued [3].

Conclusions. The paper considers some of the system's fundamentals functions, which is convenient for application in problems of interpolation. The look of interpolation polynomial in the form of (1) has a significant advantage over other forms of representation, that is, that there is no need to calculate the coefficients $c_k, k=0,1,...,N$, of the polynomial interpolation. In turn in the role of uncertain parameters in (2) advocating the value of approximate search solving in the key points $t_j,(j=1,...,N)$. It is clear that the formulation in terms of the key points allows you to associate settings with limited parts in the General case of spatial areas that include these nodal points. This fact is very useful as a study method this equation allows you to discover the range, where the convergence of decisions is slow (or live) [1].

A particular disadvantage of such a submission can be considered that a change in the grid have to transfer functions $\varphi_0(t),\varphi_1(t),...,\varphi_N(t)$ however, this disadvantage in many applied problems are inconsequential, since in the vast majority of these problems are considering only the uniform grid.

Fundamental trigonometric splines have a number of advantages in comparison with polinomial splines. Such benefits include no algorithms for constructing trigonometric splines for different values of the parameter and view their only expression (uniformly convergent trigonomentric number) on the entire range of approximation.

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Математичні моделі на основі фундаментальних тригонометричних сплайнів

В. П. Денисюк, О. В. Негоденко

Анотація. В роботі розглядаються деякі системи фундаментальних функцій, які зручно застосовувати в задачах інтерполяції. До систем фундаментальних функцій, найбільш відомих на теперішній час, належать система фундаментальних функцій Лагранжа та системи повних, парних та непарних фундаментальних тригонометричних многочленів. Проте на рівномірних сітках існують і інші системи фундаментальних функцій, зокрема системи поліноміальних фундаментальних сплайнів, а також тригонометричних повних, парних та непарних фундаментальних сплайнів . Деякі з таких систем і розглядаються в даній роботі.

Ключові слова: *фундаментальні функції, функції Лагранжа, інтерполяція, прості поліноміальні сплайни, тригонометричні сплайни.*