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Comparative method adopted by triangulating models of surfaces for solving the planning tasks on construction site

T. Honcharenko*, V. Mihaylenko

Kyiv National University of Construction and Architecture, Kyiv, Ukraine
*Corresponding author. E-mail: geocad@ukr.net

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Abstract. The publication describes a method of comparing surfaces given by triangulation models, which is proposed to be used for solving tasks of planning the for construction territory. The proposed approach uses digital elevation model constructed within the specified limits on the basis of the triangulation method. The creation of a combined triangulation grid of both surfaces allows interpolation of the functions of determining the height markings based on the localization of triangulations in one another. Comparison of functions on separate faces of combined triangulation allows us to determine the difference between surfaces. The method is proposed to be used to compare two digital elevation models – "As project" and "As built", for determination of two surfaces intersection and for calculation of the earthworks volumes on the construction site.

Keywords: *surface triangulation model, digital elevation model, TIN surface, combined triangulation grid (CTG), method of calculation of volumes of earthworks, construction site.*

Introduction and analysis of publications. The planning tasks on construction site are connected with the decision of the following tasks of modeling the relief's surface:

- calculation of the total volume of excavation works within the construction territory;
- definition of the line of intersection of two surfaces - the planned and adjacent territory;
- determining the difference between "As project" and "As built" reliefs;
- calculation of volumes of unsuitable soil, unsuitable for falling asleep.

The solution of all the above-mentioned tasks is related to the task of the comparison of surfaces. For example, the calculation of volumes of earthworks is the final stage of the solution of the problem of vertical planning of the territory for development and serves as a qualitative assessment of design relief design. Solving this problem requires the use of a method of comparing two surfaces to determine the difference in volume of soil. In the study of methods for calculating the volume of earthworks [1–3] found that the main methods are cross sections and triangular and quadrilateral prisms. The method of cross-section is used for equal, without large changes of territories, and for more accurate results, the calculation is based on triangular and quadrangular prisms. The results of this calculation are superimposed on the grid of squares of cartography of earthworks. For simplification of calculations and specification of volumes in squares containing both a recess and a mantle, when calculating, triangular prisms are used more often. All methods considered are methods of approximate calculation of volumes of excavation works.

Studies in the field of comparison of three-dimensional surfaces are conducted long ago [4–10], and there are several approaches to solving this problem. In most of the existing methods for the comparison of two surfaces, it is assumed that for each point of one surface there is a corresponding point of another surface [6, 7]. Some methods suggest using the transformation of the outputs of irregular grids in regular, after which approaches to the comparison of surfaces can be applied to regular sets of points [1–5, 10]. With this recalculation, there is a problem of choosing the optimal step of a regular grid, which leads to a significant amount of calculations to achieve an acceptable accuracy of the approximation of the surface. There is an approach using curvature maps (Curvature Maps), which is based on the construction of contour lines in the vicinity of certain points of the surface, followed by a comparison of surfaces on these maps [4, 9,10].

In the triangulation model of the surface, 3 types of components are distinguished: a node (vertices), ribs (sections), and faces (triangles). A triangulation is called a planar graph, all of its internal regions being triangles [3]. The triangulation method for constructing a digital surface model consists of connected points, lines, polygons, which are geometric forms of discrete models of spatial objects in such a way that all segments of the broken and polygons pass through the edges of the triangulation. Both surface reliefs, constructed in this way, have irregular triangulation grids of different density. There is a need for as accurate as possible a comparison of such surfaces, which are determined by the functions of two variables on different irregular grids.

Purpose of the study is development of method for comparing surfaces given by irregular grids of triangulation models of different density, for solving the set of planning tasks on the construction site.

The statement of the task of the comparison of surfaces in the tasks of planning construction territory can be formulated in this way.

There are two independent triangulation models of the surface - existing and design within the calculated territory. The digital elevation model is determined by the triangulation model and is described by the following set:

$$T_{TIN} = \{T_N, T_E, T_F\},$$

де T_N –- set of nodes, T_E - set of edges, T_F - set of faces.

Such a model consists of a set of disjoint spatial triangles, in which value is defined as a function of two variables given on a discrete irregular grid:

$$z_i = f(x_i, y_i).$$

For the purpose of calculating the volume of excavation work, it is necessary to determine the difference between the surfaces and to calculate which areas of the land plots to cut, and which, on the contrary, to fall asleep to get the desired surface. In this case, it is necessary to determine the volumes of transported masses of soil (the amount of cut and buried volumes) and the balance volume (the difference between the cut and buried volumes, that is, the surplus or lack of soil).

Mathematical statement of the task of earthworks calculation on the construction site has the following content. Specifies a comparison area that is described by a plurality of contours $\{K_i, ..., K_m\}$, m>= 1, each of which is a closed broken line connecting a set of points on a plane (vertices of polygons):

$$K_i = \left((x_1^i, y_1^i), (x_2^i, y_2^i), ..., (x_{n_i}^i, y_{n_i}^i), (x_1^i, y_1^i)\right),\ n_i \ge 3.$$

such that the cross sections and the intersection of the circuits themselves are not allowed..

In this area, triangulation models of two surfaces of TIN_1 and TIN_2 have been constructed, which are described by the corresponding sets in a three-dimensional coordinate system:

$$TIN_1 = \{(x_1^i, y_1^i, z_1^i)\}_{i=1}^{N_1} \text{ та } TIN_2 = \{(x_2^i, y_2^i, z_2^i)\}_{i=1}^{N_2},$$

де N_1 та N_2 - number of nodes in each model respectively.

Need to determine:

- The area of the polygon L^- and its corresponding volume, defining the territory within which the surface, which is described by the model TIN_1 higher surface TIN_2. This determines the volume of soil to be cut.

- The area of the polygon L^+ and its corresponding volume, which determines the area on which the surface of TIN_1 is lower than the surface of TIN_2. This determines the volume of soil to be filled.

- The area of the polygon L^0, on which the models TIN_1 and TIN_2 are coincide. Thus the area of zero volumes is determined.

The method of solving the task consists in the successive execution of the following steps:

1. Determine the irregular two-dimensional networks of triangles T_1 and T_2 the nodes of which are projections of the triangulation models TIN_1 and TIN_2 to the plane Oxy:

$$T_1 = \{(x_1^i, y_1^i)\}_{i=1}^{N_1},\ T_2 = \{(x_2^i, y_2^i)\}_{i=1}^{N_2},$$

then the value of the third coordinate value is given by the corresponding functions:

$$z_1^i = f_1(x_1^i, y_1^i),\ z_2^i = f_2(x_2^i, y_2^i).$$

2. Localize the nodes of each of the triangulation grids in the triangles of another grid. The nodes of the grid T_1 are localized in the triangulation model TIN_2, the nodes of the grid T_2 are localized in the triangulation model TIN1. In this way we create a new triangulation network of triangles T^O:

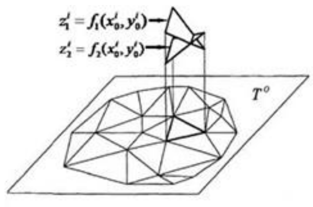


Fig.1. Combined triangulation model

$$T^o = T_1 \cup T_2.$$

To obtain a combined triangulation grid, we use a modified algorithm for merge two unbundled triangulations, proposed in [5]. As ribs in the combined triangulation network, all the edges of the triangles T1, T2 and their intersection points are introduced:

$$T^o = \{(x_0^i, y_0^i)\}_{i=1}^{N},$$

де N - number of nodes of the new combined triangulation grid (CTG).

3. For each node of CTG T^O, we determine by means of linear interpolation the values of heights of both triangulation models TIN_1 and TIN_2 by the corresponding functions z_1^i and z_2^i given in Fig.1.

Let's consider the method of linear interpolation in more detail.

On Fig. 2 for determining the value of the function Z_0 at an arbitrary point (X_0, Y_0), the triangle of the triangulation model in which it was found is determined. The triangle $Z_1 Z_2 Z_3$ in three-dimensional space is determined by three points with coordinates (X_1, Y_1, Z_1), (X_2, Y_2, Z_2) and (X_3, Y_3, Z_3). It is necessary to interpolate the coordinate Z_0 of the point (X_0, Y_0), which got into a triangle with coordinates (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3). The plane in three-dimensional space is given by the equation:

$$a \cdot X + b \cdot Y + c \cdot Z + d = 0,$$

Then in a three-dimensional space, the plane passing through the points with coordinates (X_1, Y_1, Z_1), (X_2, Y_2, Z_2) and (X_3, Y_3, Z_3) is determined by the coefficients that can be defined by the formulas:

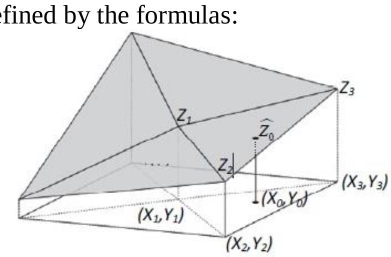


Fig.2. Linear interpolation value Z_0

$$a = Y_1(Z_2 - Z_3) + Y_2(Z_3 - Z_1) + Y_3(Z_1 - Z_2)$$
$$b = Z_1(X_2 - X_3) + Z_2(X_3 - X_1) + Z_3(X_1 - X_2)$$
$$c = X_1(Y_2 - Y_3) + X_2(Y_3 - Y_1) + X_3(Y_1 - Y_2)$$
$$d = X_1(Y_2Z_3 - Y_3Z_2) + X_2(Y_3Z_1 - Y_1Z_3) + X_3(Y_1Z_2 - Y_2Z_1)$$

The value of the function z0 at the point (x0, y0) is determined by the formula:

$$Z_0 = \frac{-a \cdot X_0 - b \cdot Y_0 - d}{c}.$$

The values of both functions are known in the nodes of the combined triangulation grid T^O.

4. It is necessary to determine the mutual spatial arrangement of triangles for each triangle CTG T^O given by the functions:

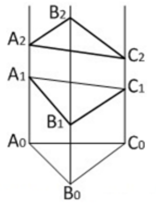


Fig.3.Triangulation faces

$z_1^i = f_1(x_0^i, y_0^i)$ and $z_2^i = f_2(x_0^i, y_0^i)$

Let's consider in more detail individual triangles from CTG T^0.

Fig.3 shows the three triangulation faces from CTG T^0 – $\Delta A_0B_0C_0$, $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$. Each corresponding vertex of all three triangles has the same coordinates on axis Ox and Oy, and the corresponding coordinates along the axis Oz are equal to the values of the functions f_1 and f_2 at these vertices:

$Z_{A1} = f_1(x_{A1}, y_{A1}$ $Z_{B1} = f_1(x_{B1}, y_{B1})$, $Z_{C1} = f_1(x_{C1}, y_{C1})$,),

$Z_{A2} = f_2(x_{A2}, y_{A2}$ $Z_{B2} = f_2(x_{B2}, y_{B2})$, $Z_{C2} = f_1(x_{C2}, y_{C2})$.),

There is a problem of determining the coordinates of the axis Oz vertices A_1, and A_2, B_1 and B_2, C_1 and C_2:

$$a = Z_{A2} - Z_{A1},\ b = Z_{B2} - Z_{B1},\ c = Z_{C2} - Z_{C1}.$$

It is necessary to compare functions f_1 and f_2 on a triangle $\Delta A_0B_0C_0$ for calculation of the difference between them and solution the above problem.

Let's consider all possible cases of the mutual arrangement of spatial triangles $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$.

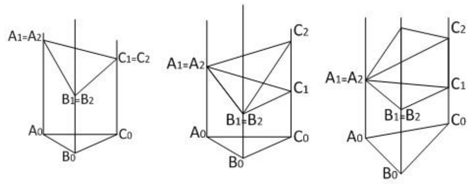
When crossing triangles different shapes are formed depending on the characters of the quantities ***a, b, c.*** There are 27 variants of combinations, since each of the three numbers may be greater, less or equal to zero. Each of the 27 variants can be attributed to one of three cases.

Let's consider more in detail these three situations.

The first situation reflects 15 variants of the location of the triangulation faces, some of which are shown in Fig.4.

All values of the set {a, b, c} simultaneously have the same signs or are equal to zero:

($a \geq 0$ & $b \geq 0$ & $c \geq 0$) or ($a \leq 0$ & $b \leq 0$ & $c \leq 0$).



1) 2) 3)

Fig. 4. Variants of the location triangulation faces:
1) $a = 0$, $b = 0$, $c = 0$; 2) $a = 0$, $b = 0$, $c > 0$; 3) $a = 0$, $b > 0$, $c > 0$

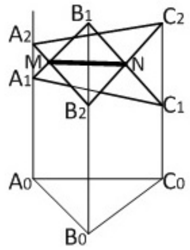


Fig.5. Crossing the faces
a>0, b<0, c>0

The formula for calculating the volume of such a figure for the first case equals the volume of the truncated triangular prism:

$$V = S_{\Delta A_0B_0C_0}\frac{a+b+c|}{3} \qquad (1)$$

The second situation reflects 6 variants of crossing triangulation faces. The two numbers in {a, b, c} have the same sign and the third one. In fig. 5 shows an option for crossing triangulation triangles $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$ by segment MN, which corresponds to this situation:

a>0, b<0, c>0.

The point's coordinates M and N are based on the formulas:

$$M_t = \frac{A_t a + B_t b}{a+b},\ N_t = \frac{B_t b + C_t c}{b+c},\ t \in \{x, y, z\}.$$

The volume of the triangular pyramid B_2MNB_1 is calculated by the formula:

$$V_{піраміди} = V_{B_2MNB_1} = \frac{1}{3}H^{B_1}_{(MNB_2)}S_{\Delta MNB_2} \qquad (2)$$

The volume of a pentagonal figure $A_1A_2MNC_1C_2$ is expediently calculated as the sum of volumes of three triangular pyramids:

$$V_{п'ятигр} = V_{A_1A_2MNC_2C_1} = V_{A_1A_2MC_1} + V_{MA_1C_1C_2} + V_{MNC_2C_1},$$

$$V_{A_1A_2MNC_2C_1} = \frac{1}{3}(H^M_{(A_1A_2C)}S_{\Delta A_1A_2C} + H^M_{(A_1C_2C_1)}S_{\Delta A_1C_2C_1} + H^M_{(NC_1C_2)}S_{\Delta NC_1C_2}) \qquad (3)$$

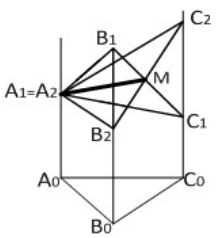


Fig.6. Crossing the faces
a=0, b<0, c>0

The third situation considers 6 variants of crossing the triangulation faces. One of the numbers of the set $\{a, b, c\}$ is 0 and the other two have different signs**.**

Fig. 6 shows the intersection of triangulation triangles $\Delta A_1B_1C_1$ and $\Delta A_2B_2C_2$ by segment A_1M, which corresponds to this situation:

a=0, b<0, c>0.

In this case, the result of crossing triangulation triangles is two triangular pyramids $B_1A_1MB_2$ and $MA_1C_2C_1$, their volumes are calculated by the formulas:

$$V_{B_1A_1MB_2} = \frac{1}{3}H^{B_1}_{(A_1MB_2)}S_{\Delta A_1MB_2} \qquad (4)$$

$$V_{MA_1C_2C_1} = \frac{1}{3}H^M_{(A_1C_2C_1)}S_{\Delta MA_1C_2C_1} \qquad (5)$$

Thus, for such a case, the crossings of the triangles separately calculates the volumes of the two pyramids and depending on the combination of values of the set *{a, b, c}* are added to the corresponding total earthworks volume and area of excavation work and of the embankment.

Table 1 provides all possible combinations of signs of the values of the set *{a, b, c}* and specifies the formula's number of volumes the figures (1)–(5) for calculation the volumes of the excavation and embankment.

Table 1.

Variants number	a	b	c	Situation number	Formula number for calculation excavation volume +( )	Formula number for calculation embankment volume - ( )
1.	> 0	> 0	> 0	1	+ (1)	
2.	> 0	> 0	= 0	1	+ (1)	
3.	> 0	> 0	< 0	2	+ (3)	- (2)
4.	> 0	= 0	> 0	1	+ (1)	
5.	> 0	= 0	= 0	1	+ (1)	
6.	> 0	= 0	< 0	3	+ (4)	- (5)
7.	> 0	< 0	> 0	2	+ (3)	- (2)
8.	> 0	< 0	= 0	3	+ (4)	- (5)
9.	> 0	< 0	< 0	2	+ (2)	- (3)
10.	= 0	> 0	> 0	1	+ (1)	
11.	= 0	> 0	= 0	1	+ (1)	
12.	= 0	> 0	< 0	3	+ (4)	- (5)
13.	= 0	= 0	> 0	1	+ (1)	
14.	= 0	= 0	= 0	1		
15.	= 0	= 0	< 0	1		- (1)
16.	= 0	<0	> 0	3	+ (5)	- (4)
17.	= 0	<0	= 0	1		- (1)
18.	= 0	<0	< 0	1		- (1)
19.	< 0	> 0	> 0	2	+ (3)	- (2)
20.	< 0	> 0	= 0	3	+ (5)	- (4)
21.	< 0	> 0	< 0	2	+ (2)	- (3)
22.	< 0	= 0	> 0	3	+ (5)	- (4)
23.	< 0	= 0	= 0	1		- (1)
24.	< 0	= 0	< 0	1		- (1)
25.	< 0	< 0	> 0	2	+ (2)	- (3)
26.	< 0	< 0	= 0	1		- (1)
27.	< 0	< 0	< 0	1		- (1)

5. Summarize the results presented in Table 1, depending on the type tasks of planning the for construction territory.

Task 1 – Calculation of total volumes of excavation within the calculated territory.

For the first situation:

- variant No. 14 – both triangulation triangles lie in the same plane, $TIN_1=TIN_2$, the surfaces coincide within triangulation triangle, the area of the triangle enters the polygon L^0.

- variants No. 1, 2, 4, 5, 10, 11, 13 - TIN_1 surface is higher TIN_2 surface within the triangulation triangle, the area of the triangle enters the polygon L^-.

- variants No. 15, 17, 18, 23, 24, 26, 27 - TIN_1 surface is lower TIN_2 surface within the triangulation triangle, the area of the triangle enters the polygon L^+.

For the second and third situations:

- all variants 1–27 – TIN_1 and TIN_2 surfaces intersect within the triangulation triangle. The section of the intersection of two space triangles is a line of zero works - the boundary between the embankment and cavity. He divides the triangle into two parts, which will be included in the resulting polygons L^+ and L^-.

Task 2 – Determination of the line of intersection of two surfaces.

For the second and third situations:

→ variants No. 2, 6, 9, 12, 16, 19-22, 25 – if all the segments of the triangles intersection in the CTG are interconnected, we obtain a broken line of intersection of two surfaces TIN_1 and TIN_2.

Task 3 – Comparison of the two surfaces TIN_1 and TIN_2 for the definition of difference reliefs "As project" and "As built".

For the first, second and third situations:

→ all variants 1–27 – for determinion the volumes difference between the TIN_1 and TIN_2 surfaces all formulas of volumes (1) – (5) need to be added.

This method can be used also to solve the problem of counting volumes of excavated soil unsuitable for falling asleep. To this end, a plurality of closed circuits are added to the triangulation model of the surface of the TIN_1, which defines the corresponding areas of soil ineligibility.

Conclusions. Comparative method adopted by triangulating models of surfaces for solving the planning tasks on construction site is characterized by a certain versatility and accuracy of the calculation in comparison with the methods discussed above. Scientific novelty consists in creation of the combined triangulation model of both surfaces given by irregular grids of different density, taking into account the constraints of the set of contours. The practical significance of the method lies in its application for solving problems that arise when planning and modeling the surface of the site for development, in particular for comparing digital elevation models - "As project" and "As built", to determine the intersection of two surfaces, to calculate the volume of earthworks on the construction site.

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