A contact problem solution with taking into account shear deformations

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Paper received 08.02.19; Accepted for publication 12.02.19.

https://doi.org/10.31174/SEND-NT2019-193VII23-20

Abstract. In the article the contact interaction of two elastic bodies with a curvilinear surface is considered, taking into account transverse deformations (changes in the surface curvature, or the excitement of the surfaces of the contact). The equations for determination of projections of deformations of the body surface points on three axes are derived. The stresses of compression in the contact area are determined. A tangential stresses caused by surface curvature changes are defined. The interaction of the profile beam of a drum and a rope and complex equations taking into account the features of the contact are considered. Normal and tangential stresses in the contact zone of a rope and a drum are defined. On the basis of the new provisions, a conclusion is made about the cause of the occurrence of tangential stress not only due to compression of the body, but also to the displacement.

Keywords: *contact interaction, curved surface, lateral deformations, compression stresses, contact area.*

Introduction. Mechanics of contact interaction is the actual area of deformed solids. Its development is stimulated by the problems of mechanical engineering, extractive and processing industries, but primarily by tribological issues.

Overview of related publications. One of the first researchers, who succeeded to get a general solution of the contact problem was Hertz H.[1]. He examined the contact of two elastic bodies with curved surfaces, that is loaded with forces operating transversal to the plane of the contact. Issues of the contact strength were dealt with by such outstanding scientists as Dinnik [2] Belyaev [3] Kovalsky B.S. [4], Pinegin [5], Phepl L. [6], Jonson K. [7] and others but some questions need explanations so far.

The basic material summary. When two elastic bodies are contacted, of which at least one has a curvilinear surface, it can be assumed that under normal load, the curvature will be "thrown", that is, the radius of curvature increases (Fig. 1).

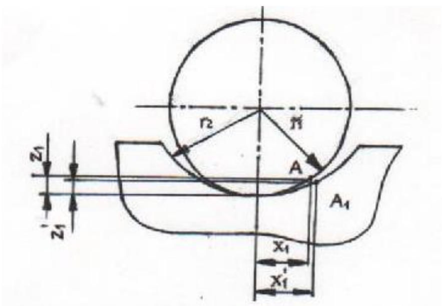

Fig. 1. Calculation scheme

Then point A, with coordinates x_1, z_1 is shifted to point A_1 having coordinates x_1', z_1'. As arc OA is equal arc OA_1 we obtain

$$r_1\varphi_1 = r_1'\varphi_1', \tag{1}$$

$$x_1 = r_1 \sin\varphi_1,$$

$$x_1' = r_1' \sin\varphi_1' \tag{2}$$

$$z_1 = r_1 - r_1\cos\varphi_1 = r_1(1-\cos\varphi_1),$$

$$z_1' = r_1'(1-\cos\varphi_1') \tag{3}$$

As we can see from these calculations, a point has not only vertical shift on axes Z_1 as before accepted by many authors but also the horizontal ones. These shifting will be maximal for the edges of the contact plane, and in the center they are equal a zero.

Find motion on axis z

$$w_1 = z_1 - z_1' = r_1(1-\cos\varphi_1) - r_1'(1-\cos\varphi_1') \tag{4}$$

From (1) we obtain

$$r_1' = \frac{r_1\varphi_1}{\varphi_1'} \tag{5}$$

Substitute this expression to equation (4) we obtain

$$w_1 = \left[(1-\cos\varphi_1) - \frac{\varphi_1}{\varphi_1'}(1-\cos\varphi_1')\right] \tag{6}$$

In a similar we obtain movement of other body

$$w_2 = \left[(1-\cos\varphi_2) - \frac{\varphi_2}{\varphi_2'}(1-\cos\varphi_2')\right] \tag{7}$$

Motion on axis x for the first body

$$u_1 = x_1 = x_1' = r_1\left(\sin\varphi_1 - \frac{\varphi_1}{\varphi_1'}\sin\varphi_1'\right) \tag{8}$$

For other body

$$u_2 = r_2\left(\sin\varphi_2 - \frac{\varphi_2}{\varphi_2'}\sin\varphi_2'\right) \tag{9}$$

For spaced body in plane YOZ transversal shifts will be equal

$$y_1 = R_1\sin\beta_1,\ y_1' = R_2\sin\beta_1', \tag{10}$$

Where R –radius of curvature of the body surface in the plane YOZ.

Then surface of the body motion on axis y is

$$v_1 = y_1 = y_1' = R_1\left(\sin\beta_1 - \frac{\beta_1}{\beta_1'}\sin\beta_1'\right) \tag{11}$$

$$v_2 = y_2 = y_2' = R_2\left(\sin\beta_2 - \frac{\beta_2}{\beta_2'}\sin\beta_2'\right) \tag{12}$$

Closing of the bodies will be equal

$$a = z_1' - z_1 + z_2' - z_2 = r_1\left[\frac{\varphi_1}{\varphi_1'}(1-\cos\varphi_1') - (1-\cos\varphi_1)\right] + \left[\frac{\varphi_2}{\varphi_2'}(1-\cos\varphi_2') - (1-\cos\varphi_2)\right] \tag{13}$$

Consider the contact stresses in the interaction zone of rope and drum.

In case when the contact occurs between rope and drum groove (Fig.2) we have

$$K_{11}=r_b;\ K_{12}=R;\ K_{21}=r_k;\ K_{22}=R,$$

$\gamma = 0$; $\gamma_1 = \gamma_2$ – deviation angle of the rope twist, then

$$\begin{cases} 2(A+B)=\frac{1}{r_b}+\frac{1}{R}+\frac{1}{r_k}+\frac{1}{R} \\ 2(A+B)=\left(\frac{1}{r_b}-\frac{1}{R}\right)\cos\gamma_1+\left(\frac{1}{r_k}+\frac{1}{R}\right)\cos\gamma_1 \end{cases} \qquad (14)$$

or

$$A=\frac{1}{2R}\left(1-\cos2\gamma_1\right)+\frac{1}{4r_b}\left(1+\cos2\gamma_1\right)+\frac{1}{4r_k}\left(1+\cos2\gamma_1\right) \qquad (15)$$

$$B=\frac{1}{2R}\left(1-\cos2\gamma_1\right)+\frac{1}{4r_b}\left(1+\cos2\gamma_1\right)+\frac{1}{4r_k}\left(1+\cos2\gamma_1\right) \qquad (16)$$

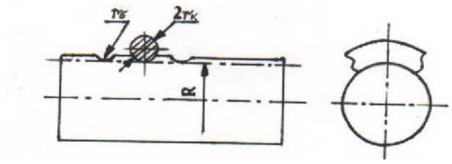

Fig 2. Layout of rope and drum contact.

Assumed the very small deviation angle ($\gamma \le 3^\circ$) and relationship $\frac{R}{r_k}=18...20$, we obtain:

$$\begin{cases} z_1=\frac{1}{2r_k}x^2+\frac{1}{R}y^2 \\ z_2=\frac{1}{r_b}x^2+\frac{1}{R}y^2 \end{cases} \qquad (17)$$

As in the middle the surface of pressure for each pair of corresponding points

$$z_1+z_2+w_1+w_2=a\ ,$$

we obtain

$$w_1+w_2=r_k(1-\cos\varphi_1)-2r_b\left(1-\cos\frac{r_k}{r_b}\varphi_1\right)-\frac{1}{2}\left(\frac{1}{r_k}+\frac{1}{r_b}\right)x^2-\frac{2}{R}y^2 \qquad (18)$$

Taking into account that the rope surface at pressure takes the point of the drum groove surface, it can be assumed that $w_1 = w_2$.

Then

$$w_1=\frac{r_k}{2}\left(1-\cos\varphi_1\right)-\frac{r_b}{2}\left(1-\cos\frac{r_k}{r_b}\varphi_1\right)-\frac{x^2}{4}\left(\frac{1}{r_k}+\frac{1}{r_b}\right)-\frac{1}{R}y^2 \qquad (19)$$

Write equation of elastic equilibrium

$$\begin{cases} \Delta^2u+\frac{1}{1-2\lambda}\frac{\partial\Delta}{\partial x}=0 \\ \Delta^2v+\frac{1}{1-2\lambda}\frac{\partial\Delta}{\partial y}=0 \\ \Delta^2w+\frac{1}{1-2\lambda}\frac{\partial\Delta}{\partial z}=0 \end{cases} \qquad (20)$$

Where u,v,w – shifts projections on coordinate axes x, y, z.

E – modulus of elasticity;

G – shear modulus;

λ – a Poisson constant

Δ^2u, Δ^2v; Δ^2w – Laplace's surgery.

Substitute expressions of deformation u and w from formulas (7) and (9) to the equations system (20) we obtain

$$v=\left(1-2\lambda\right)y\left[\frac{1}{R}+\frac{1}{2}\left(\frac{1}{r_k}+\frac{1}{r_b}\right)-\frac{z}{2r_k\left(1-\cos\varphi_1\right)}+\frac{1}{2}\right] \qquad (21)$$

Determine the tangential stresses on the body surface

$$X=G\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)=Gx\left[\frac{1}{2}\left(\frac{1}{r_k}+\frac{1}{r_b}\right)+\frac{1}{r_k\left(1-\cos\varphi_1\right)}\right] \qquad (22)$$

$$Y=G\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=Gy\left[\frac{1}{R}+\frac{1-2\lambda}{2r_k\left(1-\cos\varphi_1\right)}\right] \qquad (23)$$

Normal stresses define by formula

$$Z=2G\left(\frac{\partial w}{\partial z}+\frac{\lambda}{1-2\lambda}\Delta\right)=2G\left[\frac{1}{2}\left(\frac{x}{r_k\sin\varphi}-1\right)+\lambda\left(\frac{1}{R}+\frac{1}{2r_k}+\frac{1}{2r_b}\right)\right] \qquad (24)$$

Results and its discussion. The appearance of cracks in the depth of the body can also be explained by the fact that there is a boundary layer that separates the core of the body with a practically invariable structure and a surface layer with transverse pressure and longitudinal displacement. If the shear stresses of the surface layers, which depend on the material properties, the equilibrium state and other factors, are sufficiently large, then cracks can occur on the surface of the body that is compressed and propagate inside.

Conclusion. Such approach to the contact problem solution explains many questions. As practice has shown, tangential stresses are increased at a distance from the center of the contact, and they do not have the greatest values beyond the boundaries of the contact zones, and in the center of the contact, that is, for x = 0, y = 0, they are zero. In addition, the largest tangential stresses are at a depth, that is, at the points where the maximum material displacement occurs as a result of a change in the radius of curvature of the body surface which is in contact.

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