

## The productive methods of teaching mathematics on the basis of competence approach

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**Abstract.** The article deals with the theoretical description of productive teaching methods, providing the organization of learning and cognitive activity of students on the basis of competence approach and methodical concept for use problematic, partial search, research methods in teaching mathematics. We define the basic units of each of the above methods. Examples of formulation of problem questions, problem situation and organizing teaching and research tasks on the basis of mathematical material are also given in this article.

**Keywords:** teaching method; problem method; partial search method; research method; problem questions; problem situation; teaching and research tasks.

**Introduction.** Taking into consideration today's realities the understanding of the process of acquisition of knowledge, which is considered as a component of competence, is changing nowadays. At the present stage society put forward new requirements for knowledge, in particular to ensure its functionality in further use. It is important not memorizing specific information, but awareness of the general laws, the ability to adapt new knowledge into the system previously learned, the ability to form a system of knowledge at free practical application.

The implementation of these tasks requires a significant expansion of methodological apparatus, diversity of types and changing nature of organization of students' learning and cognitive activity that should become mobile, creative, independent. The successful learning at school and knowledge projection in a practical, activity plane are caused by a complex system of psychological factors, such as: characteristics of mental activity; attitude and motivation to learn; critical self-assessment of students' success; the level of training and learning; taking into account age and individual development; conscious choice of teaching methods.

**Overview of publications on the topic.** The works of many scientists (Yu. Babanskyi, H. Bevz, M. Burda, H. Vashchenko, M. Danylov, B. Yesypov, K. Lebedyntsev, I. Lerner, M. Makhmutov, V. Onischuk, M. Skatkin, Z. Sliepkan, N. Tarasenkova and others) are dedicated to the classification of teaching methods, where method as a

component of the study is investigated from different perspectives: historical, psychological, epistemological, logical, methodical.

In pedagogy, particular in didactics, teaching method is seen as a way to achieve the learning objectives, education and development of students in the process of common activity with a teacher; method of teaching activity and the organization of learning and cognitive activity of students in solving various didactic problems aimed at mastering the material being studied; interdependent set of thinking actions; ways of solving learning tasks, all of which are specific learning objectives, etc.

The complexity of this phenomenon, hence the possibility of different approaches to the disclosure of its essence, explains the simultaneous existence of different concepts and classifications of teaching methods (table 1).

In our study, we adhere to the classification developed by I. Lerner and M. Skatkin, noting that the first two methods are aimed at organizing students' reproductive activity and the last three are aimed at productive, creative activity.

**The aim of the article** is a theoretical description of productive teaching methods that provide the organization of students' learning and cognitive activity on the basis of competence approach and methodological concept for use problem, partial search, research methods in teaching mathematics.

Table 1

Classification features	Teaching methods classification
According to M. Danylov and B. Yesypov [2]	
The main teaching tasks that are needed to be addressed at a particular stage of learning	Acquiring knowledge, methods of formation of knowledge, skills and abilities, methods of application of received knowledge, skills and abilities
According to Yu. Babanskyi [1]	
Organization and implementation of learning and cognitive activity	Verbal explanation, story, lecture, discussion, work with a textbook; visual illustration, demonstration, independent observation; exercises, laboratory works, practical, graphic and research works
Encouraging learning activity	The formation of cognitive interests educational discussion, providing the success of learning, cognitive games, creating a situation of interest during the process of teaching, creating a situation novelty of educational material, relying on students' experience; stimulation of duty and responsibility in learning.
Methods of control and self-control	Formation of skills of oral, written and practical laboratory and practical control and self-control; individual tests; frontal tests; oral and written tests and exams; programmable tests, machine control.
According to I. Lerner and M. Skatkin [3]	
The content of education and ways of learning, ie the type (character) learning and cognitive activity.	Explanatory and illustrative (informational and receptive); reproductive, problem presentation; partial search (heuristic); research methods.

**Materials and methods.** The aim of competence approach is to develop in students the ability and willingness to act effectively in the field of educational material and outside educational subjects and situations, to solve life, social problem tasks effectively. The result of this approach not only provides a combination of knowledge, skills and abilities, but also is focused on solving real, practical, everyday problems based on mathematical modeling and integration.

Using the productive teaching methods, based on the determining factor that is the nature of student's learning and cognitive activity, is aimed at changing the nature of students' activity from performing, actively performing, actively independent activity to creatively independent activity and as a result a progressive personal development: changing student's positions from performing → to active → to the position of the subject.

Problem method of teaching mathematics, the essence of which is to ensure the effective ratio of students to form the subject mathematics and key competencies, intensive

development of self-learning and cognitive activity, individual creativity in the implementation of educational objectives and tasks, is based on the implementation of interactive learning. The interactive learning is seen as a form of learning and cognitive activity of the student, which is aimed directly at his/her participation in verbal filling of local educational and information space lesson with information, which serves as the topic of the lesson [6, p. 178].

We determine problem questions as the basic units of the problem method implementation.

The problem question is the question when the student has not got any ready answer beforehand, he/she finds the answer for them self. Unlike usual question, problem question does not provide a simple guessing or reproduction of information, the answer to this question requires the student to perform simple productive mental operations, and hence it is problematic. In practice the use of problem questions is effective after learning new material (mathematical concepts), for example (table 2).

Table 2

The formulation of the concept, grade	Problem question [4]
Divide numbers on multipliers where all multipliers are prime numbers is called the divide number into prime multipliers (Grade 6)	Is there a divide for the prime multipliers equality $5=1 \cdot 5$ ?
Only those numbers are divided by 9, whose sum of digits is divided by 9 (Grade 6)	Can we consider that the numbers, that are divided by 9, are also divide by 3?

The result of psychological research of conditions of raising and asking problem questions at different age stages (pre-primary age – 5-6 and 6-7 years old; primary school age – 8-10 years old; adolescence – 11-15 years old) was the discovery of two sensitive periods in the development of questions [8]. In the first period (5 to 7 years) there is a sharp jump in the level of search activity in the form of questions. During these years, the child has a free form of questions directed to research. During this period child's questions are projected from questions directed by adults to the problem questions aimed at independent, searching disclosure of the unknown.

At the beginning of adolescence (11 years old) growth of search activity in the form of questions is stopped. Adolescence (11-15 years old) is second psychologically favorable period in the development of questions. A teenager moves from the general problem situation to a deep examination of the problem – selective examination. Having found out the problem (unknown), a student pauses upon comprehensive examination, trying to find answers to questions by himself. This period of child's development should be used to enhance the effect influence on the level of cognitive activity and creativity of the student.

From the above, we can conclude that the problem method is an effective method of teaching mathematics, when students are in 5-6 grades, and define it as a basis for the introduction of partial-search method of teaching in grades 6-8.

Partial-search method of teaching is under the organization of students' participation in the implementation of some steps of search of problem solution. The role of the teacher is to create learning and cognitive tasks, divide it into steps, identify those steps that students will perform on their own, that is the teacher organizes partially inde-

pendent learning and cognitive activity of students based on interactive learning.

Interactive learning is seen as a form of learning and cognitive activity of the student, the implementation of which provides simultaneous execution of the same educational and cognitive tasks, characterized by problematic, informative, developing or controlling overtones, focus on the direct or indirect involvement of students in the verbal content of the local educational and information space of a lesson, which serves as the topic of the lesson and is organized in accordance with one of the three forms of interactions: co-individual; co-coherent; co-interacting [6, p. 185].

The main component of interactive learning is communication. It is considered as a multi-faceted process of establishment and development of contacts between subjects, that involves the exchange of information, certain tactics and strategies of interaction, perception and understanding of each other during communication. Cognitive communication predominates while learning, which we consider as communication that implements all the functions of knowledge on a particular subject material.

Isolating the basic unit of introducing partial-search method – problem situation which is described as an intellectual difficulty that arises when a student does not know how to explain certain facts or phenomena and cannot achieve the desired goal by already known ways, we consider some examples (table 3).

The process of management of students' solving a problem situation has four components.

1. Preparing to solve a problem situation (formulation of hypothesis (assumption); setting learning and cognitive task; an indication of the sequence of activity; distribution of differentiated didactic material).

Table 3

Topic, grade	Problem situation	Methodical comments
The sum of angles of a triangle, Grade 7	Construct a triangle whose degree measure of the interior angles respectively equal to: $50^\circ, 70^\circ, 80^\circ$ .	The sum of measures of the interior angles of a triangle is equal to $180^\circ$ . Therefore, the triangle with given data cannot be built.
Inequality of triangle, Grade 7	Construct a triangle whose sides length are equal to 5 cm, 7 cm and 8 cm.	The triangle can be constructed, if the length of side of the triangle is not bigger than the sum of the other two sides of the triangle.

2. Organization of work (reading the content of the materials and planning; dividing the task into some parts; individual performance of the task; discussing the results; additions, clarifying, summarizing the task; summing up the results).

3. Independent work of students at the problem situation (communication in a system teacher-student, student-teacher, student-student, student-computer, etc; work on the instructions, solving using the algorithm, etc.).

4. Finishing the work (general conclusion about the achievement of the goals and refutation or confirmation of the hypothesis (assumption)).

In the process, the teacher performs various functions: he/she monitors progress, answers questions, provides individual assistance to students, creates a favorable psychological background, so we organize communication the structure of which is classified by L. Friedman [5]. Communication consists of three interrelated components: 1) communication (information exchange between people in the process of communication); 2) interactive (the interaction between individuals); 3) perceptive (the process of

mutual perception of partners in communication and the establishment of the basis of emotional relationship to each other) [5, p.46]. While talking, discussing and solving problem situation, we formulate questions that are aimed at creating common methods of analysis and solving the whole class of problems. For example, the following questions can be asked: What are the processes described in the problem? What quantities is each process characterized by? What is known about each quantity? Can you find a relationship between the quantities?

Such questions will organize the work of students in the main phase of solving – analysis of the situation. Questions that form a system of benchmarks, can be used in the study of the problem situation. This changes the students’ approach the study of theoretical material. The theory is seen not only as an object that should be memorized, but also as a basis for practical application. Questions help to understand the essence of the problem situation, to establish a new relationship with the material previously studied.

For example, based on life and learning experiences set formula of square shapes shown below (table 4).

Table 4

Educational tasks	Methodical comments
	At primary school students are taught to identify shapes using square cell. To calculate the area of data pieces, you need to count the number of cells and multiply by $0.25 \text{ cm}^2$ . The total formula of finding area: $S = 0,25n$ , where $n$ – number of cells that figure are limited by.
	At primary school students learned the formula of the area of a rectangle and a square. Specifically, $S=ab$ and $S=aa=a^2$ . Thus, to calculate these figures it is enough to find the lengths of sides of the rectangle and square.
	To calculate the area of a parallelogram can be possible if we use the recommendations of the first task. But to apply this method to determine the area of the second parallelogram is impossible. We recommend to carry out series of reforms: lower BK height; "cut" AVK triangle and move it, so that the CD side coincides with AB side, AC side with the continuation of AD side; as a result of changes rectangle is formed the area of which we can find. $S=ah$ .
	To calculate the area of a trapezoid we use completion elements. We extend smaller trapezoid basis of the length of each corresponding larger base. We obtain a parallelogram, which consists of two equal trapezoids. $S=\frac{1}{2}(a+b)h$ .

The research method of teaching mathematics lies in the introduction of general and partial methods of scientific research in the learning process at all stages (from perception to use in practice); organization of activities at the

lesson and after lessons; updating interdisciplinary connections; complication of content and improvement of procedural and substantive aspects of learning and cognitive activity. There is a change of character of relations

“teacher – student – group of students” in the direction of cooperation. A method of learning: interactive learning inter-change (mutual learning) and internal learning (I teach myself).

The internal learning is considered as an individual form of learning and cognitive activity, which includes independent accomplishment of the task, part of the task, exercises, etc. [6, p. 158].

Learning and cognitive activity of the student seen in his/her: 1) attitude towards the content and the learning process; 2) ambition of effective mastery of knowledge by means of optimum time; 3) mobilizing moral and educational efforts to achieve educational goals.

In academic writings, there are three levels of cognitive activity of students: first level – reproducing activity, characterized by student’s desire to understand, memorize and reproduce knowledge, to master the ways of their application on the model; second level – interpreting activity, characterized by student’s desire to know connections between phenomena and processes, master the ways of its application in modified situations, conditions; third level –

creative activity, characterized by interest and desire not only to penetrate deeply into the nature of the phenomena studied, but also for find a new way for this [7].

So, using the research method in training, a teacher increases the level of students’ activity and provides their learning and cognitive activity in object-sensual, visual-imagery, language-sign base in close connection with empirical and theoretical activities.

The basic unit of implementation of the research method is the teaching and research task. These tasks are closely linked to content (theoretical) generalization, they lead the student to the formation of skills and abilities to summarize and organize training material, to learn new ways of action and formation of learning experience.

When performing teaching and research tasks students have to achieve its creative potential – knowledge, skills, abilities, feel a situation of success, believe in their ability to solve given tasks.

Example of teaching and research objectives using matrix method (table 5).

Table 5

Name of teaching and research tasks	Measurements on the ground.
Task description	Determine the width of the river without using measuring devices.
Subject of study	The circle and the radius of the circle.
The aim of the study	Define the methods by which we can determine the width of the river and experimentally verified.
The hypothesis (assumption) Research	I think that we can measure the distance _____
Equipment and materials	Cap, ruler.
Course of study	
1. Measure the length of the step. <i>Note.</i> In order to measure the length of the step we need to make some mark on the floor. Make a few steps. Measure the distance. The number divided by the number of steps. The second method. The length of the step is equal to growth divided into four and to the received number add 37 cm. The third way. The distance from the eye to the feet is divided by two. 2. Approach the river bank. 3. Put the cap on or put the hand to the forehead, imitating a visor. 4. Tilt your head so that the line coincides with the peak of the second line the bank. 5. Without changing the angle, turn right (left) and identify the object, which coincides with the line visor. 6. Approach the defined subject and count the number of steps. 7. Number of steps is multiplied by the length of the step.	
The overall conclusion	
I believe that we can measure the distance _____. So the hypothesis (assumption) _____. <div style="text-align: right;">confirmed or refuted</div>	

Work on the implementation of teaching and research tasks requires teacher’s coordinating actions. In the first stage of the implementation of tasks teacher: 1) performs overall planning of students’ activity and behaviour; 2) encourages them to perform specific action (highlights the purpose and object of research, stimulates the interest); 3) ascertains the degree of understanding of the problem; 4) explains the work requirements, methods, tools and principles to achieve certain results; 5) predicts performance. Student’s actions and behaviour are diverse: 1) understanding of the task and the desire to fulfill it; 2) questions to the teacher about the content of the task; 3) request to repeat explanations of ways, means and principles of achieving the goal; 4) interpretation of the meaning and content of the task; 5) assessment of complexity, the ability to complete the task; 6) refusal to perform tasks. During the second phase of the assignment, the relationship is changed. Teacher’s activity is based on: 1) clarifying or further explanation; 2) providing practical

help and support efforts (actions) of the student; 3) stimulating cognitive activity, independence (questions, comments, advice); 4) evaluation of the quality of work; 5) an explanation of errors. The following items are typical for student’s activity: questions, treatment, self-esteem of actions and words, clarifying, debate with the teacher.

**Conclusions.** Thus, the impact of training depends on the situation of communication during common activities, in the interaction when the task requires concerted actions from the student. The structure of mutual relations between a student and a teacher depends on: 1) choice of teaching methods; 2) the content of educational task; 3) stage of its implementation; 4) efficiency and quality of the work results; 5) evaluation, self-assessment of the task.

It is important for children to feel teacher’s interest in their common teaching and cognitive activity at the lesson, that allows students to develop their creative thinking, ways of learning and creates a personal active position.

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#### Продуктивные методы обучения математике на основе компетентностного подхода

Л.С. Голодюк

**Аннотация.** В статье раскрывается теоретическое описание продуктивных методов обучения, обеспечивающих организацию учебно-познавательной деятельности учащихся на основе компетентностного подхода и методический концепт по использованию проблемного, частично-поискового, исследовательского методов в процессе обучения математике. Установлены базовые единицы каждого указанного выше метода. Приведенные примеры формулирования проблемного вопроса, проблемной ситуации и составления учебно-исследовательской задачи на основе математического материала.

**Ключевые слова:** метод обучения; проблемный метод, частично-поисковый метод, исследовательский метод, проблемный вопрос; проблемная ситуация; учебно-исследовательская задача.