The notion of "digit": semiotic aspect

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Abstract. In the article, based on analysis of works on philosophy, linguistics, psychology, pedagogy and methodology of mathematics the place of digits in the classification is studied from the point of view of semiotics – the science of signs. The author establishes common and different features of the concepts of "sign" and "symbol" as interpreted by different sciences. The analysis of the scientific and methodological sources justified the need for the use of the term "sign-symbolic means" and enabled its definition. It is noted that in the process of solving a series of mathematical problems using digits that refer to sign-symbolic means one performs functions of substitution, encoding, decoding, schematization and modeling. It is concluded that the ability to use digits as a sign-symbolic means for solving mathematical problems, including calculations, is a measure of a child's intellectual development.

Keywords: *sign, symbol, digit, number, semiotics, sign-symbolic means.*

Introduction. Learning math is not possible without the use and operation of the system of signs and symbols, including the ten Arabic numbers. In studying Mathematics in elementary school children operate and use them in practical computing activities, and the first visual introduction took place back in preschool.

Literature review. The fact that the study of *signs* and *symbols* is reflected in the works of linguists (I. Arzamastseva, Yu. Stepanov), culturologists (E. Kassirer), philosophers (R. Descartes, M. Zhukov), psychologists (L. Vygotsky, L. Zankov, L. Zorin, H. Kostiuk, Zh. Piaget, N. Salmina), psychiatrists (J. Lacan), pedagogues and methodologists (D. Bohoyavlenskii, I. Bohatyrov, N. Istomina, N. Menchynska, N. Tarasenkova) shows that signs and symbols are multi-vector.

Purpose. As the use of signs and symbols is not only an integral part of primary school arithmetic, but also mathematical and scientific activities in general, there is a need for analysis of the theoretical principles of the use of signs in different areas of human activity with the aim of understanding the process of their perception.

Results and discussion. In this context it is necessary to determine the nature of these concepts, which are interpreted differently by representatives of various scientific fields.

Thus the term "*symbol*" is defined as:

− the conditional nomination of any magnitude, the concept introduced to some science (*in mathematics*);

− sign means something secret, hidden, often supernatural or divine that acquires the status of symbol only when a connection between the sign and its value is established (*in the history of culture*);

− one of the central concepts that is essential for the construction of any theory; the most controversial concept for understanding (*in philosophy*);

− the minimum amount of data that can be transmitted in digital form (*in the general sense*);

− mark or entity that designates another entity; digits are symbols that indicate the number (*in literary criticism*).

The above-mentioned shows that the terms "symbol" and "sign" are sometimes used interchangeably, especially when it comes to the formula as special signs used in "scientific languages" – mathematical, physical, chemical, logic, etc. The sign has such an important function as replacement, and a set of *signs forms a sign system*.

By definition of J. Malafiyik the sign is universal and the symbol is a specific manifestation of the sign [9, p. 104].

In the cultural study "Essay on Man", E. Kassirer concluded that without symbols a person's life is likened to a prisoner's life of Plato's cave limited by his biological needs and practical interests. Then the individual will not be able to access the "ideal world", which opens from different sides – religion, art, philosophy and science [7, p. 487].

In her work on psychology, N. Salmina notes the difference in these concepts. Thus, the sign indicates the content and the symbol reveals it. However, the author notes that these functional differences of signs and symbols are not essential in learning and they can be viewed collectively as sign-symbolic means that combine a set of signs and symbols [10, p. 4].

Given the demand for the use of signs in different sciences (mathematics, logic, linguistics, philosophy, psychology, biology, anthropology, aesthetics and sociology), they have become the central object of study in science that studies the specific signs and their systems – semiotics. Its founder Charles Peirce believed that all communication means are signs involved in the formation and development of human consciousness.

Ch. Peirce granted a special status to semiotics because such sciences as logic, mathematics and others are sign systems. Therefore, semiotics, as the science of signs, should become the meta science, which allows to describe general principles and methods of scientific knowledge.

In the context of studying the sign theory semiotics is associated with such sciences as logic, philosophy, psychology and others. This is because signs determine their learning and thinking in terms of understanding the mechanisms of perception and transmitting information, patterns and characteristics reflecting reality, design features of thinking and intellectual activity of the individual.

By definition of Ch. Morris "human civilization is impossible without signs and sign systems; the human mind is inseparable from the functioning of signs" [11, p. 37].

We can infer from the concept of the sign developed by Ferdinand de Saussure and Charles Pierce that the ability to operate signs relates to properties of the organism to interact with nature, which in turn is a sign of intelligence.

Charles Peirce's assertion that we "think only by signs" that take the form of words, images, imagination,

actions or objects gains importance. Each of these forms has no inner meaning and they become signs only when we put value in them.

This means that the use of symbols allows a person to operate in his/her mind with the "substitutes" for objects of the external world, to create iconic model of reality, to identify characteristics and relationships between these objects.

Defining the role of signs in the human mind we should indicate the place of digits in semiotics. Thus, according to the classification proposed by A. Korshunov and V. Mamontov all the signs are divided into linguistic and non-linguistic (indexes, iconic signs, symbols). If we consider the language consisting of mathematical terms, numbers, letters, formulas and various expressions to be called mathematical, then digits relate to the language of signs. The feature of such signs is that they come in the system and function as signs only in it, according to the existing rules in it – both explicit and implicit. Language signs can be defined as material objects, intended for use as signs.

By the type of language, they distinguish signs of natural and artificial languages. In this division digits relate to artificial languages.

By the classification of the founder of semiotics Charles Peirce, due to the way of connection with the signified object mathematical symbols (digits and signs of actions) are sign-symbols.

By type of sign systems ten Arabic digits (0 to 9) form a simple semiotics. By the ability to be a part (subsystem) in a complex system of mathematical symbols that includes a number of subsystems they relate to a multi-level semiotics. In such systems there is a hierarchy of signs, i.e. signs of a higher level can be combined to form new signs (record of many digital numbers, numeric expressions, etc.). Overall Arabic digits belong to simple multilevel semiotics [2, p. 29-30].

In mathematical language the use of signs in sign systems is based on identification of abstraction, which lies in the perception of essential properties of objects and ignoring insignificant their properties. On its basis the same-type signs that were created under different conditions, in different times, by different means and that have different material nature become identified. In particular, the digit represented as different fonts printed is considered the same digit, and various options for its record are considered as different examples of copy of the same sign.

An interesting feature is the weight of the sign by which A. Solomonyk understands predetermined place of the sign, its function and value relative to other signs. It is important to study the digit in the semiotic perception, as the same digits recorded in various categories of the natural series of numbers, significantly change their weight in relation to other digits [12; 13].

Returning to learning math using semantic-symbolic means N. Tarasenkova indicates their developmental role that is worth mentioning only in a situation where they are meaningful for students forms [14, p. 17]. The researcher proposes to cover and analyze the process and learning outcomes in terms of specificity – inherent activity with the semantic-symbolic means, which serve students and teacher during recording and converting the content of educational material in mathematics, and how a semiotic development of students correlates with the improvement of their mathematical education and personality formation [14, p. 5]. By I. Bohatyrov's definition the semiotic development of students is the process that enables them to deliberate sign-symbolic activity (substitution, encoding, decoding, schematization and modeling) while solving math problems. We find important the scientist's belief that learning should be built to ensure conditions for the pupils to master and use freely the sign-symbolic means in the learning process. Because it helps to neutralize a large number of conflicts between logical and visual, as well as to compare the process of students' semiotic development with the semiotic function of their psychic under which I. Bohatyrov understands the ability to own and operate freely the sign-symbolic shell of mathematical content [3, p. 8**]**.

The importance of the ability to use sign-symbolic means in everyday life has been emphasized by many experts. Thus, N. Salmina believes that by the time your child goes to school he/she should have generated such kind of sign-symbolic activity as substitution (use of alternates that perform the same function as the substituted subject and through which a sequence of visual learning is built and conditions for proper perception and understanding of new knowledge content by students are created), encoding (the translation of reality, or text describing reality, into sign-symbolic language, which is followed by content decoding) – decoding (the ability to recognize content), schematization and modeling [10; 14, p. 21-22].

In the field of sign-symbolic means, I. Malafiyik formulated the following principles of coding and decoding:

− conciseness (for encoding information they use only those means that are necessary to inform its essence);

− synthesis and unification (the used means should not be split);

− focus on key semantic elements (the most significant items are noticeable by shape, size, colour or disproportion);

− autonomy (parts that transmit independent messages should be accentuated and separated one from the other);

− structure (each autonomous unit of the complex should have a clear differentiated structure);

− stages (there must be the spatial distribution of all information for its consistent perception and appropriate placement) [9, p.108].

According to N. Istomina, the awareness of the difference between the number and the digit in the study of single-digit numbers is quite difficult for a child. In some cases, even teachers have difficulty associated with the use of these terms.

Finding the match between the numerical figure (subject model), word-numeral (verbal model) and the sign-digit (symbolic model) helps children to understand that the digit is a sign that identifies the number of different items [6, p. 63-64].

According to the research of H. Kostiuk, children are becoming aware of number at the end of the third year of life and in the process of communicating with adults. The result is that the activities carried out through verbal communication with adults [7, p. 300-301]. However,

they equate the number of the corresponding figure a little later.

Since the perception of a digit as a semiotic category is associated with the concept of number, we should analize the development history of the latter. Thus, M. Zhukov states that people at first did not separate the number of objects from them and used for this "named numbers" – hands and fingers. Only at the higher stage of their development people began to separate the concept of number from objects, using natural numbers. With the further development of society they learned to speak them with articulate sounds, and with the advent of writing, to record them with appropriate signs – digits. We find interesting the conclusion of F. Engels that in order to figure out one not only needs to have items to count, but also the ability to be distracted when considering these items from all other their properties but number. This ability is the result of a long historical development, based on experience. Thus, in the complex process of natural number establishment the primary importance was attached to the identification of abstraction that is not confined to mathematics [5, p. 11-12].

Using sign-symbolic means in mathematics has significant advantages compared to the verbal form of representation. In particular, the representation of the number of items as an appropriate digit is percieved better (the research of D. Bohoyavlenskii and N. Menchinskii shows that first-graders overlook the word-numeral in the problem instruction if it is represented not as a digit to which they are accustomed) and multi-digit numbers are perceived much faster. Learners immediately view them holistically, which simplifies and accelerates the overall perception of the text [4, p. 169].

These processes are not only legitimate, the evidence of a child's development, but are important in further study of mathematics.

According to Zh. Adamar, "in the process of mental activity the more difficult and complex a question is, the less you can trust the words, the more clearly you must understand that you should control this dangerous ally" [1, p. 76].

The result of any research can be expressed only through signs, because people express their thoughts using only signs that are specifically created for this. They have no other ways of expression [12; 13].

Interesting is the comparison of the two systems of signs – digits and the alphabet – because of their genesis they are not separated from each other.

According to S. Stepanov, alphabets and arithmetic have the same bases, as evidenced by the alphabetical reflection of counting and numbers in various writings [11].

There are well-known facts relating to feedback, when in some cultures (e.g. in Slavic under the Greek influence) they originally used letters, not digits, for numbering and marking amounts.

In ancient Greece and Rome, they used Roman digits to indicate quantity. However, those digits were bulky and not abstract, but their significant disadvantage was that they impeded the process of computing [12, p. 66].

A significant advantage of using Arabic digits in arithmetic, where each number except verbal designation is an appropriate representation of the digits, is that they are involved in the computation algorithms. For example, when doing the task type presented in verbal form "twenty-five multiplied by seventeen" you should put it (at least mentally) as follows: 25×17, using written multiplication algorithm. That is, any problem can be presented verbally, but it can be solved only by using required symbols [, p. 90].

Conclusion. Using digits as one of the types of the sign-symbolic expression of content is a necessary component of theoretical thinking, which helps to distinguish forms from content. It is an important tool for proper learning and the development of theoretical and abstract thinking, and as a result – the intellectual development of children in general.

The ability to use digits lies the paramount ability of the human psychics to the development and training of the student, which appears in the ability to operate them.

From the perspective of the semiotic approach to mathematics digits, from the standpoint of abstraction, are involved in the identification of abstract thought operations and influence on the development of computing skills.

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Понятие "цифра": семиотический аспект

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Аннотация. В статье на основе анализа трудов по философии, лингвистики, психологии, педагогики и методики математики определяется место цифр в классификации семиотики – науке о знаках. Устанавливается общее и отличное в трактовке понятий "знак" и "символ" различными науками. В результате анализа научно-методических источников обосновывается необходимость использования термина "знаково-символические средства" и подается его определения. Отмечается, что в процессе решения ряда математических задач с использованием цифр, относящихся к знаково-символическим средствам выполняются функции замещения, кодирования-декодирования, схематизации и моделирования. Делается вывод, что способность использовать цифры как знаково-символические средства при решении математических задач, в том числе и вычислений, является показателем интеллектуального развития ребенка.

***Ключевые слова**: знак, символ, цифра, число, семиотика, знаково-символические средства.*