Calculation of integrals by Monte Carlo in the illumination problem of synthesized objects

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Abstract. The problem of calculation of multidimensional integrals by the Monte Carlo method, which is the most often used in the calculation of global illumination synthesized objects created by different software system knownas thesingletask of rendering.The equation global illumination transformed to a form suitable for the application of the Monte Carlo method. We are proposed method and algorithm for calculation ofsuch integrals.The problems of constructing a solution of the algorithm on a personal computer are considered. Modeling results are proposed.

Keywords: *Monte Carlo method, illumination problem, synthesized objects, equation of illumination.*

Introduction. Automatic generation of images on the screen is the most common problem arising in connection with the development of computer vector graphics and in image recognition tasks. Create photorealistic images requires the construction of a large number of objects that have some skeleton structure, on which stretches the texture, and the realism is achieved illumination created pictures. Illumination estimated by image brightness in the direction of the observer-generated visibility at the illuminated surface. Illumination model is a complex integral equation, which does not solving a quadrature. The basic technique of its decision is an approximation. In computer graphics, this equation is known as rendering equation[1].

A typical solution of the rendering equation is considered a series expansion. The main difficulty expansions of this type is to provide the convergence of the given series.This achieved by the choice ofradiusspectraldecomposition of the operator, which for convergence must be less than one. The series consist of large number of components that are determined by the number of re-emission of the light source to the observation point. One of the most attractive methods for the solution of this equation is the use of the Monte Carlo method, the foundations of which were proposed by J. von Neumann in problem of passes the neutrons through various material [2].

The purpose of this paper is to develop a method and algorithm for solving the problem of the object illumination synthesized by computer means to produce an image is sufficiently close to the reality.

Publications review. Theoretical foundations of the Monte Carlo method is available in [1-4]. In [5] proposedthe calculation of global illumination of the scene represented by a Fredholm integral equation of the second kind. After a sets of transformations of this equation reduces to the form of Riemann, which is solved by the classical Monte Carlo method.

The authors of [6] are proposes the construction of a local assessment of illumination at a given point of the scene that is modeled using a diffuse reflection function defined by the observer points and observation points on the assumption that Markov's ray walking on the stage. Dual local assessment as opposed to the previous approach estimation takes into account the direction of arrival of the secondary rays, which allows them to appreciate the value of brightness at any point of the three-dimensional scene.

In both cases, a positive result obtained by illumination evaluation that represents the convolution of the weighting coefficients of a Markov chain the core function integral equations with the all-possible rays. However, in accordance with the strong proof about of the number of tests given in [7], a conclusion high precision of the applied approach is premature.

Decreasing of computational costs when addressing the problem of illumination of the scene in [8] is achieved using the method of quasi – Monte Carlo and the properties of the integrand.

Software development simulates of light passing through the multilayer materials Monte Carlo method in the C standard proposed in [9].

Problem formulation. The model of the light propagation that determined in accordance with [1, 10] integral equation of the form

$$L(x,\omega,\lambda,t) = L_0(x,\omega,\lambda,t) + \int_\Omega f_r(x,\omega,\omega',\lambda,t) L_i(x,\omega',\lambda,t)(-\omega',n)d\omega', \quad (1)$$

wherein $L(\cdot)$ ist he amount of light that are acts on the observer's point of ξ; ω ist he direction from which the viewer is seeingthe point$x$in the wavelength range of visible lightλat time t; $L_0(\cdot)$ ist he amount of light that emitted by the source; $L_i(\cdot)$ are the amount of light that comes from the directionωin time t; $f_r(\cdot)$ is a bi-directional light distribution function reflections with areas ω' in ω; (−ω'· n) ist he absorption coefficient of the Ω surface; Ωis the illuminated region, its view is not defined.

As a light source is an anisotropic source with uniform lighting of surface object and the illuminated object is a uniform. These conditions make it possible to introduce the following assumptions

$$L_0(\cdot) = const,\ f_r(\cdot) = const,\ (-\omega', n) = const. \quad (2)$$

By (2), the equation (1) transformed to the form

$$L'(x,\omega,\lambda,t) = \int_\Omega L'_i(x,\omega',\lambda,t)d\omega', \quad (3)$$

where $L'_i(\cdot)$ differs from $L_i(\cdot)$ scale factors, and the value of $L'(\cdot)$ and $L_0(\cdot)$ shift defined from (1) and (2). Such transformations to simplify the equation (1) without changing the entity of the model.

In this work formulated and solved the problem of developing a method and the algorithm for calculating the integral (3).

The main solution. Rendering equation (3) is a Fredholm integral equation of the 2nd kind, which has no solution by conventional methods (analytical and quadrature). To solving it is used a gaming approach based on the Monte Carlo method as for the cast dice, the roulette game or firing "hit or miss", which are gives an approximate solution. Immediately it should be noted about the need for a uniform random variable within the interval integration. The principle of integration is to replace real integrand random number associated with the original function of the interval of integration, and counting the number of hits in the interval of integration called acting out of a random variable. This leads to the fact that instead of calculating the integral

$$I = \int_{\Omega} f(x)dx, \tag{4}$$

which is quite difficult computationally, therefore we calculates its assessment by statistical methods [4].

We have suppose a random values ξ determinates uniform density distribution $p_\xi(x)$ so as

$$\int_{\Omega} p_\xi(x)dx = 1. \tag{5}$$

We can find so value of η that related with ξ this relation

$$\eta = f(x) / p_\xi(x) \,. \tag{6}$$

Then the expected value of η

$$M(\eta) = \int_{\Omega} \left[\frac{f(x)}{p_\xi(x)}\right] p_\xi(x)dx = \int_{\Omega} f(x)dx \tag{7}$$

it allows you to apply to the calculation of the integral (1) statistical methods

$$I \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i, \tag{8}$$

where $\eta_i = f(\xi_i) / p(\xi_i)$, and N – a large sample of random variables ξ_i, $N\to\infty$ using the law of large numbers (Khintchine's law).

Modeling. As is well known [11, 12], the color of the objects is obtained by mixing a simple (not represented by a combination of the other) colors in certain proportions. In describing the object's color characteristics is convenient to use the concept of the color space, which allows you to describe the color in the color coordinates, i.e. to work with color in the form of a mathematical model.

The most popular in computer graphics is a model RGB for represent the color of objects. Curves observation intensity the XYZ color coordinate depending on the wavelengths recommended by the International Commission on Illumination (CIE) shown in Fig. 1.

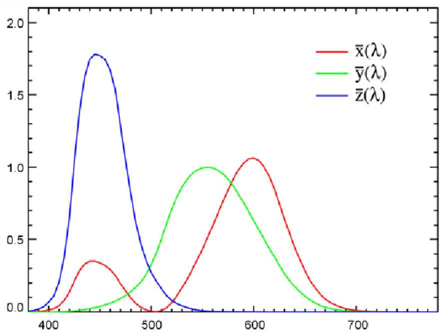

Fig. 1. The color coordinates $\bar{x}$ (λ), $\bar{y}$ (λ), $\bar{z}$ (λ).

Visible lightisintherange of 380-730 nm. The curves of light's reflection from different surfaces presented in Fig. 2, and in Fig. 2, a) shows the reflection surfaces, which values are slightly changed or are remained constant in changing wavelength. These objects called achromatic. It is such surfaces aschalk, white fabric,snow, zinc oxide.The curves of reflection surfaces that have colorpoor saturatedshows in Fig. 2b. As anexamples howndifferent degreesofoak leaf yellowing. The figures howsthatthe color saturation increases as leaf yellowing. In Fig. 2, c) shown a few examples, from which it follows that the color can be very much reflect in one region of the spectrum, and having a significant absorption in the other. The curvesare continuous functions of the wavelength ρ (λ).

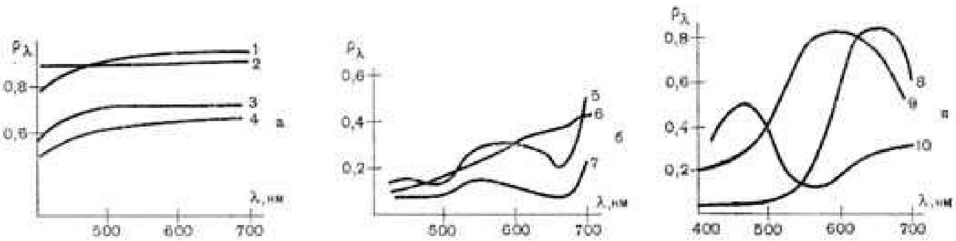

Fig. 2. The curves reflect different surfaces:
a) achromatic: 1 – white matter; 2 – snow; 3 – zinc oxide; 4 – chalk;
b) poor saturated (oak leaf): 5 – yellow; 6 – brown; 7 – green;
c) saturated: 8 – cinnabar; 9 – cadmium yellow; 10 – Cobalt Blue.

To convert these curves to the tri-color values needs to calculate the integrals such as

$$X = \int_{\lambda_1}^{\lambda_2} S_e(\lambda)\bar{x}(\lambda)d\lambda, \quad (9)$$

$$Y = \int_{\lambda_1}^{\lambda_2} S_e(\lambda)\bar{y}(\lambda)d\lambda, \quad (10)$$

$$Z = \int_{\lambda_1}^{\lambda_2} S_e(\lambda)\bar{z}(\lambda)d\lambda. \quad (11)$$

In (9) – (11) $\bar{x}, \bar{y}, \bar{z}$ – standard surveillance function color coordinate, which form is shown in Figure 1, and $S_e(\lambda)$ – the spectral density of intensity. As a pattern image we are use McBeth chart (Fig. 3) that its represents a 24 flat cardboard, on which exemplary colors that are commonly used in computer graphics to determine the color balance or the optical density of any color system.

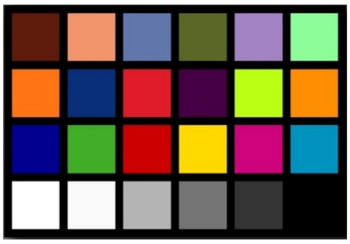

Fig. McBeth chart

In integrating,we used ofspectraldata the McBeth chart [12]. Calculation of integrals (9)-(11) is carried out in accordance with (8) as a

$$I_x = \frac{(\lambda_2-\lambda_1)}{N}\sum_{i=0}^{N-1} S_e(\lambda)\bar{x}(\lambda), \quad (12)$$

where $S_e(\lambda)$ – the spectral distribution of light, $\bar{x}(\lambda)$ – value monitoring functions from CIE. Then the calculation integral (4) by Monte Carlo method in accordance with (12) isleads to a random selection of the wavelength λ from the visible light range, the definition of $S_e(\lambda)$ and values $\bar{x}(\lambda)$, multiplying these values, the repetition of last operation *N* time and the end result is multiplied on $(\lambda_2-\lambda_1)/N$ factor. From *XYZ* coordinate system, we allowedtransition to RGB model by matrix multiplication, because the models are linear and additive

$$\begin{vmatrix} R \\ G \\ B \end{vmatrix} = T \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}, \quad (13)$$

where *T* – the transformation matrix of the form [13]

$$T = \begin{vmatrix} 0{,}41847 & -0{,}15866 & -0{,}082835 \\ -0{,}091169 & 0{,}25243 & 0{,}015708 \\ 0{,}00092090 & -0{,}0025498 & 0{,}17860 \end{vmatrix}.$$

To render McBeth chart each color area its presented as square with 64 × 64 pixels. After determining a random value λ the wavelength of the visible region of the spectrum in the range of 10 *nm*, for it is determined the value of the spectral density from given strictly defined table, pass on the values of curves observations by step of 5 nm. In those cases, when the wavelength skipstable's value,in the calculation we are usedinterpolationbetween the two adjacent to it. Image quality depends on the number of *N*at (12). The final goal of integration to obtain a chart Fig. 3, but using means computer graphics, simulation integrating the Monte Carlo method.

The final values of XYZ normalized and converted to RGB values. Describedprocessing cycle closed for each pixel square. As can be seen from Fig. 3 of the squares in the table 24 (column – 6, and the lines – 4). Next, the data saved up in a separate file on disk. Model values for *RGB* channelsobtained according to (13) are in the range[0, 1]. View of the result image on the monitor screen needs nonlinear transformation for example is gamma correction and convert it into *sRGB* model.

sRGB model it can be obtained by following transformation with each of the *RGB* values

$$C_{sRGB} = \begin{cases} 12{,}92 C_{lin}, \text{если } C_{lin} \le 0{,}0031308, \\ (1+a) C_{lin}^{\frac{1}{2,4}} - a, \text{если } C_{lin} > 0{,}0031308, \end{cases} \quad (14)$$

where a = 0,55. The obtained values in (14) are also in the range [0, 1], thatits transformed to the range [0, 255] by multiplied on a 255, further its rounded to obtain integer values.

Results integration McBeth chart for N = 32, 128, 512,1024 are shown in Fig. 4-7.

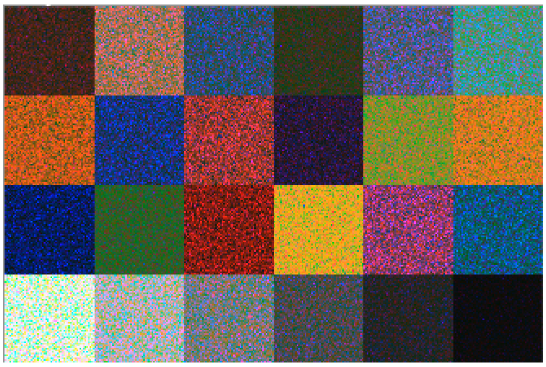

Fig. 4. N=32.

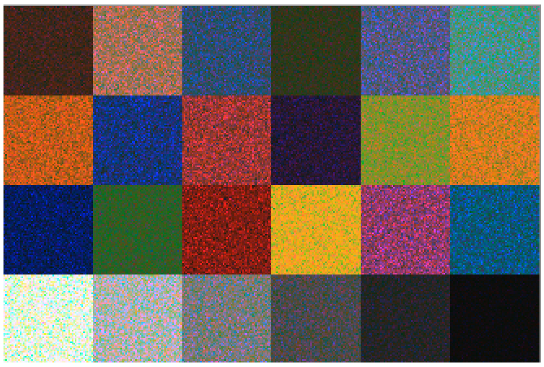

Fig. 5. N=128.

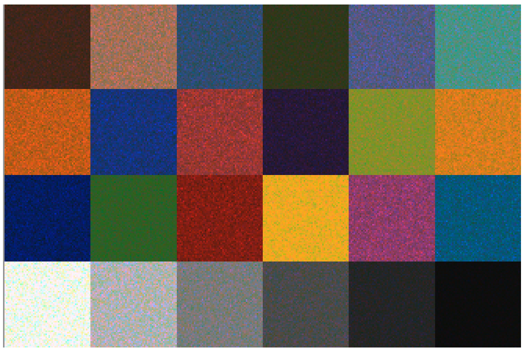

Fig. 6. N=512.

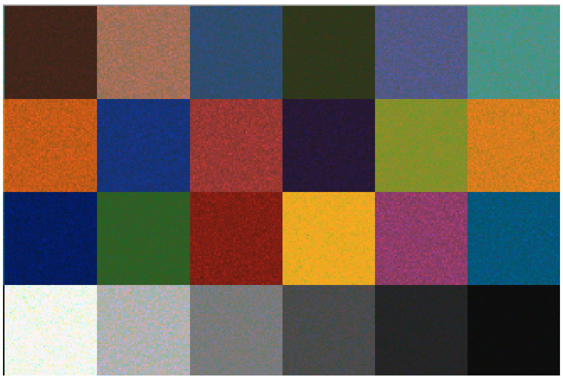

Fig. 7. N=1024.

Analysis of these Figures leads to the conclusion about the level of achieved noise. In order to reduce the noise in the picture in 2 times, the number N needs to increase in 4 times. Thus, it is possible to consider Monte Carlo technique as a possible means for reducing noise gene-rated on the image.

Conclusions. The problem of lighting objects synthesized by software is considered. The decision stochastic of rendering equation by Monte Carlo method is proposed. A whole mechanism, allowing to solve a wide range of computer graphics tasks and lighting technics, such as synthesize of photo-realistic images, performs calculations of global illumination, creation optical effects, complex texture, shading, surface landslides, post-processing effects is given.

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