Applications as a way of implementation of interdisciplinary connections of mathematical and economic disciplines

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Abstract. The article indicates the important role of Mathematical Sciences in the training of a modern economist. It is emphasized the need for applications in the study of Higher Mathematics. It is discussed the peculiarities of interdisciplinary connections between mathematical and basic economic disciplines that encourage to improve fundamental training of bachelors in Economics. A number of practical tasks with the use of Mathematics in Economics are shown.

Keywords: *application tasks, tasks of economic content, interdisciplinary communication*

Mathematical education lays the foundation for successful future activities under production conditions as a subsystem of professional training of students of Economics. A modern economist should be able to use mathematical concepts and methods of investigation of economic processes. But in the present conditions of teaching Mathematics in a sufficiently large amount in a so-called "pure" form it is a rather inefficient process and difficult for students to perceive. Therefore, the style of teaching Higher Mathematics for economists should be directed into the way that would provide the implementation of the links of Mathematics and Economics, and solving actual applied problems in Economics.

The questions of applied orientation in teaching Mathematics is the subject of research by many Ukrainian scientists: G.P. Bevz, M.Y. Ignatenko, T.V. Krylova, L.L Panchenko, Z.I. Sliepkan, L.O. Sokolenko, V.O. Shvets and others.

Applied orientation of Higher Mathematics teaching is realized most effectively when solving applied problems. Under applied problems one mostly means the problems arising out of the sphere of Mathematics but which are solved by mathematical methods. The future specialists must learn to build mathematical models, choosing the appropriate mathematical methods and algorithms, and apply them to solving the problems.

It is advisable to follow certain requirements while selecting and calculating applied problems:

– the tasks should be of real practical content which corroborates the practical significance of the acquired mathematical knowledge;
– the tasks should be formulated in an accessible and understandable language, and new terms have to be explained to students;
– numerical data in the applied tasks should be realistic, correspond to those ones existing in practice;
– a personal experience of students and local material should be reflected in the task content if possible; it allows to show effectively the use of mathematical knowledge of students and cause their cognitive interest;
– in applied tasks the situations of industrial production, trade, economy and other sciences should be reflected to illustrate the application of mathematical knowledge in specific professions.

Applied tasks can be used in the classroom of Higher Mathematics at the stage of motivation of educational activity; in the formation of skills and abilities; at the stage of generalization and systematization of knowledge; during updating of basic knowledge.

The standard scheme for solving the vast majority of applied economic problems can be reduced to the following:

1. Problem analysis and interpretation of the data within the mathematical theory (a construction of the mathematical problem model).
2. Search (choice) of an algorithm for solving the problem (a study of the model).
3. Solving the problem by using the algorithm found.
4. The interpretation of the obtained results in terms of this problem.

The main direction in the realization of the interdisciplinary connections is considered to be calculating of the applied problems with the use in economy. Let us examine some of its points according to the working programs on Higher Mathematics.

In the study of "Elements of Linear Algebra" module it is necessary to provide students with the models of the balance sheet analysis and linear sharing.

Task 1.

The following table gives the intersectoral balance of a three-branch model of economy:

Branch of Industry	Consumption Sector			Final Product Y	Gross Output X	New Final Product $\overline{Y}$
	1	2	3			
1	10	5	40	45	100	100
2	30	0	30	40	100	50
3	20	40	0	140	200	80

Find the following economic indicators:

1) direct costs coefficients a_{ij} (direct costs matrix A);
2) full cost coefficients s_{ij} (full cost matrix S);
3) gross output $\overline{X} = (\overline{x_1}, \overline{x_2}, \overline{x_3})$ of the branches, that provides a new final product $\overline{Y} = (\overline{y_1}, \overline{y_2}, \overline{y_3}) = (100,50,80)$

The solution. Direct costs coefficients a_{ij} are determined by the formula: $a_{ij} = \frac{x_{ij}}{x_j} (i, j = \overline{1,n})$, and the matrix of direct costs is as follows:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

In our case $(i, j = \overline{1,3})$ we have:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} \frac{10}{100} & \frac{5}{100} & \frac{40}{200} \\ \frac{30}{100} & \frac{0}{100} & \frac{30}{200} \\ \frac{20}{100} & \frac{40}{100} & \frac{0}{200} \end{pmatrix} = \begin{pmatrix} 0,1 & 0,05 & 0,2 \\ 0,3 & 0 & 0,15 \\ 0,2 & 0,4 & 0 \end{pmatrix}.$$

Matrix A meets one of the performance criteria, which says that matrix A is productive, if $a_{ij} \geq 0 \quad \forall ij = \overline{1,n},$ $\max\limits_{j=1,n} \sum\limits_{i=1}^{n} a_{ij} \leq 1$, then it is number j, which is $\sum\limits_{i=1}^{n} a_{ij} < 1$.

In our case we have: *max*{0,1+0,3+0,2; 0,05+0+0,4; 0,2+0,15+0}=max{0,6; 0,45; 0,35}=0,6<1 and all the elements of matrix A are inseparable. Therefore, for the final product Y you can find the necessary amount of gross output X by the formula: $X = (E - A)^{-1} Y$.

$S = (E - A)^{-1}$ where $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ – is an identity matrix.

$$E - A = \begin{pmatrix} 0,9 & -0,05 & -0,2 \\ -0,3 & 1 & -0,15 \\ -0,2 & -0,4 & 1 \end{pmatrix}$$

Matrix $S = (E - A)^{-1}$ – is the inverse of *E-A* matrix. Let's find the matrix by the formula:

$(E - A)^{-1} = \frac{1}{\det(E - A)} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$, where A_{ij} are algebraic additions to matrix elements

$(E - A) = \begin{pmatrix} 0,9 & -0,05 & -0,2 \\ -0,3 & 1 & -0,15 \\ -0,2 & -0,4 & 1 \end{pmatrix}$, and *det(E-A)* – is a determinant of this matrix. After completing all the necessary calculations, one obtains the inverse matrix: $S = (E - A)^{-1} = \begin{pmatrix} 1,23 & 0,7 & 0,27 \\ 0,43 & 1,12 & 0,255 \\ 0,42 & 0,48 & 1,16 \end{pmatrix}$. Let's remind of the required volume of the gross output in each industry, which is as follows: $X = (E - A)^{-1} Y = \begin{pmatrix} 100 \\ 100 \\ 200 \end{pmatrix}$.

The gross output required for a given final product is obtained from the ratio

$\overline{X} = (E - A)^{-1} \overline{Y} = S\overline{Y} = \begin{pmatrix} 1,23 & 0,17 & 0,27 \\ 0,43 & 1,12 & 0,255 \\ 0,42 & 0,48 & 1,16 \end{pmatrix} \begin{pmatrix} 100 \\ 50 \\ 80 \end{pmatrix} = \begin{pmatrix} 153 \\ 120 \\ 159 \end{pmatrix}$, тобто $\overline{X} = (\overline{x_1}, \overline{x_2}, \overline{x_3}) = (153;120;159)$.

Thus, gross output, providing a new final product in areas 1 and 2 should be increased to 153 and 120 conventional units, respectively, and in area 3 should be decreased to 159 conventional units. While teaching "Elements of Analytical Geometry" module one should consider the examples of analysis of diminution and profitability of production, profitableness of transportation, which is expressed by the lines of the first and second order.

Task 2. Two similar enterprises A and B produce products at the same wholesale selling price *m* for a product per one kilometer of transportation to the enterprise. However, vehicle fleet, which gives services to enterprise A, is equipped with newer and more powerful cars. Therefore, the transportation costs of one product constitute: for company A – 10 currency units, and for company B – 20 monetary units for one kilometer of transportation. The distance between the enterprises is 300 km. How should the market be geographically divided between the two enterprises in order to make consumer costs for shipping and transportation be the same?

The solution. Let's denote by S_1 and S_2 the distances to the market from points A and B respectively. Then consumer expense amounts to: $Y_1 = m + 10S_1$, $Y_2 = m + 20S_2$. Let's find the set of points (*x; y*) for which $Y_1 = Y_2$: $m + 10S_1 = m + 20S_2$ or $S_1 = 2S_2$

Let's find S_1 and S_2:

$S_1 = \sqrt{x^2 + y^2}$,

$S_2 = \sqrt{(300 - x)^2 + y^2}$, $\qquad \sqrt{x^2 + y^2} = 2\sqrt{(300 - x^2) + y^2}$,

or, after the elevation of the left and right sides of the last equality to the square, grouping and selecting the full square at variable *x* i *y*, one has $(x - 400)^2 + y^2 = 200^2$.

These are equation circles. Thus, it is more profitable for a "within a circle" consumer to buy in point B, for an "outside the circle" consumer – in point A, and for "on the circle" one – is equally profitable both in points A and C.

In the study of "Introduction to the Mathematical Analysis" module (learning sequences, function boundaries, functional dependencies between two or more variables, etc.) it is necessary to distinguish the important features that are used in Economics (production, functions of supply and demand, cost, utility, production).

Task 3. Peter Kolobkov grows cucumbers in his own greenhouse. Then he sells the whole of the harvest in the city market. It is known that the market price of cucumbers has been established at the level of p = 5 grn. per 1 kg. At the same time there are certain costs associated with the purchase of fertilizers, material for a greenhouse, etc. As a result, the overall cost of the grown cucumbers (C) from the quantity (in kg) of the grown cucumbers (x) is up this: $C(x) = \frac{1}{4}x^2 + 4$. Tell Peter, how many kilograms of cucumbers he has to collect from his greenhouse in a season to get a maximum profit? What is the size of the profits?

The solution. Let's write the expression for the profit function:

$$P(x) = R(x) - C(x) = p \cdot x - C(x) = 5 \cdot x - \left(\frac{1}{4}x^2 + 4\right) = -\frac{1}{4}x^2 + 5x - 4$$

Since $x > 0$, the task is to study the parabolic function $P(x) = -\frac{1}{4}x^2 + 5x - 4$ on the highest value in the interval $[0;+\infty)$. The profit function looks like a quadratic function, so one can use its properties. The schedule of the function $P(x) = -\frac{1}{4}x^2 + 5x - 4$ is a parabola, whose branches directed downward. The greatest value of the function will become a point which is the apex of the parabola: $m = -\frac{b}{2a}, n = -\frac{b^2 - 4ac}{4a}$ (parabola vertex coordinates). The coordinates of the top of the parabola:

$m=-\frac{5}{2\cdot\left(-\frac{1}{4}\right)}=10$, $n=-\frac{25-4\cdot\left(-\frac{1}{4}\right)\cdot(-4)}{4\cdot\left(-\frac{1}{4}\right)}=21$. Thus, the profit of the greatest significance in $[0;+\infty)$ equals 21, and it is achieved when *x=10*. Peter Kolobkov will get a maximum profit, if he gathers in the greenhouse 10 kg of cucumbers, herewith the profit will be 21 grn. [1].

More opportunities arise in the course of teaching sections "Differential Calculus" and "Integral Calculus", on the basis of which the concept of economic substance (marginal cost, profit-income, elasticity features of the function, profit maximization, the calculation of discount income, income from the interest of a deposit, etc.) are entered.

Task 4. For the production of some products two types of resources in quantities *x* and *y* are used. The production function is like this:

$$Q(x,y)=100-0,9x^2-0,4y^2+18,4x+16,2y.$$

The price of the unit of the first resource is 4 monetary units, the second –2 monetary units and the price of the unit of the product is 10 monetary units. Find the combination of resources and production so that the production income was maximized.

The solution. A profit is the difference between the proceeds from the sale of the product and the cost of the purchasing resources:

$PR(x,y)=10(100-0,9x^2-0,4y^2+18,4x+16,2y)-4x-2y=$
$=1000-9x^2-4y^2+180x+160y$.

Let's find the point of maximum of the profit function. The stationary point is:

$$\begin{cases}PR'_x(x,y)=-18x+180=0,\\ PR'_y(x,y)=-8x+160=0\end{cases}\Rightarrow\begin{cases}x=10,\\ y=20.\end{cases}$$

Let's satisfy ourselves that the profit function has a maximum at the stationary point found:

$$\left[\begin{array}{l}PR''_{xx}(x,y)=-18=A,\\ PR''_{xy}(x,y)=0=B, \quad \Rightarrow\Delta=AC-B^2=144.\\ PR''_{yy}(x,y)=-8=C\end{array}\right.$$

If Δ = 144 > 0 і *A* = –18 < 0, the function *PR*(*x*, *y*) has a maximum $PR_{\max}=PR(10;20)=3500$ at the points (10; 20).

So, we will get a maximum profit in the amount of 3500 monetary units if we produce Q (10; 20) = 358 units of production, using 10 units of the first resource and 20 units of the second resource.

Task 5. According to compute net investment $I(t)=50000t$ let's calculate the capital gains from the first to the third year and determine for how many years the capital increase will amount to 2500000 converted monetary units.

The solution. To determine the capital gains in the time interval from $t_1=1$ до $t_2=3$ let's use the formula $\Delta K=K(t_2)-K(t_1)=\int_{t_1}^{t_2}I(t)dt$.

$$\Delta K=K(3)-K(1)=\int_1^3 50000t\,dt=25000t^2\Big|_1^3=200000$$

To determine in how many years the capital increase will be 2500000 converted monetary units, it is necessary to equate capital gains $\Delta K=K(T)-K(0)$ with 2500000 converted monetary units, that is $25000t^2\,|_0^T=2500000$, $25000T^2=2500000$, $T^2=100, T=10$.

So one needs ten years in order that the capital increase reaches 2.5 million converted monetary units.

The study of "Differential Equations" module leads us to consider the economic and mathematical models, such as: natural growth model, output growth model, dynamic model of Keynes [2].

Task 6. Let supply and demand for goods be defined by correlation:

$$q=2p''-p'-p+15,\ p=3p''+p'+p+5,$$

where *p* – a product price, p' – a pricing tendency; p'' – a rate of change of prices. Let also at the initial time $p(0)=6$, $q(0)=s(0)=10$. Proceeding from the compliance requirements of supply and demand, find the price dependence from time.

The solution. Proceeding from the compliance requirements of supply and demand, we have $q=s$. So, $2p''-p'-p+15=3p''+p'+p+5$. Hence we obtain inhomogeneous linear second-order differential equations with constant coefficients: $p''+2p'+2p=10$. We solve the corresponding homogeneous equation: $p''+2p'+2p=0$.

We form the characteristic equation: $k^2+2k+2=0$. The roots of the characteristic equation are $k_{1,2}=-1\pm i$. The general solution of homogeneous equation is $p*(t)=C_1e^{-t}\cos t+C_2e^{-t}\sin t$.

Let's find a particular solution of the inhomogeneous equation. The right side of equation $f(x)=10$, hence the particular solution of inhomogeneous equation will look as $p_{ch}=A$. Differentiating p_{ch} twice, we obtain: $p'_{ch}=0, p''_{ch}=0$. Substituting these values to the differential equation, we get $2A=10\Rightarrow A=5\Rightarrow p_{ch}=5$

The overall solution is as follows:

$$p(t)=e^{-t}(C_1\cos t+C_2\sin t)+5.$$

Taking into account the initial conditions, we find a particular solution $p(0)=6\Rightarrow 6=C_1+5$, $C_1=1$. Considering that $q=2p''-p'-p+15$ і $q(0)=10$, we find

$$p'(t)=-e^{-t}(\cos t+C_2\sin t)+e^{-t}(-\sin t+C_2\cos t)\Rightarrow$$
$$\Rightarrow p'(0)=C_2-1;$$
$$p''(t)=e^{-t}(-2C_2\cos t)+2\sin t\Rightarrow p''(0)=-2C_2;$$
$$q(0)=2(-2C_2)-(C_2-1)-6+15\Rightarrow 10=-5C_2+10\Rightarrow$$
$$\Rightarrow C_2=0$$

Therefore: $p(t)=5+e^{-t}\cos t$.

Thus, the analysis of applications has shown that it is emerged a certain level of interdisciplinary connections between mathematical and economic disciplines, the establishing of which improves professional training of the future economists.

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