Dzyuba V.A., Steblyanko P.O.

Use of splines in the calculation of deflections for plates of variable thickness

Dzyuba Victoria, postgraduate student
Steblyanko Pavel, Doctor of Physical and Mathematical Sciences, Professor
Department of Mathematics and methods of teaching mathematics
Cherkasy National University named by Bohdan Khmelnitsky, Cherkasy, Ukraine

Abstract. The paper presents new results to construct a method of calculating of the mechanical characteristics of plates and shells of variable thickness. It is focused on the need to develop a method of obtaining reliable results of high accuracy.
Keywords*: iterative method, numerical solution, the stress-strain state (SSS), a plate of variable thickness*

Introduction. During the design of machines it is necessary to calculate the stress-strain state (SSS) of thin-walled structural elements with the aim of rational choice of their size [1]. The high level of modern technology requires the preservation of reliability and durability of structural systems, which leads to the need of development of new theories and methods of calculation of shells and plates [1,2]. They should consider the different modes of operation, the actual material properties, loading conditions, technological, operational and other factors. .

Analysis of recent research and publications. Recently, the search for effective methods of calculation of cylindrical shells get great popularity among researchers. The substantial progress in this direction is impossible without the use of modern computers. In particular, new research results can be found in the works of G. Berikelashvili, O.T. Becker, A.T. Vasilenko, G.G. Vlaikov, Ya.M. Grigorenko, A.Ya. Grigorenko, P.A. Steblyanko etc.

Analysis of shell theory research indicates the need to develop a method of high accuracy for the calculation of thin-walled structures within a single species, as well as creating opportunities for a wide range of application of the method and summarize the results. There is a need to solve these problems in a spatial setting [5]. Therefore, to our mind, the methods of calculation of plates and shells of variable thickness require further researching.

Purpose. Development of numerical and analytical research method variant of mechanical characteristics of plates and shells of variable thickness increased accuracy**.**

Materials and researching methods. Quite often, when solving the specific problems, scholar faces with a problem of transition from three-dimensional elasticity problems to two-dimensional. There are, at present, methods of construction of two-dimensional theories that can be classified as follows: methods of making different hypotheses and methods of regular approximation to process within the solution of the three-dimensional problem of elasticity theory. Each theory has its own characteristics, it can be explained by the fact that different hypotheses concerning the deformation of membranes and changing metrics of shell thickness are used, which in turn calls into question the credibility of determining SSS membranes, since it leads to significant errors in the calculations.

We consider the displacement and stress-strain state of a thin-walled body (plate, shell) loaded on the surface of effort $-q_x, q_y, q_z$ in the middle surface. In the formulation it is assumed that the thickness of the body h can vary in both directions. The key unknowns of this problem are: u, v, w – displacement of the generating, directing and normal to the middle surface.

Introduce to consider differential equations of the theory of shells Mushtari-Donnell-Vlasov

$$C_{11}\frac{\partial^2 u}{\partial x^2}+C_{66}\frac{\partial^2 u}{\partial y^2}+\left(C_{12}+C_{66}\right)\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial C_{11}}{\partial x}\cdot\frac{\partial u}{\partial x}+\frac{\partial C_{66}}{\partial y}\cdot\frac{\partial u}{\partial y}+$$
$$\frac{\partial C_{66}}{\partial y}\cdot\frac{\partial v}{\partial x}+\frac{\partial C_{12}}{\partial x}\cdot\frac{\partial v}{\partial y}+\frac{\partial}{\partial x}\left(\frac{C_{11}}{R_1}+\frac{C_{12}}{R_2}\right)w+\left(\frac{C_{11}}{R_1}+\frac{C_{12}}{R_2}\right)\frac{\partial w}{\partial x}=q_x,$$

$$D_{11}\frac{\partial^4 w}{\partial x^4}+D_{22}\frac{\partial^4 w}{\partial y^4}+\left(2D_{12}+4D_{66}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+2\frac{\partial D_{11}}{\partial x}\frac{\partial^3 w}{\partial x^3}+2\frac{\partial D_{22}}{\partial y}\frac{\partial^3 w}{\partial y^3}+$$
$$\left(2\frac{\partial D_{12}}{\partial y}+4\frac{\partial D_{66}}{\partial y}\right)\frac{\partial^3 w}{\partial x^2\partial y}+\left(2\frac{\partial D_{12}}{\partial x}+4\frac{\partial D_{66}}{\partial x}\right)\frac{\partial^3 w}{\partial x\partial y^2}+$$
$$\left(\frac{\partial^2 D_{11}}{\partial x^2}+\frac{\partial^2 D_{12}}{\partial y^2}\right)\frac{\partial^2 w}{\partial x^2}+\left(\frac{\partial^2 D_{12}}{\partial x^2}+\frac{\partial^2 D_{22}}{\partial y^2}\right)\frac{\partial^2 w}{\partial y^2}+4\frac{\partial^2 D_{66}}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+$$
$$\left(\frac{C_{11}}{R_1}+\frac{C_{12}}{R_2}\right)\frac{\partial u}{\partial x}+\left(\frac{C_{12}}{R_1}+\frac{C_{22}}{R_2}\right)\frac{\partial v}{\partial y}+\left(\frac{C_{11}}{R_1^2}+\frac{2C_{12}}{R_1R_2}+\frac{C_{22}}{R_2^2}\right)w=q_z,$$

$$C_{66}\frac{\partial^2 v}{\partial x^2}+C_{22}\frac{\partial^2 v}{\partial y^2}+\left(C_{12}+C_{66}\right)\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial C_{22}}{\partial y}\cdot\frac{\partial v}{\partial y}+$$
$$\frac{\partial C_{66}}{\partial x}\cdot\frac{\partial v}{\partial x}+\frac{\partial C_{66}}{\partial x}\cdot\frac{\partial u}{\partial y}+\frac{\partial C_{12}}{\partial y}\cdot\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\left(\frac{C_{12}}{R_1}+\frac{C_{22}}{R_2}\right)w+\left(\frac{C_{12}}{R_1}+\frac{C_{22}}{R_2}\right)\frac{\partial w}{\partial y}=q_y,$$

where $x, y, z -$ the coordinates relative generating, directing and thickness of the shell, respectively, $R_1, R_2 -$ the radii of curvature in the direction of the coordinate axes of shell $x, y,$ and the notation are introduced

$$C_{11} = C_{22} = \frac{Eh}{1-\nu^2},\ C_{12} = \frac{\nu Eh}{1-\nu^2},\ C_{66} = Gh = \frac{Eh}{2(1+\nu)},$$

$$D_{11} = D_{22} = \frac{Eh^3}{12(1-\nu^2)},\ D_{12} = \frac{\nu Eh^3}{12(1-\nu^2)},\ D_{66} = \frac{Eh^3}{24(1+\nu)}\ .$$

In this case, $E -$ Young modulus and $\nu -$ Poisson□□s coefficient.

In a closed form solving the following system of differential equations in partial derivatives is not possible. Usually when solving the following problems approximate methods are used.

In order to reduce the order of a complete system of equations the additional unknown quantities are introduced to the first one

$$u'(x,y) = \frac{\partial u(x,y)}{\partial x},\ v'(x,y) = \frac{\partial v(x,y)}{\partial x},\ w'(x,y) = \frac{\partial w(x,y)}{\partial x},$$

as well as

$$w''(x,y) = \frac{\partial w'(x,y)}{\partial x},\ w'''(x,y) = \frac{\partial w''(x,y)}{\partial x}.$$

Based on this boundary value problem is formulated as

$$\frac{d\vec{Y}}{dx} = A(x)\cdot\vec{Y} + \vec{f}(x),\quad 0 \le x \le L,\ \vec{Y}(0) = \vec{Y}_0,\ \vec{Y}(L) = \vec{Y}_L. \qquad (1)$$

Unknown quantities are the components of the vector

$$\overline{Y} = \left\{\overline{y_1}, \overline{y_2}, ..., \overline{y_8}\right\}^T \equiv \left\{\overline{u}, \overline{u'}, \overline{v}, \overline{v'}, \overline{w}, \overline{w'}, \overline{w''}, \overline{w'''}\right\}^T,\ \overline{y_m} = \left\{\overline{y_{m_0}}, \overline{y_{m_2}}, ..., \overline{y_{m_N}}\right\}^T,\ \left(m = \overline{1,8}\right).$$

Based on the conditions defined on the edges, we can formulate the boundary conditions for the system of equations (1). Consider the case where one end of shell (plate) is fixed rigidly, and the second one is fixed hinge, so that the boundary conditions take the form:

$$x = 0,\ u = v = w = 0,\ w' = \frac{\partial w}{\partial x} = 0;\ x = L,\ u = v = w = 0,\ w'' = \frac{\partial^2 w}{\partial x^2} = 0. \qquad (2)$$

The matrix $A(x)$ $8N \times 8N$ has the form

$$A(x) = \begin{Vmatrix} 0 & E & 0 & 0 & 0 & 0 & 0 & 0 \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} & L_{26} & 0 & 0 \\ 0 & 0 & 0 & E & 0 & 0 & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & E \\ 0 & L_{82} & L_{83} & 0 & L_{85} & L_{86} & L_{87} & L_{88} \end{Vmatrix}. \qquad (3)$$

Here 0 i E are accordingly zero and single matrices $N \times N$, $N -$ amount of knots in direction to a coordinate y. Through L_{ij} the auxiliary matrices $N \times N$ are marked, the elements of which are expressed through geometrical and mechanical characteristics. Except for it they depend a substantial rank on the method of approximation of derivatives from the basic unknown functions by a coordinate y.

We introduce a net by a coordinate y as follows

$$y_{i+1} = y_i + h_3;\ i = 1, 2, ...N;\ y_1 = 0;\ y_{N+1} = 2\pi R.$$

Solution of boundary value problems for the system (1) is possible to find as interpolation expression, got with the use of stressed splines.

$$u(x,y) = \sum_{n=-2}^{2} \alpha_n(t)\cdot u_{i+n}(x);\ v(x,y) = \sum_{n=-2}^{2} \alpha_n(t)\cdot v_{i+n}(x);\ w(x,y) = \sum_{n=-2}^{2} \alpha_n(t)\cdot w_{i+n}(x), \qquad (4)$$

де $t = \frac{1}{3h_2}[y - y_i],\quad y \in [y_i; y_{i+3}].$

Coefficients $\alpha_n(t)$ from formulas (4) are resulted in process [4].

The expression (4) is used for description of solution of the system (1) and all derivatives on a coordinate Y. The part derivatives from the unknown functions u, v, w are determined on the basis of formulas (4) by direct differentiation of the proper expressions. As a result, the moving w is possible to write down for the first and the second derivatives as follows:

$$\frac{\partial w(x,y)}{\partial y} = \frac{1}{3h_2} \cdot \sum_{n=-2}^{2} \alpha'_n(t) \cdot w_{i+n}(x), \quad \frac{\partial^2 w(x,y)}{\partial y^2} = \frac{1}{9h_2^2} \cdot \sum_{n=-2}^{2} \alpha''_n(t) \cdot w_{i+n}(x).$$

By an analogical method the third and the fourth derivatives are determined from the normal moving

$$\frac{\partial^3 w(x,y)}{\partial y^3} = \frac{1}{27h_2^3} \cdot \sum_{n=-2}^{2} \alpha'''_n(t) \cdot w_{i+n}(x), \quad \frac{\partial^4 w(x,y)}{\partial y^4} = \frac{1}{81h_2^4} \cdot \sum_{n=-2}^{2} \alpha^{1Y}_n(t) \cdot w_{i+n}(x).$$

Using formulas (4) we will write down the final expressions for the unknown functions and their derivatives by a circular coordinate in a point $y = y_i, t = 1/3$.

We will enter denotations

$$u(x, y_i) \equiv u_i(x), \quad v(x, y_i) \equiv v_i(x), \quad w(x, y_i) \equiv w_i(x)$$

$$\frac{\partial u(x,y_i)}{\partial y} \approx \frac{1}{12h_2} \cdot [k_0 u_{i-2}(x) - n_0 u_{i-1}(x) + n_0 u_{i+1}(x) - k_0 u_{i+2}(x)],$$

$$\frac{\partial v(x,y_i)}{\partial y} \approx \frac{1}{12h_2} \cdot [k_0 v_{i-2}(x) - n_0 v_{i-1}(x) + n_0 v_{i+1}(x) - k_0 v_{i+2}(x)],$$

$$\frac{\partial w(x,y_i)}{\partial y} \approx \frac{1}{12h_2} \cdot [k_0 w_{i-2}(x) - n_0 w_{i-1}(x) + n_0 w_{i+1}(x) - k_0 w_{i+2}(x)],$$

$$\frac{\partial^2 u(x,y_i)}{\partial y^2} \approx \frac{m_0}{h_2^2} \cdot [u_{i-1}(x) - 2u_i(x) + u_{i+1}(x)],$$

$$\frac{\partial^2 v(x,y_i)}{\partial y^2} \approx \frac{m_0}{h_2^2} \cdot [v_{i-1}(x) - 2v_i(x) + v_{i+1}(x)], \qquad (5)$$

$$\frac{\partial^2 w(x,y_i)}{\partial y^2} \approx \frac{m_0}{h_2^2} \cdot [w_{i-1}(x) - 2w_i(x) + w_{i+1}(x)].$$

$$\frac{\partial^3 w(x,y_i)}{\partial y^3} = \frac{m_1}{h_2^3} \cdot [-w_{i-2}(x) + 2w_{i-1}(x) - 2w_{i+1}(x) + w_{i+2}(x)].$$

High-quality new result consists in the fact that by stressed splines (4) it is possible to calculate the fourth derivative from w by direct differentiation, namely

$$\frac{\partial^4 w(x,y_i)}{\partial y^4} \approx \frac{m_2}{h_2^4} \cdot [w_{i-1}(x) - 2w_i(x) + w_{i+1}(x)]. \qquad (6)$$

Here

$$m_0 = 0{,}982, \; m_1 = 0{,}473, \; m_2 = 0{,}218,$$
$$n_0 = 7{,}9136, \; k_0 = 0{,}95681, \; k_1 = 11{,}2646, \; k_2 = 18{,}4641,$$
$$k_3 = 9{,}1344, \; k_4 = 1{,}9349, \; k_5 = 3{,}0870, \; k_6 = 5{,}9787.$$

By an analogical method it is possible to write down the expression for additional functions u', v', w' and their derivatives on a circular coordinate in a point $y = y_i, t = 1/3$.

$$u'(x,y_i) \equiv u'_i(x), \; v'(x,y_i) \equiv v'_i(x), \; w'(x,y_i) \equiv w'_i(x), \; w''(x,y_i) \equiv w''_i(x), \; w'''(x,y_i) \equiv w'''_i(x),$$

$$\frac{\partial u'(x,y_i)}{\partial y} \approx \frac{1}{12h_2} \cdot [k_0 u'_{i-2}(x) - n_0 u'_{i-1}(x) + n_0 u'_{i+1}(x) - k_0 u'_{i+2}(x)],$$

$$\frac{\partial v'(x,y_i)}{\partial y} \approx \frac{1}{12h_2} \cdot [k_0 v'_{i-2}(x) - n_0 v'_{i-1}(x) + n_0 v'_{i+1}(x) - k_0 v'_{i+2}(x)],$$

$$\frac{\partial w''(x,y_i)}{\partial y} \approx \frac{1}{12h_2} \cdot [k_0 w''_{i-2}(x) - n_0 w''_{i-1}(x) + n_0 w''_{i+1}(x) - k_0 w''_{i+2}(x)], \qquad (7)$$

$$\frac{\partial^2 w''(x,y_i)}{\partial y^2} \approx \frac{m_0}{h_2^2} \cdot [w''_{i-1}(x) - 2w''_i(x) + w''_{i+1}(x)].$$

The method of numeral decision consists in the fact that it is necessary to enter a net $x_{j+1} = x_j + h_1;$ $j = 1, 2, ..., M-1;\ x_1 = 0;\ x_M = L$ on a coordinate x.

The eight auxiliary matrices $L_{22}, L_{25}, L_{26}, L_{44}, L_{45}, L_{46}, L_{82}, L_{88}$ have the most simple diagonal form, namely

$$L_{22} = h' \cdot \|E\|,\ L_{25} = h' \cdot r' \cdot \|E\|,\ L_{26} = -r' \cdot \|E\|,\ L_{44} = h' \cdot \|E\|,$$

$$L_{45} = \frac{h'r'}{h\nu} \cdot \|E\| - r'' \cdot \|E_{1n}\|,\ L_{82} = -\frac{12r'}{h^2} \cdot \|E\|,\ L_{88} = 6h' \cdot \|E\|, \tag{8}$$

where the next denotations are entered

$$h' = -\frac{1}{h}\frac{\partial h}{\partial x},\ h'' = -\frac{1}{h}\frac{\partial h}{\partial y},\quad r' = \frac{\nu}{R},\ r'' = \frac{1}{\nu R},\ r''' = -\frac{1}{h^2 R}.$$

The differential operators of different orders by a variable y from the unknown quantities enter into the complement of the other ten auxiliary matrices $L_{21}, L_{23}, L_{24}, L_{41}, L_{42}, L_{43}, L_{83}, L_{85}, L_{86}, L_{87}$ by a size $N \times N$. Approximation of these operators in knots y can be different depending on that, what interpolation expression for this purpose is used. Using expressions (5) – (7), we will get

$$L_{23} = -\frac{1}{C_{11}} \cdot \frac{\partial C_{12}}{\partial x} \cdot \|E_{1n}\|,\ L_{41} = -\frac{1}{C_{66}} \cdot \frac{\partial C_{66}}{\partial y} \cdot \|E_{1n}\|,\ L_{83} = -\frac{1}{D_{11}} \cdot \left(\frac{C_{12}}{R_1} + \frac{C_{22}}{R_2}\right) \cdot \|E_{1n}\|, \tag{9}$$

where

$$E_{1n} = \begin{Vmatrix} 0 & n_0 & -k_0 & 0 & \dots & 0 & 0 & k_0 & -n_0 \\ -n_0 & 0 & n_0 & -k_0 & 0 & \dots & 0 & 0 & k_0 \\ k_0 & -n_0 & 0 & n_0 & -k_0 & 0 & \dots & 0 & 0 \\ 0 & k_0 & -n_0 & 0 & n_0 & -k_0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & k_0 & -n_0 & 0 & n_0 & -k_0 & 0 \\ 0 & 0 & \dots & 0 & k_0 & -n_0 & 0 & n_0 & -k_0 \\ -k_0 & 0 & 0 & \dots & 0 & k_0 & -n_0 & 0 & n_0 \\ n_0 & -k_0 & 0 & 0 & \dots & 0 & k_0 & -n_0 & 0 \end{Vmatrix},$$

In matrices $L_{21}, L_{24}, L_{42}, L_{43}, L_{86}, L_{87}$

$$L_{24} = \begin{Vmatrix} \alpha_1 & 8\alpha_2 & -\alpha_2 & 0 & \dots & 0 & 0 & \alpha_2 & -8\alpha_2 \\ -8\alpha_2 & \alpha_1 & 8\alpha_2 & -\alpha_2 & 0 & \dots & 0 & 0 & \alpha_2 \\ \alpha_2 & -8\alpha_2 & \alpha_1 & 8\alpha_2 & -\alpha_2 & 0 & \dots & 0 & 0 \\ 0 & \alpha_2 & -8\alpha_2 & \alpha_1 & 8\alpha_2 & -\alpha_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \alpha_2 & -8\alpha_2 & \alpha_1 & 8\alpha_2 & -\alpha_2 & 0 \\ 0 & 0 & \dots & 0 & \alpha_2 & -8\alpha_2 & \alpha_1 & 8\alpha_2 & -\alpha_2 \\ -\alpha_2 & 0 & 0 & \dots & 0 & \alpha_2 & -8\alpha_2 & \alpha_1 & 8\alpha_2 \\ 8\alpha_2 & -\alpha_2 & 0 & 0 & \dots & 0 & \alpha_2 & -8\alpha_2 & \alpha_1 \end{Vmatrix},$$

$$L_{42} = \begin{Vmatrix} \alpha_3 & 8\alpha_4 & -\alpha_4 & 0 & \dots & 0 & 0 & \alpha_4 & -8\alpha_4 \\ -8\alpha_4 & \alpha_3 & 8\alpha_4 & -\alpha_4 & 0 & \dots & 0 & 0 & \alpha_4 \\ \alpha_4 & -8\alpha_4 & \alpha_3 & 8\alpha_4 & -\alpha_4 & 0 & \dots & 0 & 0 \\ 0 & \alpha_4 & -8\alpha_4 & \alpha_3 & 8\alpha_4 & -\alpha_4 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \alpha_4 & -8\alpha_4 & \alpha_3 & 8\alpha_4 & -\alpha_4 & 0 \\ 0 & 0 & \dots & 0 & \alpha_4 & -8\alpha_4 & \alpha_3 & 8\alpha_4 & -\alpha_4 \\ -\alpha_4 & 0 & 0 & \dots & 0 & \alpha_4 & -8\alpha_4 & \alpha_3 & 8\alpha_4 \\ 8\alpha_4 & -\alpha_4 & 0 & 0 & \dots & 0 & \alpha_4 & -8\alpha_4 & \alpha_3 \end{Vmatrix},$$

$$L_{21}=\begin{Vmatrix} -2\alpha_6 & \alpha_8 & -\alpha_5 & 0 & \dots & 0 & 0 & \alpha_5 & \alpha_7 \\ \alpha_7 & -2\alpha_6 & \alpha_8 & -\alpha_5 & 0 & \dots & 0 & 0 & \alpha_5 \\ \alpha_5 & \alpha_7 & -2\alpha_6 & \alpha_8 & -\alpha_5 & 0 & \dots & 0 & 0 \\ 0 & \alpha_5 & \alpha_7 & -2\alpha_6 & \alpha_8 & -\alpha_5 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \alpha_5 & \alpha_7 & -2\alpha_6 & \alpha_8 & -\alpha_5 & 0 \\ 0 & 0 & \dots & 0 & \alpha_5 & \alpha_7 & -2\alpha_6 & \alpha_8 & -\alpha_5 \\ -\alpha_5 & 0 & 0 & \dots & 0 & \alpha_5 & \alpha_7 & -2\alpha_6 & \alpha_8 \\ \alpha_8 & -\alpha_5 & 0 & 0 & \dots & 0 & \alpha_5 & \alpha_7 & -2\alpha_6 \end{Vmatrix},$$

$$L_{43}=\begin{Vmatrix} -2\alpha_0 & \beta_2 & -\alpha_9 & 0 & \dots & 0 & 0 & \alpha_9 & \beta_1 \\ \beta_1 & -2\alpha_0 & \beta_2 & -\alpha_9 & 0 & \dots & 0 & 0 & \alpha_9 \\ \alpha_9 & \beta_1 & -2\alpha_0 & \beta_2 & -\alpha_9 & 0 & \dots & 0 & 0 \\ 0 & \alpha_9 & \beta_1 & -2\alpha_0 & \beta_2 & -\alpha_9 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \alpha_9 & \beta_1 & -2\alpha_0 & \beta_2 & -\alpha_9 & 0 \\ 0 & 0 & \dots & 0 & \alpha_9 & \beta_1 & -2\alpha_0 & \beta_2 & -\alpha_9 \\ -\alpha_9 & 0 & 0 & \dots & 0 & \alpha_9 & \beta_1 & -2\alpha_0 & \beta_2 \\ \beta_2 & -\alpha_9 & 0 & 0 & \dots & 0 & \alpha_9 & \beta_1 & -2\alpha_0 \end{Vmatrix},$$

$$L_{86}=\begin{Vmatrix} -2\beta_4 & \beta_6 & -\beta_3 & 0 & \dots & 0 & 0 & \beta_3 & \beta_5 \\ \beta_5 & -2\beta_4 & \beta_6 & -\beta_3 & 0 & \dots & 0 & 0 & \beta_3 \\ \beta_3 & \beta_5 & -2\beta_4 & \beta_6 & -\beta_3 & 0 & \dots & 0 & 0 \\ 0 & \beta_3 & \beta_5 & -2\beta_4 & \beta_6 & -\beta_3 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \beta_3 & \beta_5 & -2\beta_4 & \beta_6 & -\beta_3 & 0 \\ 0 & 0 & \dots & 0 & \beta_3 & \beta_5 & -2\beta_4 & \beta_6 & -\beta_3 \\ -\beta_3 & 0 & 0 & \dots & 0 & \beta_3 & \beta_5 & -2\beta_4 & \beta_6 \\ \beta_6 & -\beta_3 & 0 & 0 & \dots & 0 & \beta_3 & \beta_5 & -2\beta_4 \end{Vmatrix},$$

$$L_{87}=\begin{Vmatrix} \beta_9 & \omega_2 & -\beta_7 & 0 & \dots & 0 & 0 & \beta_7 & \omega_1 \\ \omega_1 & \beta_9 & \omega_2 & -\beta_7 & 0 & \dots & 0 & 0 & \beta_7 \\ \beta_7 & \omega_1 & \beta_9 & \omega_2 & -\beta_7 & 0 & \dots & 0 & 0 \\ 0 & \beta_7 & \omega_1 & \beta_9 & \omega_2 & -\beta_3 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \beta_7 & \omega_1 & \beta_9 & \omega_2 & -\beta_7 & 0 \\ 0 & 0 & \dots & 0 & \beta_7 & \omega_1 & \beta_9 & \omega_2 & -\beta_7 \\ -\beta_7 & 0 & 0 & \dots & 0 & \beta_7 & \omega_1 & \beta_9 & \omega_2 \\ \omega_2 & -\beta_7 & 0 & 0 & \dots & 0 & \beta_7 & \omega_1 & \beta_9 \end{Vmatrix}, \qquad (10)$$

where for the thin-walled plate of variable thickness

$$\alpha_1=-\frac{1}{C_{11}}\cdot\frac{\partial C_{66}}{\partial y},\ \alpha_2=-\frac{1}{C_{11}}\cdot(C_{12}+Gh)\cdot\frac{k_0}{12h_2},\ \alpha_3=-\frac{1}{C_{66}}\cdot\frac{\partial C_{12}}{\partial y},\ \alpha_4=-\frac{1}{C_{66}}\cdot(C_{12}+Gh)\cdot\frac{k_0}{12h_2},$$

$$\alpha_5=-\frac{k_0}{12h_2C_{11}}\cdot\frac{\partial C_{66}}{\partial y},\ \alpha_6=-\frac{m_0C_{66}}{h_2^2C_{11}},\ \alpha_7=\alpha_6+\frac{n_0}{12h_2C_{11}}\cdot\frac{\partial C_{66}}{\partial y},\ \alpha_8=\alpha_6-\frac{n_0}{12h_2C_{11}}\cdot\frac{\partial C_{66}}{\partial y},$$

$$\alpha_0=-\frac{m_0}{h_2^2C_{66}}\cdot C_{22},\beta_1=\alpha_0+\frac{n_0}{12h_2C_{66}}\cdot\frac{\partial C_{22}}{\partial y},\beta_2=\alpha_0-\frac{n_0}{12h_2C_{66}}\cdot\frac{\partial C_{22}}{\partial y},\ \alpha_9=-\frac{k_0}{12h_2C_{66}}\cdot\frac{\partial C_{22}}{\partial y},$$

$$\beta_4 = -\frac{2m_0}{h_2^2 D_{11}} \cdot \left(\frac{\partial D_{12}}{\partial x} + 2\frac{\partial D_{66}}{\partial x} \right), \ \beta_5 = \beta_4 + \frac{n_0}{3h_2 D_{11}} \cdot \frac{\partial^2 D_{66}}{\partial x \partial y},$$

$$\beta_6 = \beta_4 - \frac{n_0}{3h_2 D_{11}} \cdot \frac{\partial^2 D_{66}}{\partial x \partial y}, \ \beta_5 = -\frac{k_0}{3h_2 D_{11}} \cdot \frac{\partial^2 D_{66}}{\partial x \partial y},$$

$$\beta_7 = -\frac{k_0}{6h_2 D_{11}} \cdot \left(\frac{\partial D_{12}}{\partial y} + 2\frac{\partial D_{66}}{\partial y} \right), \ \beta_9 = -\frac{1}{D_{11}} \left[\frac{\partial^2 D_{11}}{\partial x^2} + \frac{\partial^2 D_{12}}{\partial y^2} - \frac{4m_0}{h_2^2}\left(D_{12} + 2D_{66}\right) \right],$$

$$\omega_1 = -\frac{1}{h_2 D_{11}} \left[\frac{2m_0}{h_2}\left(D_{12} + 2D_{66}\right) - \frac{n_0}{6}\left(\frac{\partial D_{12}}{\partial y} + 2\frac{\partial D_{66}}{\partial y} \right) \right],$$

$$\omega_2 = -\frac{1}{h_2 D_{11}} \left[\frac{2m_0}{h_2}\left(D_{12} + 2D_{66}\right) + \frac{n_0}{6}\left(\frac{\partial D_{12}}{\partial y} + 2\frac{\partial D_{66}}{\partial y} \right) \right].$$

$$L_{85} = \begin{Vmatrix} \omega_6 & \omega_{10} & \omega_7 & 0 & 0 & \dots & 0 & 0 & 0 & \omega_7 & \omega_9 \\ \omega_9 & \omega_6 & \omega_{10} & \omega_7 & 0 & 0 & \dots & 0 & 0 & 0 & \omega_7 \\ \omega_7 & \omega_9 & \omega_6 & \omega_{10} & \omega_7 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \omega_7 & \omega_9 & \omega_6 & \omega_{10} & \omega_7 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \omega_7 & \omega_9 & \omega_6 & \omega_{10} & \omega_7 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \omega_7 & \omega_9 & \omega_6 & \omega_{10} & \omega_7 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \omega_7 & \omega_9 & \omega_6 & \omega_{10} & \omega_7 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \omega_7 & \omega_9 & \omega_6 & \omega_{10} & \omega_7 \\ \omega_7 & 0 & 0 & 0 & \dots & 0 & 0 & \omega_7 & \omega_9 & \omega_6 & \omega_{10} \\ \omega_{10} & \omega_7 & 0 & 0 & 0 & \dots & 0 & 0 & \omega_7 & \omega_9 & \omega_6 \end{Vmatrix}. \quad (11)$$

Denotations are entered here

$$\omega_6 = \frac{2m_0}{D_{11}h_2^2} \cdot \left(\frac{\partial^2 D_{12}}{\partial x^2} + \frac{\partial^2 D_{22}}{\partial y^2} \right) - \frac{2m_2}{h_2^4} D_{22}, \ \omega_9 = -\frac{1}{D_{11}h_2^2} \left[m_0 \cdot \left(\frac{\partial^2 D_{12}}{\partial x^2} + \frac{\partial^2 D_{22}}{\partial y^2} \right) + \frac{4m_1}{h_2} \cdot \frac{\partial D_{22}}{\partial y} + D_{22}\frac{m_2}{h_2^2} \right],$$

$$\omega_{10} = -\frac{1}{D_{11}h_2^2} \left[m_0 \cdot \left(\frac{\partial^2 D_{12}}{\partial x^2} + \frac{\partial^2 D_{22}}{\partial y^2} \right) - \frac{4m_1}{h_2} \cdot \frac{\partial D_{22}}{\partial y} + D_{22}\frac{m_2}{h_2^2} \right], \omega_7 = -\frac{2m_1}{D_{11}h_2^3} \cdot \frac{\partial D_{22}}{\partial y}, \omega_8 = \omega_7 .$$

Zero values (zero iteration) are selected in quality of initial values of the basic unknown quantities. As maximum boundary conditions have the form (2), it is possible to write down

$$u_i(0) = 0, \ v_i(0) = 0, \ w_i(0) = 0, \ w'(0) = 0, \ i = 1, 2, 3, \dots, N.$$

Other unknowns $u_i'(0), v_i'(0), w_i''(0), w_i'''(0)$ are determined by the decision of the system of type (1), where

$$\vec{Y} = \begin{Vmatrix} u' \\ v' \\ w'' \\ w''' \end{Vmatrix} = \begin{Vmatrix} y_2 \\ y_4 \\ y_7 \\ y_8 \end{Vmatrix}, \ A(0) = \begin{Vmatrix} h'E & L_{24} & 0 & 0 \\ L_{42} & h'E & 0 & 0 \\ 0 & 0 & 0 & E \\ \frac{12r'}{h^2}E & 0 & L_{87} & 6h'E \end{Vmatrix}.$$

Results and their discussions. Searching for the solution of the system (1) can be carried out under the tridiagonal matrix algorithm. The calculations were made in two directions [2, 3] on every iteration in knots $x_{j+1} = x_j + h_1$; $j = 1, 2, \dots, M-1$; $x_1 = 0$; $x_M = L$.

At first it was with the use of formulas

$$\vec{Y}_{k+1}\left(x_{i+1}\right) = \vec{Y}_k\left(x_i\right) + h_1\left[\left(A(x_i) \cdot \vec{Y}\right)_k + \vec{f}(x_i)\right], \ k = 0, 1, 2, \dots \quad (12)$$

for increasing values $j = 1, 2, \dots, M-1$ and then in the opposite direction $j = M-1, M-2, \dots, 2, 1$.

Here

$$\vec{Y}_{k+1}(x_{i-1}) = \vec{Y}_k(x_i) + h_1\left[\left(A(x_i)\cdot\vec{Y}\right)_k + \vec{f}(x_i)\right],\ k = 0,1,2,\ldots \quad (13)$$

Half the sum of values of unknown quantities in knots, got by formulas (12) and (13) allows to get solution on an iteration step.

Consider a numeral example, where plate, square in a plan, $x \in (0;L), y \in (0;L)$ of variable thickness is locally loaded with normal surface effort $q_z - const$, $x \in \left(\frac{L}{2};\frac{3L}{4}\right), y \in \left(\frac{5L}{12};\frac{7L}{12}\right)$. The thickness of plate changes after a law $h = h_0\left[1+\alpha\left(1-6x+6x^2\right)\right]$ [1].

The figure 1 shows the typical results of numerical solutions for the dimensionless $\left(Ew/10^4 q_z\right)$ normal displacement of the middle surface of the plate.

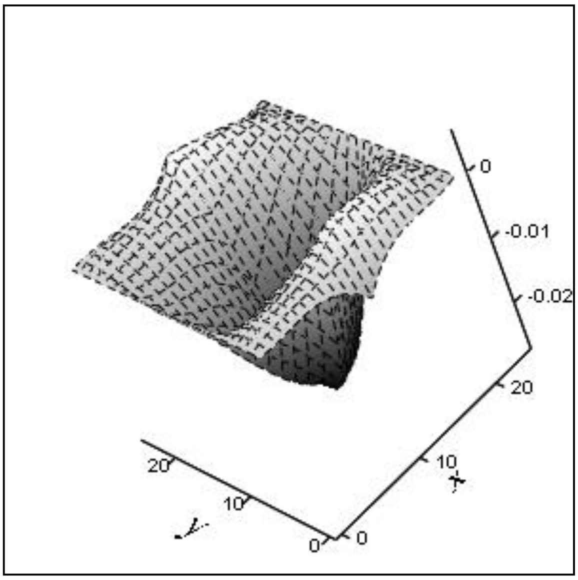

Fig.1

The basic unknowns are determined after numeral solution of the system (1). On the next stage it is possible directly by geometrical and physical relations to find according to deformation, curvatures, specific efforts and specific moments.

The method of calculation offered in the article opens new possibilities in the plan of solutions of problems in relation to optimization by the thickness of form of the thin-walled plates and shells.

Conclusions. The new effective iteration approach to the solution of problem of calculation of the thin-walled plates and shells of variable thickness is offered. The method of calculation of the stress-strain state of the thin-walled bodies of variable thickness is developed. The method of calculation is taken to the decision of SLAE (The System of Linear Algebraic Equations) with the use of iteration methods. The Seidel method is offered in this work to application. To improve the convergence of the Seidel method, that is required to reduce the required number of iterations the accuracy to the minimum proposed by the introduction of a specially developed derivative approximation, which provides a maximum diagonal coefficients in the original algebraic equations in comparison to neighboring elements [3, 5].

REFERENCES TRANSLATED AND TRANSLITERATED

1. Grigorenko Ya.M., Vlaykov G.G., Grigorenko A.Ya. Numerically-analytical solution of problems of mechanics of shells on the basis of various models. – Kiev: Akademperiodika, 2006. – 472 p.
2. Godunov S.K. About the numerical solution of boundary value problems for systems of linear ordinary differential equations – 1961. – B.16. – Issue 3 (99). – P. 171 – 174.
3. Dzyuba V.A., V.A. Dzyuba, P.A. Steblyanko. The construction method of high accuracy solution of the problem for a cylindrical shell of variable thickness based on iterative methods // Zbirnyk naukovykh prats Dniprodzerzhynskoho derzhavnoho tekhnichnoho universytetu / Dniprodzerzhynsk: DDTU. – 2014. – Issue 1 (24). – P. 216–221
4. Steblyanko P.A., Dzyuba A.P., Safronov O.O. Using of spline approximation for solving the problem of calculation of the cylindrical shell with variable stiffness // Problemy obchislyuvalnoyi mekhaniky y mitsnosti konstruktsiy. Zbirnyk naukovykh prats . – Issue 15. – Dnipropetrovsk , 2011. – P. 170–182.
5. Traub Dzh. Iterative methods for solving equations / Traub Dzh.; Transl. from English by I. A. Glinkin; under red. of A.G. Sukharev. – M.: Mir, 1985. – 264 p.

ДзюбаВ. А., Стеблянко П.А. Применение сплайнов при исчислении прогибов для пластины переменной толщины

Аннотация. В статье приведены новые результаты для построения метода расчета перемещений и напряженно-деформированного состояния пластины переменной толщины. Сосредоточено внимание на необходимости разработки метода повышенной точности получения достоверных результатов.

Ключевые слова: *итерационный метод, численное решение, напряженно-деформированное состояние (НДС), тонкостенные тела переменной толщины*