Mathematical model of flat vertical oscillations of the trolley for transporting dangerous cargo with usage of pneumatic elements in the second stage of suspension

Abstract. We are introducing the construction of a mathematical model of plane vertical oscillations of the trolley for transporting dangerous cargo, which has a two-stage spring suspension using high quality pneumatic elements.

Keywords: explosive cargo, two stage spring suspension, vibrations.

Statement of the problem. For transportation of dangerous, particularly explosive cargo from a location to the point of utilization it was designed the construction of a special trolley [1], which spring suspension has characteristics that satisfy the requirements for safe transportation, and absence of an engine and transmission makes a simple and reliable design of the construction (Fig. 1).

The main feature of the design of the trolley, as opposed to the traditional for automobile manufacturing single spring suspension, is the usage of the additional second stage with the stiffness corrector [2-4], the dynamic characteristics of which provide conditions for safe transportation.

Some features of this design [5] in a real operation that may significantly complicate preparations for the transport of dangerous cargo are solved using canned one-ridged elastic elements [6] in reference points of the load platform and the described turntable platform of the first axis bound significantly improves driving performance of the trolley, especially on curved sections of roads.

Determination of the required parameters of the introduced spring suspension, which is strongly dependent on its dynamic properties, estimates should be provided on a mathematical model of plane vertical oscillations of the described design.

Analysis of recent research and publications. General theoretical foundations of mathematical models and calculation methods of spring suspension of modern vehicles are set out in the work [7-10], and the design of mathematical models of the trolley for transportation of dangerous cargo with different constructions of elastic elements in the second stage in the works [11-12].

Calculation of thermodynamic processes within designing of the air spring suspension system paths, based on the theory of “filling-emptying” and the quasistationary method of determining the parameters of the air condition that are reviewed in the works [13-15].

Statement of the problem and its solution. Keeping in mind that the vertical oscillations in the longitudinal plane are the main influence on the dynamic properties of the transport system it is appropriate to perform their calculation on two-axis model (Fig. 2).

To construct an appropriate mathematical model trolley is considered as a system involving four elastic-connected solid bodies:
- load platform with cargo and brought it to the weight of the second stage spring suspension, which is denoted by the weight \( M_2 \);
- support platform with reduced her weight by parts of the second and first stage spring suspension, a lot of which is denoted by \( M_1 \);
- trolley wheels, dual weight are denoted by \( M_{01} \) and \( M_{02} \).

Analysis of the mathematical model of the trolley, computational part is shown in the Figures 1 and 2.

Analysis of the mathematical model of the trolley, computational part is shown in the Figures 1 and 2.

The flat vertical mechanical model of the trolley. \( M_2 \) – weight load platform, \( C_{2}^{\text{eq}} \) – equivalent stiffness of the elastic element of the second stage, \( M_1 \) – weight of the support platform, \( C_i \) – stiffness of torsions of the first stage of suspension, \( R_i \) – viscous friction in the suspension of the first stage, \( M_0 \) – weight of the trolley wheels, \( C_0 \) – the equivalent stiffness of tires, \( R_0 \) – viscous friction in tires, \( \eta \) – profile of the road.

Scheme of the modified elastic element of the second stage pneumatic suspension with corrector stiffness in the transverse plane is shown on the Figure 3.
The second stage of the spring suspension, which is consisted of four elastic pneumatic elements and two stiffness correctors, is simulated using research results posted in earlier works [18-20].

The total stiffness of the corrector springs is denoted by $C_2$, their length in the static position – $L$, and the initial deformation – $\delta$.

Analysis of the structural features of the second stage of spring suspension showed that the volume of the air pipe is much smaller than the other components of pneumatic system that eliminates it from the mathematical model of the process [16].

In view of the above, the differential equations of motion of the trolley oscillatory processes consist of static equilibrium with respect to the provisions of relevant weights using the general provisions of the speakers.

The following groups of equations are used for the mathematical model:

- Kinetostatics equation for the elements of the mechanical system;
- Geometric dependencies that determine the relative position and movement of the mechanical elements of the system;
- Analytical and experimental characteristics of elastic elements of the mechanical system;
- Thermodynamic equations that define the processes in the system pneumatic spring suspension.

Powers that influence the components of the mechanical model of the trolley shown on Figures 5 and 6.

The second stage of the spring suspension, which is consisted of four elastic pneumatic elements and two stiffness correctors, is simulated using research results posted in earlier works [18-20].

The total stiffness of the corrector springs is denoted by $C_2$, their length in the static position – $L$, and the initial deformation – $\delta$.

Analysis of the structural features of the second stage of spring suspension showed that the volume of the air pipe is much smaller than the other components of pneumatic system that eliminates it from the mathematical model of the process [16].

In view of the above, the differential equations of motion of the trolley oscillatory processes consist of static equilibrium with respect to the provisions of relevant weights using the general provisions of the speakers.

The following groups of equations are used for the mathematical model:

- Kinetostatics equation for the elements of the mechanical system;
- Geometric dependencies that determine the relative position and movement of the mechanical elements of the system;
- Analytical and experimental characteristics of elastic elements of the mechanical system;
- Thermodynamic equations that define the processes in the system pneumatic spring suspension.

Powers that influence the components of the mechanical model of the trolley shown on Figures 5 and 6.
Fig. 6. - Powers that influence elements of the second stage of spring suspension of the trolley in the transverse plane. \( M_2 \) - weight of the load platform. \( F_{G2} \) - double power in the second stage pneumatic elements. \( F_{K2} \) - power springs corrector stiffness in the second stage. \( M_2 \) - weight of the support platform.

The elastic powers \(- F_{o2} F_{12}, F_{o2} F_{12} \), acting on the tires and torsion suspension equal to the first stage \( F_{o2} = C_o A_{o2}, F_{12} = C_o A_{12}, F_{o2} = C_o A_{o2}, F_{12} = C_f A_{12} \),

where: \( \Delta A_{o2} A_{12} \Delta A_{12} \) - deformation of elastic elements constitute

\[
A_{o1} = \dot{\eta}_1 - Z_{o1}, \quad A_{11} = Z_{o1} - \dot{Z}_1 + \phi_3 \dot{\alpha}, \\
A_{o2} = \dot{\eta}_2 - Z_{o2}, \quad A_{12} = Z_{o2} - \dot{Z}_2 - \phi_3 \dot{\alpha}.
\]  

The elastic powers \( F_{21}^H, F_{22}^H \) of the second stage of suspension should be determined by the powers \( F_{l1}, F_{l2}, F_{K1}, F_{K2} \), respectively. pneumatic elements and spring stiffness corrections.

These powers are equal in the pneumatic springs \( F_{l1} = P_{l1} S_{l1}, F_{l2} = P_{l2} S_{l2} \),

\[
F_{l2} = P_{l2} S_{l2},
\]

in the correctors \( F_{K1} = C_1^L (L + \delta - \sqrt{L^2 + A_{12}^{2}}) \), \( F_{K2} = C_2^L (L + \delta - \sqrt{L^2 + A_{12}^{2}}) \),

where \( P_{l1}, P_{l2} \) - excess pressure caused by thermodynamic processes in pneumatic membranes and determined decision of the appropriate equations,

\( S_{l1}, S_{l2} \) - effective area of pneumatic membranes, depending on their working height is determined experimentally and is introduced into the equation mathematical model geometric dependencies

\[
S_{l1} = f(A_{l1}), \quad S_{l2} = f(A_{l2})
\]

where \( A_{l1}, A_{l2} \) - deformations of pneumatic springs

\[
A_{l1} = Z_{l1} - \dot{Z}_1 - \phi_1 \dot{\alpha}, \quad A_{l2} = Z_{l2} + \phi_1 \dot{\alpha}.
\]

Dissipative powers in tires and first stage of suspension are modeled by viscous friction, which is proportional to the relative velocity corresponding elements

\[
R_{o1} = k_o \dot{A}_{o1}, \quad R_{o2} = k_o \dot{A}_{o2}, \quad R_{l1} = k_1 \dot{A}_{l1}, \quad R_{l2} = k_1 \dot{A}_{l2},
\]

where: \( k_o, k_1 \) - binary viscous friction coefficient, respectively, in tires and torsion,

\( \dot{A}_{o2} \dot{A}_{12} \dot{A}_{12} \) - corresponding relative velocities:

\[
\dot{A}_{o1} = \dot{\eta}_1 - Z_{o1}, \quad \dot{A}_{11} = Z_{o1} - \dot{Z}_1 + \phi_3 \dot{\alpha}, \quad \dot{A}_{o2} = \dot{\eta}_2 - Z_{o2}, \quad \dot{A}_{12} = Z_{o2} - \dot{Z}_2 - \phi_3 \dot{\alpha}.
\]

Dissipative powers in the second stage pneumatic elements hanging defined energy dissipation in flowing the air from one volume to another through the throttling orifice with square hole \( S_P \) and defined and included in the appropriate thermodynamic equations.

Differential equations of motion oscillatory processes trolley consisting given above according to the distribution of elastic and dissipative powers four-weight model (Fig. 4-6).

For the wheels of the trolley

\[
M_o \dot{Z}_{o1} = F_{o1} - F_{l1} + R_{o1} - R_{l1}, \quad M_o \dot{Z}_{o2} = F_{o2} - F_{l2} + R_{o2} - R_{l2}.
\]

For the support platform

\[
M_i \dot{Z}_i = F_{i1} + F_{i2} + R_{i1} + R_{i2} - F_{z1}^H - F_{z2}^H, \quad I_1 \dot{\phi}_1 = -F_{i1} a + F_{i1} \dot{a} - R_{i1} a + R_{i1} \dot{a} + F_{z1}^R b + F_{z2}^R b,
\]

or after the appropriate transformations

\[
M_i \dot{Z}_i = F_{i1} + F_{i2} + R_{i1} - R_{i2} - F_{o1} (sin \frac{\Delta Z}{L}) \cdot sign \Delta Z - F_{o2} (sin \frac{\Delta Z}{L}) \cdot sign \Delta Z.
\]

For the load platform

\[
M_i \dot{Z}_i = F_{i1} + F_{i2} + F_i (sin \frac{\Delta Z}{L}) \cdot sign \Delta Z - F_{i2} (sin \frac{\Delta Z}{L}) \cdot sign \Delta Z.
\]

Additional geometric parameters of pneumatics: \( H_1 \) - surface area rubber-cord shell, \( H_2 \) - additional surface area of the tank.

Additional geometric parameters of pneumatic: \( H_1 \) - surface area rubber-cord shell, \( H_2 \) - additional surface area of the tank, \( S_P \) - Square hole throttling orifice.

A mathematical model of the thermodynamic process in the compression stroke the air springs in which the pressure in them more than additional tanks, i.e. with \( P_{l1} > P_{l2} \) and \( P_{l2} > P_{l1} \), is determined by the following system of equations.

Element 1. \( P_{l1} > P_{l2} \)

Pneumatic spring:

- the amount of the air flowing through the throttling orifice with The air in an additional tank

\[
dG_{l1} = \mu S_P \sqrt{2 \mu P_{l1} (P_{l1} - P_{l2}) / \mu}.
\]

- the law of energy storage

\[
RT_{l1} dG_{l1} - k_1 (T_{l1} - T_0) h - C_1 G_{l1} dT_{l1} - P_{l1} dV_{l1} = 0.
\]

- the equation of state of the air

\[
P_{l1} dV_{l1} + V_{l1} dP_{l1} - RT_{l1} dG_{l1} - RG_{l1} dT_{l1} = 0.
\]
The internal volume of the pneumatic membrane depends on its working height and is determined experimentally and is introduced into the equation mathematical model geometry dependence

\[ V_{\text{III}} = f(A_{\text{II}}), \]  

which enables every step solution of differential equations of the mathematical model to determine the \( dV_{\text{III}} \).

**Additional tank:**

- Note that for an additional tank at \( V_{\text{III}} = \text{const} \),

\[ dV_{\text{III}} = 0. \]

- the weight balance equation

\[ dG_{\text{III}} + dG_{\text{II}} = 0, \]

(22)

- the law of energy storage

\[ C_{T_{\text{II}}}, T_{\text{II}} = -C_{T_{\text{II}}} dG_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) \]

(23)

- the equation of state of the air

\[ V_{\text{II}} dP_{\text{II}} + RT_{\text{II}} dG_{\text{II}} = -RT_{\text{II}} dt_{\text{II}} = 0. \]

(24)

Element 2. \( P_{\text{II}}>P_{\text{I}} \)

Pneumatic spring:

- the amount of the air flowing through the throttling orifice with the air in an additional tank

\[ dG_{\text{II}} \equiv -\mu S_{\text{D}} \sqrt{2P_{\text{II}} (P_{\text{II}} - P_{\text{I}})} dt, \]

(25)

- the law of energy storage

\[ RT_{\text{II}} dG_{\text{II}} = -k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) \]

(26)

- the equation of state of the air

\[ P_{\text{II}} dV_{\text{II}} + V_{\text{II}} dP_{\text{II}} = -RT_{\text{II}} dt_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) dt = 0. \]

(27)

The internal volume of the pneumatic membrane depends on its working height and is determined experimentally and is introduced into the equation mathematical model geometry dependence

\[ V_{\text{I}} = f(A_{\text{I}}), \]

(28)

which enables every step solution of differential equations of the mathematical model to determine the \( dV_{\text{I}} \).

**Additional tank:**

- Note that for an additional tank at \( V_{\text{I}} = \text{const} \),

\[ dV_{\text{I}} = 0. \]

- the weight balance equation

\[ dG_{\text{I}} + dG_{\text{II}} = 0, \]

(29)

- the law of energy storage

\[ C_{T_{\text{II}}}, T_{\text{II}} = -k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) \]

(30)

- the equation of state of the air

\[ V_{\text{II}} dP_{\text{II}} + RT_{\text{II}} dG_{\text{II}} = -RT_{\text{II}} dt_{\text{II}} = 0. \]

(31)

Similarly (mutatis mutandis signs and direction of the air flow) are based equations defining enlargement beat. The air pressure at which it is less than the extra tank, i.e. at \( P_{\text{II}}<P_{\text{I}} \) and \( P_{\text{II}}>P_{\text{I}} \).

**Element 1.** \( P_{\text{II}}>P_{\text{I}} \)

Pneumatic spring:

- the amount of the air flowing through the throttling orifice with extra tanks in the air

\[ dG_{\text{II}} \equiv +\mu S_{\text{D}} \sqrt{2P_{\text{II}} (P_{\text{II}} - P_{\text{I}})} dt, \]

(32)

- the law of energy storage

\[ RT_{\text{II}} dG_{\text{II}} = -C_{T_{\text{II}}}, T_{\text{II}} dT_{\text{II}} - P_{\text{II}} dV_{\text{II}} - C_{T_{\text{II}}}, T_{\text{II}} dG_{\text{II}} + C_{T_{\text{II}}}, T_{\text{II}} dG_{\text{II}} = 0 \]

(33)

- the equation of the state of the air

\[ P_{\text{II}} dV_{\text{II}} + V_{\text{II}} dP_{\text{II}} = -RT_{\text{II}} dt_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) dt = 0. \]

(34)

**Additional tank:**

- the weight balance equation

\[ dG_{\text{I}} + dG_{\text{II}} = 0, \]

(35)

- the law of energy storage

\[ RT_{\text{II}} dG_{\text{II}} = -C_{T_{\text{II}}}, T_{\text{II}} dT_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) dt = 0. \]

(37)

- the equation of state of the air

\[ P_{\text{II}} dV_{\text{II}} + V_{\text{II}} dP_{\text{II}} = -RT_{\text{II}} dt_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) dt = 0. \]

(38)

**Element 2.** \( P_{\text{II}}>P_{\text{I}} \)

Pneumatic spring:

- the amount of the air flowing through the throttling orifice with extra tanks in the air

\[ dG_{\text{II}} \equiv +\mu S_{\text{D}} \sqrt{2P_{\text{II}} (P_{\text{II}} - P_{\text{I}})} dt, \]

(39)

- the law of energy storage

\[ RT_{\text{II}} dG_{\text{II}} = -C_{T_{\text{II}}}, T_{\text{II}} dT_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) \]

(40)

- the equation of state of the air

\[ P_{\text{II}} dV_{\text{II}} + V_{\text{II}} dP_{\text{II}} = -RT_{\text{II}} dt_{\text{II}} - k_{H_{\text{II}}} (T_{\text{II}} - T_{\text{I}}) dt = 0. \]

(41)

**Conclusions.** The above mathematical model of plane vertical oscillations of the trolley for transporting dangerous cargo, which has a high quality two-stage spring suspension, consists of mechanical systems (10,11,14-17), power (2,4,5,8), geometrical (3, 6,7,9,21,28) and thermodynamical (18-20, 22-27, 29-44) equations that define the powered motion of the system during the vibrations that are caused by geometrical irregularities quite hard road profile of given configuration \( \eta = \eta (\xi) \).

**REFERENCES (TRANSLATED AND TRANSLITERATED)**


5. Lagutin V.L. Nekotorye osobennosti raboty vtoroy stupeni resornogo podveshivaniya nesamokhodnoy telezhki dlya...
Лагутин В.Л. Математическая модель плоских вертикальных колебаний прицепа для транспортировки опасных грузов с использованием пневматических элементов во второй ступени подвешивания

Аннотация. Рассматривается построение математической модели плоских вертикальных колебаний прицепа для транспортировки опасных грузов, который имеет двухступенчатое рессорное подвешивание повышенного качества с использованием пневмоэлементов.

Ключевые слова: взрывоопасный груз, двухступенчатое рессорное подвешивание, колебания.