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**Development a theory for colors synthesis by four inks**

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**Abstract.** The autotype equation of color synthesis with four inks was established, one of the inks serves as a parameter. A computer program was developed. A color gamut of autotype colors synthesis was built using computer program. The results of theoretical analysis were confirmed by means of experimental data Fogra

**Keywords:** autotype synthesis of colors, autotype equations, color separation, computer program

The autotype synthesis is a basis of color reproduction in printing. The autotype equations of colors synthesis on a paper in four-ink halftone printing were described in the Neuberger D. [5] and G. Neyhebauer [4] works. Theoretical model of autotype synthesis were confirmed experimentally and proved its appropriateness by numerical methods [3, 7]. However, this model does not apply for digital images separations because of the autotype equations by Neuberger-Neyhebauer have no analytical solution. This is due to the fact that the coordinate transformation of the original RGB space with three variables to color space of imprint CMYK with four unknowns is not a direct and unambiguous. Therefore, the color management systems use the bases of experimental data, the so-called ICC-profiles. The problem of images color separation in classical scheme is that the task of determining the percentage of inks CMYK (cyan, magenta, yellow, black) for the synthesis on a paper a color in color space RGB (red, green and blue channels) is multivalued.

To resolve the problem of separation analytically, in works [8, 9] have been proposed a new information technology for images color separation based on analytical

model of color synthesis on an imprint. The advantage of the technology is the analytical calculation of the relative dot areas of CMYK inks for each color of digital image with optimal ratios.

The aim of this work is to develop a theory of colors synthesis based on Neuberger-Neyhebauer's equations for classical separation technology and define the role of the third colored ink in the autotype synthesis of colors on an imprint.

**Theory.** A color space ICaS was used for a theoretical description of the autotype synthesis of colors. Using this color space analytical solutions of autotype equations were obtained.

The advantage of using color space ICaS is that each color of image is described by three coordinates – achromatic coordinate  $I$ , which describes the "dynamic range" of color images, and two chromatic coordinates  $C$  and  $S$ , which give detailed information about the color characteristics of the image. Transformation of RGB color space coordinates into color space ICaS based on orthogonal transformation Hartley [1]:

$$\hat{\mathbf{H}} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \cos(2\pi/3) & \cos(4\pi/3) \\ 1 & \cos(4\pi/3) & \cos(2\pi/3) \end{bmatrix}, \quad \text{cas}(\pi x) = \cos(\pi x) + \sin(\pi x) \quad (1)$$

It was established that the transformation (1) is the only symmetric orthogonal transformation of color images, in which are stored all color and metric characteristics of color space RGB [6].

$$\mathbf{C}_{ICaS}^{(3)} = (1 - \sigma_K) \mathbf{C}_{ICaS}^{(2)}, \quad \mathbf{C}_{ICaS}^{(2)} = \hat{\mathbf{H}} \mathbf{F}_W (1 - \sigma_n)(1 - \sigma_m) + \hat{\mathbf{H}} \mathbf{F}_n \sigma_n (1 - \sigma_m) + \hat{\mathbf{H}} \mathbf{F}_m \sigma_m (1 - \sigma_n) + \hat{\mathbf{H}} \mathbf{F}_{n,m} \sigma_n \sigma_m, \quad (2)$$

where  $\mathbf{C}_{ICaS}^{(3)}$  – vector of color in color space ICaS, synthesized by two colored and black inks;  $\mathbf{C}_{ICaS}^{(2)}$  – vector of color synthesized by two colored inks;  $\hat{\mathbf{H}} \mathbf{F}_W$  – vector of white paper;  $\hat{\mathbf{H}} \mathbf{F}_n, \hat{\mathbf{H}} \mathbf{F}_m, \hat{\mathbf{H}} \mathbf{F}_{n,m}$  – vectors of  $n$ -th and  $m$ -th colored inks and their double overlapping in color space ICaS;  $\sigma_K$  – relative dot area of black ink;  $\sigma_n, \sigma_m$  – relative dot area of  $n$ -th and  $m$ -th colored inks. In four-colored printing as  $n$ -th and  $m$ -th inks stands for cyan and magenta (CM), or magenta and yellow (MY), or cyan and yellow (CY).

There is an autotype equation of color that is synthesized by two colored and black inks with addition a third one:

$$I_{AU}^{(4)} = (1 - \sigma_K) \left\{ I_{AU}^{(2)} + \sigma_j \left[ I_j (1 - \sigma_n)(1 - \sigma_m) + I_{n,j} \sigma_n (1 - \sigma_m) + I_{m,j} \sigma_m (1 - \sigma_n) + I_{n,m,j} \sigma_n \sigma_m \right] - I_W (1 - \sigma_n)(1 - \sigma_m) - I_n \sigma_n (1 - \sigma_m) - I_m \sigma_m (1 - \sigma_n) - I_{n,m} \sigma_n \sigma_m \right\} \quad (4)$$

In works [8, 9] the equation of autotype synthesis of color by two colored and black inks were described:

$$\mathbf{C}_{ICaS}^{(4)} = (1 - \sigma_K) (\mathbf{C}_{ICaS}^{(2)} + \sigma_j \mathbf{C}_{ICaSnew}^{(3)}), \quad (3)$$

where  $\mathbf{C}_{ICaS}^{(4)}$  – color in space ICaS synthesized by four inks (CMYK),  $\mathbf{C}_{ICaSnew}^{(3)}$  – color synthesized by three colored inks (CMY). If a value of relative dot area of fixed ink  $\sigma_j = 0$  will be added  $\mathbf{C}_{ICaS}^{(2)}$  that corresponds to the synthesis of color by two colored and black inks, in the case where fixed quantity of a third ink is 1, reduced by the components of the second addend.

An equation of achromatic coordinate  $I_{AU}^{(4)}$  takes the form (4) in the presence of a third colored ink:

where  $I_{AU}^{(2)}$  corresponds to the achromatic color coordinate which synthesized by two colored and black inks, the next addend also has the structure of  $I_{AU}^{(2)}$  coordinate, but accounted a basic vector of third colored ink and vector of overlapping the other two colored inks and vector double overlapping of all colored inks. The chromatic coordinates are similar  $C_{AU}^{(4)}, S_{AU}^{(4)}$ .

$$\left. \begin{aligned} & \left( \begin{matrix} I_{AU}^{(4)} & I_W \\ C_{AU}^{(4)} & C_W \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_j - I_W \\ C_{AU}^{(4)} & C_j - C_W \end{matrix} \right) (1 - \sigma_n)(1 - \sigma_m) + \left( \begin{matrix} I_{AU}^{(4)} & I_n \\ C_{AU}^{(4)} & C_n \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_{n,j} - I_n \\ C_{AU}^{(4)} & C_{n,j} - C_n \end{matrix} \right) \sigma_n(1 - \sigma_m) + \\ & \left( \begin{matrix} I_{AU}^{(4)} & I_m \\ C_{AU}^{(4)} & C_m \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_{m,j} - I_m \\ C_{AU}^{(4)} & C_{m,j} - C_m \end{matrix} \right) \sigma_m(1 - \sigma_n) + \left( \begin{matrix} I_{AU}^{(4)} & I_{n,m} \\ C_{AU}^{(4)} & C_{n,m} \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_{n,m,j} - I_{n,m} \\ C_{AU}^{(4)} & C_{n,m,j} - C_{n,m} \end{matrix} \right) \sigma_n \sigma_m = 0 \\ & \left( \begin{matrix} I_{AU}^{(4)} & I_W \\ S_{AU}^{(4)} & S_W \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_j - I_W \\ S_{AU}^{(4)} & S_j - S_W \end{matrix} \right) (1 - \sigma_n)(1 - \sigma_m) + \left( \begin{matrix} I_{AU}^{(4)} & I_n \\ S_{AU}^{(4)} & S_n \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_{n,j} - I_n \\ S_{AU}^{(4)} & S_{n,j} - S_n \end{matrix} \right) \sigma_n(1 - \sigma_m) + \\ & \left( \begin{matrix} I_{AU}^{(4)} & I_m \\ S_{AU}^{(4)} & S_m \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_{m,j} - I_m \\ S_{AU}^{(4)} & S_{m,j} - S_m \end{matrix} \right) \sigma_m(1 - \sigma_n) + \left( \begin{matrix} I_{AU}^{(4)} & I_{n,m} \\ S_{AU}^{(4)} & S_{n,m} \end{matrix} + \sigma_j \begin{matrix} I_{AU}^{(4)} & I_{n,m,j} - I_{n,m} \\ S_{AU}^{(4)} & S_{n,m,j} - S_{n,m} \end{matrix} \right) \sigma_n \sigma_m = 0 \end{aligned} \right\} (5)$$

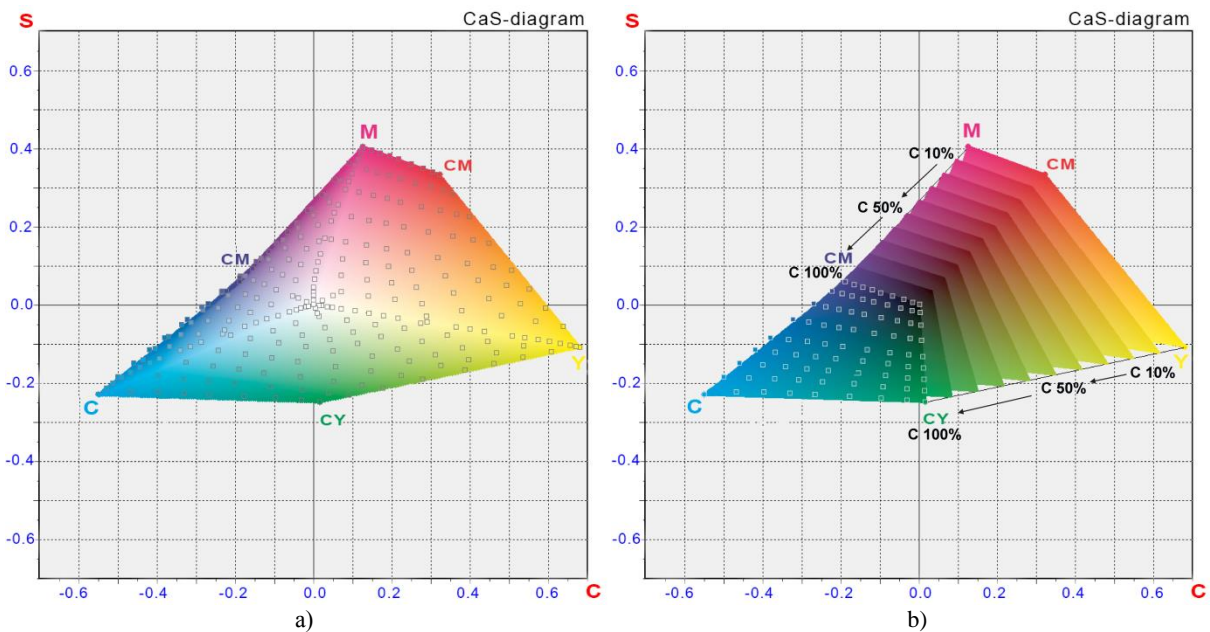
The system of autotype equations (5) has an analytical solution for two colored and black inks, in which the third  $j$ -th colored ink has a fixed value of the relative dot area. Thus, the development a theory for colors synthesis by four inks is an obtained equation of color synthesis in which one of the colored inks is fixed. This makes possible to apply the analytical solutions for the classic images color separation and get a range values of the relative dot areas of inks for each color reproduction of the original.

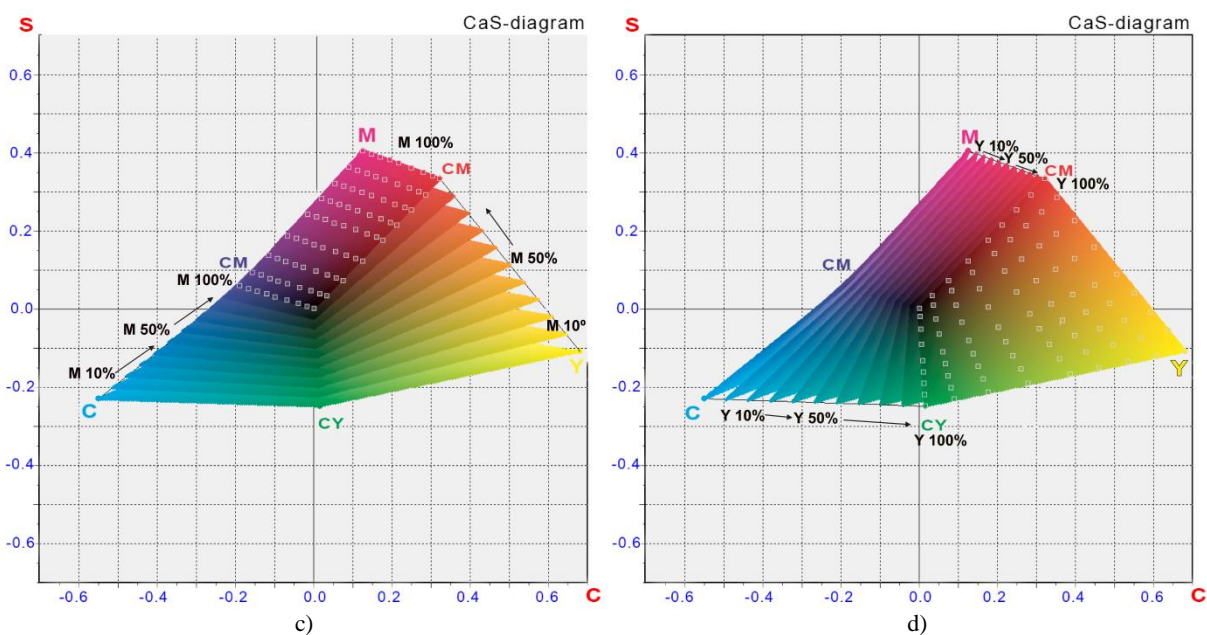
**Materials and Methods.** An algorithm of modeling of autotype synthesis of colors based on the derived equa-

A relative dot area of black ink  $\sigma_K$  based on formula (4), substituted the expression in autotype equation for chromatic coordinates ( $C_{AU}^{(4)}, S_{AU}^{(4)}$ ) and a system of non-linear homogeneous equations (5) were obtained. The coefficients in the equations (5) have a new addend with a common multiplier  $\sigma_j$ :

tions was described. A computer program "ICaS-Color Gamut Inks" allows analyzing and processing the characteristic data in the new color space ICaS and simulation gamut inks. Characteristic data Fogra39 was selected for offset printing on coated paper under the standard ISO 12642-2:2004 [2].

**Discussion of results.** The colors from characteristic data Fogra that are synthesized by two colored inks with a fixed value of the third colored ink were selected. The result presented with points on the chromatic CaS-diagrams (Figures 1a-d).



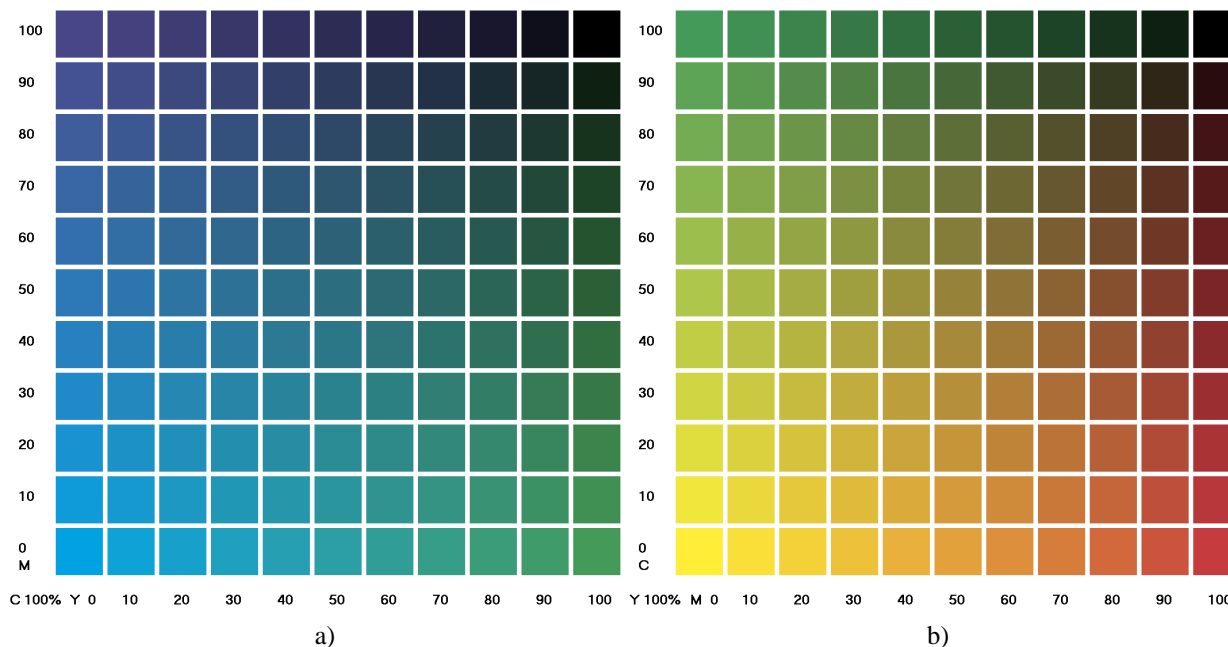


**Fig. 1.** The color surfaces on the CaS-diagram synthesized by two colored inks (a) and adding a fixed percentage of the third cyan (b), magenta (c), yellow (d) ink: points – experimental data Fogra 39.

An autotype synthesis of colors according to equation (2) was modeled and three projections from a largest color surface were received (Fig. 1 a). Also, by modeling an autotype synthesis according to equation (3) were obtained projections of color surfaces synthesized by magenta and yellow inks with a fixed value of 0 to 100 % of cyan in increments of 10% ( Fig. 1 b). Similarly received projections of color surfaces that synthesized by cyan and yellow with fixed value of magenta (Fig. 1 c) and those synthesized by cyan and magenta inks with a fixed value

of yellow ( Fig. 1 d). Theoretical gamut matches with the experimental data. Projections of color surfaces with the addition of a third colored ink cross the origin of CaS-diagram, moving region of achromatic colors in two different color areas, confirming the equation (3).

The developed computer program "ICaS-Color Gamut Inks" makes it possible to generate a control scale based on equation (3) to evaluate and analyze the synthesized colors that are formed by double overlapping of two colored inks with a fixed value of the third colored ink.



**Fig. 2.** Control scale of double overlapping: magenta and yellow with adding 100% cyan ink (a), cyan and magenta colors with adding 100% yellow ink.

**Conclusions.** The autotype equation of color synthesis with four inks with a fixed value of one of the colored inks was established. This makes it possible to apply the analytical solutions for the classic images color separation

and get a range of values of the relative dot areas of inks for each color reproduction of the original.

A computer program "ICaS-Color Gamut Inks" was developed by means of which, on the basis of derived equations can be carried out modeling synthesis of color,

and also have the ability to generate a control scale for the evaluation and analysis of colors which are synthesized

by double overlapping of two colored inks with a fixed value of the third one.

#### REFERENCES (REFERENCES TRANSLATED AND TRANSLITERATED)

1. Брейсуэл Р. Преобразование Хартли. М.: Мир, 1990, 175 с. (Русский перевод: Bracewell R.N. The Hartley Transform. Oxford University Press, New York, 1986.)  
*Breisuel R. Preobrazovanie Hartli. M.: Mir, 1990, 175 s. (Ruskiy perevod: Bracewell R.N. The Hartley Transform. Oxford University Press, New York, 1986.)*
2. Characterization data for standardized printing conditions. Web resource: <http://www.fogra.org/en/fogra-standardization/fogra-characterizationdata/a-icc-en.html>.  
*Characterization data for standardized printing conditions. Web resource: <http://www.fogra.org/en/fogra-standardization/fogra-characterizationdata/a-icc-en.html>.*
3. Ганиев Д. Х. Возможности автотипной колориметрии в высокой печати : автореф. дис. на здобуття ступеня канд. техн. наук: спец. 05.02.15 "Машины, агрегаты и процессы полиграфического производства" / Д. Х. Ганиев.– М., 1975.– 30 с.  
*Ganiev D. H. Vozmognosti avtotipnoy kolorimetrii v vysokoy pechati [Opportunities of autotype colorimetry in letterpress]: avtoref. dis. na zdobuttya stupenya kand. tech. nauk: spec. 05.02.15 "Mashiny, agregaty i processy poligraficheskogo proizvodstva" / D. H. Ganiev.– M., 1975.– 30 s.*
4. Neugebauer H. E. J. Die theoretischen Grundlagen des Mehrfarbendrucks // Zeitschrift für wissenschaftliche Photographie, Photophysik und Photochemie.– 1937. – Vol. 36, p. 36–73; reprinted in Neugebauer Memorial Seminar on Color Reproduction (14–15 Dec. 1989, Tokyo, Japan) // Proc. SPIE, Vol. 1184, P. 194–202 (1990).  
*Neugebauer H. E. J. Die theoretischen Grundlagen des Mehrfarbendrucks // Zeitschrift für wissenschaftliche Photographie, Photophysik und Photochemie.– 1937. – Vol. 36, p. 36–73; reprinted in Neugebauer Memorial Seminar on Color Reproduction (14–15 Dec. 1989, Tokyo, Japan) // Proc. SPIE, Vol. 1184, P. 194–202 (1990).*
5. Нюберг Н.Д. Теоретические основы цветной репродукции / Нюберг Н.Д.– М.: Советская наука, 1947.– 176 с.  
*Nyuberg N.D. Teoreticheskie osnovy cvetnoy reprodukcii [Theoretical foundations of color reproduction] – M.: Sovetskaya nauka, 1947.– 176 s.*
6. Предко Х. Б. Рівняння хроматичних координат кольору // Х. Б. Предко, М. Р. Крик, М. В. Шовгенюк // Технологія і техніка друкарства : зб. наук. праць. – К., 2010. – С. 28–37.  
*Predko Ch. B. Rivnyannya chromatychnyh koordynat kolyoru [Equation of chromatic coordinates of color] // Ch. B. Predko, M. R. Kryk, M. V. Shovgenyuk // Technologiya i tehnika drukarstva drukarstva : zb. nauk. prats. – K., 2010. – S. 28–37.*
7. Селиванов Ю. П. Основы моделирования и оптимального программирования автотипного процесса / Селиванов Ю. П.– М.: Книга, 1979.– 238 с.  
*Selivanov Y. P. Osnovy modelirovaniya i optimalnogo programmirovaniya avtotipnogo processa [The basics of modeling and optimal programming of autotype process] – M.: Kniga, 1979.– 238 s*
8. Шовгенюк М. В. Аналітичний розв'язок рівнянь автотипного синтезу зображення в колірному просторі ICaS / М. В. Шовгенюк, М. Р. Крик // Доповіді Національної академії наук України : наук.-теор. журн. Президії НАН України – К., 2012.– №11 – С. 81–86.  
*Shovgenyuk M. V. Analitichnyi rozvyazok rivnyan avtotipnogo syntezu zobrajennya v kolirnomu prostori ICaS [The analytical solution of autotype synthesis equations of image in color space ICaS] / M. V. Shovgenyuk, M. R. Kryk // Dopovidi Nacionalnoi akademii nauk Ukrainy: nauk.-teor. jurn. prezydii NAN Ukrainy – K., 2012.– №11 – S. 81–86.*
9. Шовгенюк М. В. Аналітичний розв'язок систем автотипних рівнянь / М. В. Шовгенюк, М. Р. Крик // Наукові записки УАД : наук.-техн. зб. – Львів : УАД, 2010.– №2(18).– С. 37–48.  
*Shovgenyuk M. V. Analitichnyi rozvyazok system avtotypnyh rivnyan [The analytical solution of autotype equations] / M. V. Shovgenyuk, M. R. Kryk // Naukovi zapysky UAD : nauk.-techn. zb. – Lviv : UAD, 2010.– №2(18).– S. 37–48.*

#### Шовгенюк М.В., Семенов В.В., Ковальский Б. М. Развитие теории синтеза цветов четырьмя красками

**Аннотация.** Выведено автотипное уравнения синтеза цветов четырьмя красками, из которых одна цветная краска выступает в качестве параметра. Разработана компьютерная программа, с помощью которой построен цветовой охват автотипного синтеза цветов на оттиске. Полученные результаты теоретического анализа подтверждены экспериментальными данными Fogra.

**Ключевые слова:** автотипный синтез цветов, автотипное уравнения, цветоделение, компьютерная программа.